1.051 Structural Engineering Design

Problem Set 6 Solutions

BEAMS

1. Problem 3.2 in textbook (Galambos et al.)

Given: L = 32 ft
w_u = 8 kips/ft
F_y = 50 ksi
The beam is laterally supported by the floor slab

Design moment and shear:

$$M_u = \frac{w_u L^2}{8} = \frac{(8)(32^2)}{8} = 1024 \text{ kips-ft} = 12288 \text{ kips-in}$$

$$V_u = \frac{w_u L}{2} = \frac{(8)(32)}{2} = 128 \text{ kips}$$

Required plastic section modulus:

$$M_u = f_b M_n = f_b M_p = f_b Z_x F_y \quad Þ \quad Z_x = \frac{M_u}{f_b F_y} = \frac{12288}{(0.9)(50)} = 273.1 \text{ in}^3$$

Select a W section which satisfies the required section modulus and has the minimum weight (note that there are no restrictions on member height)

Select W30x90 Z=283 in^3

Check compactness of flange:

$$\frac{b_f}{2t_f} = \frac{10.4}{(2)(0.61)} = 8.5 < \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{50}} = 9.2 \qquad \text{OK}$$

Check compactness of web flexure:

$$\frac{h}{t_w} = 57.5 < \frac{640}{\sqrt{F_y}} = 90.5 \quad \text{OK}$$

Lateral stability is OK since support provided by floor slab.

Check shear compactness:

$$\frac{h}{t_w} = 57.5 < 260 \quad OK$$

$$\frac{h}{t_w} = 57.5 < \frac{418}{\sqrt{F_y}} = 59.1 \quad OK$$

Design shear strength:

$$\phi V_n = 0.9V_n = (0.9)(0.6F_y A_w) = (0.9)(0.6)(50)(29.53)(0.47) = 374.7 \text{ kips}$$

$$V_u = 128 < \phi V_n \qquad OK$$

W30x90 section can be used.

2. Problem 3.3 in textbook

Intermittent lateral supports equivalent to full lateral support:

$$L_b \le L_p$$

For W30x90, $r_y = 2.09$ in

$$L_p = \frac{300 r_y}{\sqrt{F_y}} = \frac{(300)(2.09)}{\sqrt{50}} = 88.7 \text{ in}$$

Minimum number of segments required

$$n_s = \frac{L}{L_p} = \frac{(32)(12)}{88.7} = 4.3 \Rightarrow n_s = 5$$

Then, divide the span into 5 segments, each with an unbraced length:

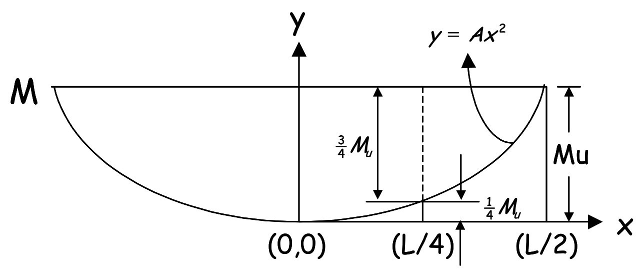
$$L_b = \frac{L}{n_s} = \frac{(32)(12)}{5} = 76.8 \text{ in}$$

3. Problem 3.4 in textbook

No lateral support

$$M_{max} = M_B = M_u = 1024 \text{ kips-ft}$$

Since the loading is uniform, the shear diagram must be a line, and the moment diagram a parabola, i.e. $M=f(x^2)$. Then from the figure below, the moment at the ¼ and ¾ points must be ¾ M_u.



$$M_A = M_C = \tfrac{3}{4} M_u = 768 \text{ kips-ft}$$

$$\begin{aligned} C &= \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} \\ &= \frac{(12.5)(1024)}{(2.5)(1024) + (3)(768) + (4)(1024) + 3(768)} \\ &= 1.14 \end{aligned}$$

Note that Cb values for certain loading and bracing configurations can readily be obtained from tables.

Check if the currently used section is adequate

For W30x90:

$X_1 = 1430$ ksi $\quad X_2 = 47 \times 10^{-3}\ (1/\text{ksi})^2$ (from section tables)

$r_y = 2.09$ in

$F_r = 10$ ksi (Comp. residual stress in flange for rolled sections)

$$\begin{aligned} L_r &= \frac{X_1 r_y}{F_y - F_r} \sqrt{1 + \sqrt{1 + X_2 (F_y - F_r)^2}} \\ &= \frac{(1430)(2.09)}{50 - 10} \sqrt{1 + \sqrt{1 + (47 \times 10^{-3})(50 - 10)^2}} \\ &= 233 \text{ in} = 19.4 \text{ ft} \end{aligned}$$

The unbraced length, $L_b=L$, is greater than L_r, thus, beam buckles in the elastic range, and failure is governed by M_{cr}. Select a larger section.

Select W44x262 (through iteration using a computer program for Lr)

$$r_y = 3.46 \text{ in} \qquad X_1 = 1930 \text{ ksi} \qquad X_2 = 12.3 \times 10^{-3}\ (1/\text{ksi})^2$$

$$\begin{aligned}
L_r &= \frac{X_1 r_y}{F_y - F_r}\sqrt{1+\sqrt{1+X_2(F_y - F_r)^2}} \\
&= \frac{(1930)(3.46)}{50-10}\sqrt{1+\sqrt{1+(12.3\times 10^{-3})(50-10)^2}} \\
&= 393 \text{ in} = 32.8 \text{ ft} > L_b \quad \text{OK}
\end{aligned}$$

$$\phi_b M_p = \phi_b Z_x F_y = (0.9)(1270)(50) = 57150 \text{ kips-in} = 4762 \text{ kips-ft}$$

$$\begin{aligned}
\phi_b M_r &= \phi_b S_x (F_y - F_r) \\
&= (0.9)(1120)(50-40) \\
&= 40320 \text{ kips-in} = 3360 \text{ kips-ft}
\end{aligned}$$

