Final Exam

14.04, Fall 2020
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There are 80 points in total. You have 120 minutes to complete the exam. a

1 Short Questions (25 points)

Explanations are needed for True/False questions to receive full points.

a) (5 pts) Explain why externalities lead to a failure of the first welfare theorem (FWT). What is one possible solution to prevent externalities from leading to a failure of the FWT?

Solution: Externalities lead to an extra commodity (e.g. pollution) that has no market. Therefore MRS in equilibrium will not equal the social price ratio (there is either too much or too little of the unpriced extra commodity). One possible solution is to create a market for the extra commodity such as pollution permits or assigning property rights for that extra commodity. Another solution is to tax or subsidize the commodity which will take into account the externalities by "pricing" the difference between private and social costs.

b) (5 pts) True or False: To check whether a given mixed strategy profile is a (mixed) Nash equilibrium, it is sufficient to see if there are pure strategies that lead to higher utility.

Solution: True. A mixed strategy profile is a mixed strategy Nash Equilibrium if and only if for each player i, $u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*)$ for all $s_i \in S_i$.

c) (5 pts) Given prices p, wealth w, and that $x \in \mathbb{R}_+^L$ is the demand vector, write out the equation for elements of the Slutsky matrix and interpret the terms as income and substitution effects.

Solution: The elements of the Slutsky matrix follow

$$s_{lk}(p, w) = \frac{\partial x_l(p, w)}{\partial p_k} + \frac{\partial x_l(p, w)}{\partial w} x_k(p, w) \quad \forall l, k$$

The Slutsky element represents the substitution effect where demand for a good x_l changes as the price of another good p_k changes given that utility is kept constant. The second term represents the income effect where $\frac{\partial x_l(p,w)}{\partial w}$ is how Marshallian demand for the good x_l changes as wealth changes, and $x_k(p, w) = \frac{\partial e(p,u)}{\partial p_k}$ from Shepard's lemma. Therefore $x_k(p, w)$ represents how wealth changes when the price of another good p_k changes.

d) (5 pts) In what way is Kakutani's fixed point theorem more general than Brower?

Solution: Kakutani's fixed point theorem allows for correspondences while Brower's theorem is defined for functions.

e) (5 points) Explain the nature of the externality problem that is present in platform markets. In the model studied by Jain and Townsend (2016), how does the market proposed by the authors attempt to mitigate this externality?

Solution: In platform markets, an externality arises because each agent's value for using a particular platform depends on the composition of other agents using the platform. For example, owning a credit card from a particular company would have little value to a consumer if no retailers accepted that credit card. Therefore, the choice of a particular agent to use a platform directly affects the utility of others using the platform in a way that is not addressed by conventional markets.

In Jain and Townsend (2016), there is an intermediary who sells contracts which entitle the buyer to use a platform of a specific size and composition with a certain probability. An equilibrium condition is that demand for each contract must equal supply for each contract and that the platforms agents end up using in equilibrium must be consistent with what was promised in the contracts.

2 Identification and Falsification (15 points)

There is a consumer purchasing 3 goods. You observe her consumption choices in 3 periods listed below.

$$x^1 = (3,1,7) \quad p^1 = (2,3,3)$$
$$x^2 = (7,3,1) \quad p^2 = (3,2,3)$$
$$x^3 = (1,7,3) \quad p^3 = (3,3,2)$$

Her expenditure in each period is

$$p^1 \cdot x^1 = p^2 \cdot x^2 = p^3 \cdot x^3 = 30$$

The cost of the alternative bundles are:

$$p^1 \cdot x^2 = p^2 \cdot x^3 = p^3 \cdot x^1 = 26$$
$$p^1 \cdot x^3 = p^2 \cdot x^1 = p^3 \cdot x^2 = 32$$

a) (5 pts) What does it mean for her choices to satisfy the Weak Axiom of Revealed Preference (WARP)?

Solution: Suppose consumption bundle x_a is chosen over x_b at price p_a and x_b is chosen over x_a at price p_b. These choices satisfy WARP if, whenever $p^a x^a > p^a x^b$, then we must have that $p^b x^a > p^b x^b$.

b) (5 pts) Do her choices satisfy WARP? Show your work.

Solution: Yes, her choices satisfy WARP. In the first period, we have that $x^1 R^D x^2$ with $p^1 \cdot x^1 > p^1 \cdot x^2$. And as $p^2 \cdot x^2 = 30 < p^2 \cdot x^1 = 32$, these choices satisfy WARP.
In the second period, we have that $x^2 R^D x^3$ with $p^2 \cdot x^2 > p^2 \cdot x^3$. And as $p^3 \cdot x^3 < p^3 \cdot x^2$, these choices satisfy WARP.
In the third period, we have that $x^3 R^D x^1$ with $p^3 \cdot x^3 > p^3 \cdot x^1$. And as $p^1 \cdot x^1 < p^1 \cdot x^3$, these choices satisfy WARP.

c) (5 pts) Are these preferences directly revealed to be transitive? What does this suggest about whether the consumer has rational preferences?

Solution: No, her preferences are directly revealed to be intransitive as $x^1 R^D x^2$, $x^2 R^D x^3$, and $x^3 R^D x^1$ forming a cycle. These revealed preferences indicate that she is not a rational consumer.

3 Aggregation and Gorman Form (20 points)

a) (5 points) Suppose a household has a Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

and wealth w. Explain how to solve for the household's indirect utility function, which is given by

$$v(p, w) = \left(\frac{\alpha}{p_1}\right)^{\alpha} \left[\frac{1-\alpha}{p_2}\right]^{1-\alpha} w.$$

Solution: The utility maximization problem is given by

$$\max_x x_1^{\alpha} x_2^{1-\alpha}$$

subject to the budget constraint $p_1 x_1 + p_2 x_2 \leq w$.

The FOC is

$$\frac{\alpha x_2}{(1-\alpha)x_1} = \frac{p_1}{p_2}$$

Combine FOC with budget constraint to get the demand functions

$$x_1(p, w) = \frac{\alpha w}{p_1}$$

$$x_2(p, w) = \frac{(1-\alpha)w}{p_2}$$

Plug these demands back into the utility function to obtain the indirect utility function:

$$v(p,w) = \left(\frac{\alpha}{p_1}\right)^{\alpha} \left[\frac{1-\alpha}{p_2}\right]^{1-\alpha} w$$

b) (3 points) Do the household's preferences satisfy Gorman form?

Solution: This is of Gorman Form where $a(p) = 0$ and $b(p) = \left(\frac{\alpha}{p_1}\right)^{\alpha} \left[\frac{1-\alpha}{p_2}\right]^{1-\alpha}$

c) (3 points) Suppose we have an economy with households $i = 1, \ldots, I$ with wealth levels $\omega = (w_1, \ldots, w_I) \in \mathbb{R}^I_+$ and individual demand functions $x^i(p, w_i)$ given prices p. Households have the same Cobb-Douglas utility function $u(x)$. Define the utility function u^* that corresponds to the positive representative household for this economy (if it exists).

Solution:

The utility function u^* corresponds to the positive representative household for this economy if and only if the aggregate demand function $X(p, \omega)$ satisfies

$$X(p,\omega) = \sum_i x^i(p, w_i) = \arg\max_x u^*(x)$$
$$s.t. \quad p \cdot x \leq \sum_i w_i$$

Note that the utility function of the representative household does not have to be the same as the utility function for the individual households.

d) (3 points) Suppose the individual households have different levels of wealth. Is there a positive representative household? Why or why not?

Solution: Yes, because the individual households preferences are Gorman form with the same $b(p)$ term.

e) (3 points) Suppose there exists a household that is a positive and normative representative household for the economy. Policymakers are planning to provide transfers in response to the COVID recession to increase aggregate demand. Should they worry about who gets those transfers and whether those individuals have different levels of wealth to begin with?