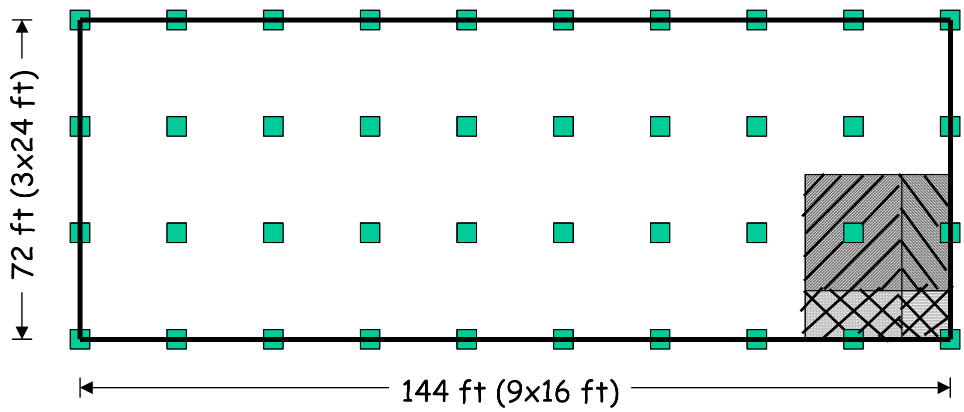
Both ϕMp and ϕM_r are considerably larger than Mu, thus, the optimum solution is to select an intermediate section, which will lead to failure by buckling in the elastic range, at M=Mcr. For the purposes of this problem, the selected section conservatively satisfactory.

COLUMNS

4. Problem 4.3 in textbook

Given:

20 story building
Plan dimensions: 72x144 (3x9 bays, 4x10 columns)
w_{Di} = 80 lb/ft^2 (interior dead load)
w_{Dw} = 15 lb/ft^2 (exterior walls)
w_L = 80 lb/ft^2
KL = 14 ft (for first floor columns)



Note that there are four types of columns: interior column, edge columns, and corner column. Loadings on these columns are:

Interior columns (per floor):

$P_D = (16)(24)(80) = 30720 \text{ lbs } = 30.7 \text{ kips}$

$P_L = (16)(24)(80) = 30720 \text{ lbs } = 30.7 \text{ kips}$

$P_u = 1.2P_D + 1.6P_L = (1.2)(30.7) + (1.6)(30.7) = 86 \text{ kips}$

Edge columns along the longitudinal direction (per floor):

$P_D = (16)(12)(80) + (16)(10)(15) = 17760 \text{ lbs } = 17.8 \text{ kips}$

$P_L = (16)(12)(80) = 15360 \text{ lbs } = 15.4 \text{ kips}$

$P_u = 1.2P_D + 1.6P_L = (1.2)(17.8) + (1.6)(15.4) = 46 \text{ kips}$

Edge columns along the transverse direction (per floor):

$P_D = (8)(24)(80) + (24)(10)(15) = 18960 \text{ lbs } = 19.0 \text{ kips}$

$P_L = (8)(24)(80) = 15360 \text{ lbs } = 15.4 \text{ kips}$

$P_u = 1.2P_D + 1.6P_L = (1.2)(19.0) + (1.6)(15.4) = 47.4 \text{ kips}$

Corner columns (per floor):

$$P_D = (8)(12)(80) + (8+12)(10)(15) = 10680 \text{ lbs} = 10.7 \text{ kips}$$

$$P_L = (8)(12)(80) = 7680 \text{ lbs} = 7.7 \text{ kips}$$

$$P_u = 1.2P_D + 1.6P_L = (1.2)(10.7) + (1.6)(7.7) = 25.2 \text{ kips}$$

Note that in the above calculations, the floor height is assumed as h=10 ft when calculating the loads due to exterior walls. Only given height is KL=14 ft for the first floor. For sidesway permitted columns verify that the effective length factor, K=1-2 (1.2-2.1 in practice) assuming that the base is fixed. In this case, this factor is conveniently and appropriately assumed as K=1.4.

Loading on the interior column controls design. Only this column will be considered here to illustrate the procedure. Note that although the axial loads on the edge and corner columns are lower, the moment on these columns will be higher, which partly compensates for the difference in axial loading compared with the interior columns.

Design Load on the first floor interior columns

$$P_u = (20)(55.3) = 1106 \text{ kips} \qquad KL = 14 \text{ ft}$$

As a first approximation, assume $F_{cr} = 0.8F_y$ where $F_y = 36$ ksi

$$P_u = \phi_c P_n \le \phi_c A_g F_{cr} \Rightarrow A_g \ge \frac{P_u}{\phi_c F_{cr}} = \frac{1106}{(0.85)(0.8)(36)} = 45.2 \text{ in}^2$$

From section tables, select W14x159, A_g = 46.7 in^2 r_y = 4.0 in

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{(14)(12)}{4\pi}\sqrt{\frac{36}{29000}} = 0.47 < 1.5$$

$$F_{cr} = (0.658^{\lambda_c^2})F_y = (0.658^{0.47^2})(36) = 32.8 \text{ kips}$$

$$P_u = \phi_c P_n \le \phi_c A_g F_{cr} \Rightarrow A_g \ge \frac{P_u}{\phi_c F_{cr}} = \frac{1106}{(0.85)(32.8)} = 39.7 \text{ in}^2$$

A second iteration can be made, but the selected section is sufficiently close to the required section area.

Use W14x159