Solution: No. For aggregate demand, policymakers should care about total wealth only if a positive representative household exists.

f) (3 points) What if the policymaker's goal is to increase social welfare?

Solution: Maximizing the welfare of the normative representative household generates a Pareto-optimal allocation. However, policymakers also care about which households receive the transfers as different households may have different Pareto-weights depending on wealth, so they should design transfers taking into account initial wealth levels.

4 General Equilibrium and Welfare Theorems (10 points)

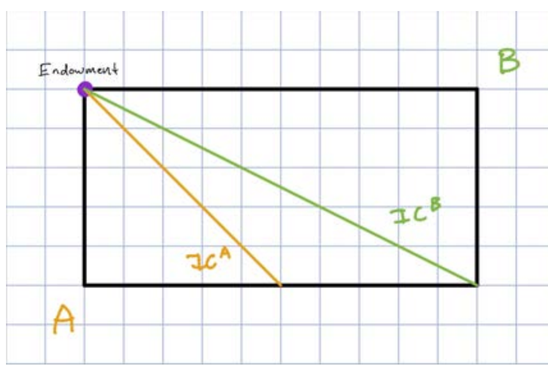
Consider an exchange economy with two consumers and two goods. Consumer A has preferences over goods x and y given by

$$u^A(x, y) = x + y$$

Consumer B has preferences given by

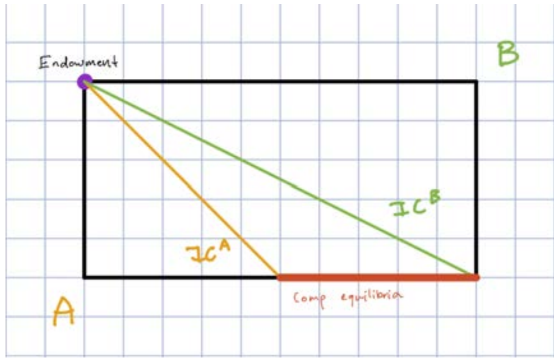
$$u^B(x, y) = x + 2y$$

Consumer A has 0 units of good x and 1 unit of good y. Consumer B has 2 units of good x and 0 units of good y.



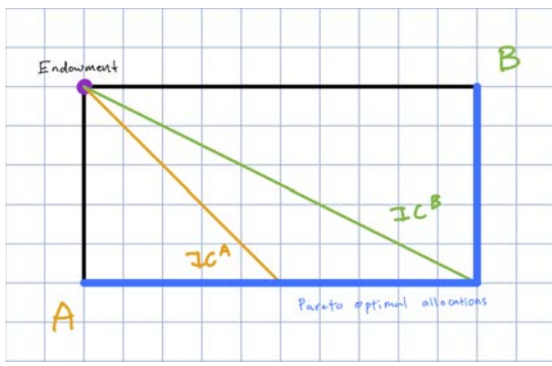
a) (5 points) An Edgeworth Box with the endowment and indifference curves passing through the endowment are provided for you above. Draw the set of competitive equilibria in the Edgeworth Box.

Solution: The competitive equilibria is the part of the bottom edge between the indifference curves going through the endowment. These equilibria correspond to price ratios p_x/p_y between 1/2 and 1.



b) (5 points) Draw the set of Pareto-optimal allocations in your Edgeworth Box. From the First Welfare Theorem, how should the set in (a) be related to this set?

Solution: The FWT says that competitive equilibria are a weak subset of the Pareto-optimal allocations.



5 Cryptocurrency and Bubbles (10 points)

a) (5 points) Suppose money is only a bubble. Explain why policy makers may not want to eliminate the bubble.

Solution: In a dynamically inefficient economy, as in the OLG model of Tirole (1985), bubbles can exist in steady state and be welfare improving by acting as a substitute to capital investment.

b) (5 points) Suppose money is a bubble and policy makers have decided to keep the bubble. Would they want to intervene with monetary policy and why?

Solution: When there are nonbinding nonnegativity constraints on money, a monetary equilibrium would require some intervention to be an optimal allocation.