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IAN BALL: Today we're going to talk about imperfect competition. I want to start with just a small review of what we've done, and then we'll move to imperfect competition. I think this fits in with a broader theme of the course where or a broader approach in the course where we're going to start with some simple examples to try to motivate the theory.

Then we'll dig deep into the theory and maybe prove some theorems, and then we'll apply the theory to some substantive economic applications. Today is going to be one of those days where we dig into a substantive economic application. So if you don't have much of an econ background, some of what we covered today may be new to you. If you have take a lot of econ courses, this may be a bit of a review.

But just a reminder of what we've done so far, we've been studying strategic form games, and we've looked at various solution concepts. And we've said that, in a strategic form game, we applied this procedure of iterated elimination of strictly dominated strategies. And we generated these nested sets of strategies. And at the end, we got to S infinity.

And remember, what we said is that stronger assumptions about rationality and higher-order knowledge of rationality allowed us to get sharper predictions about how players will play. And at the end, when we get to S infinity, what we further know is that a subset of the rationalizable strategies are-- maybe I'll jump down to what we showed last time. We showed these two relationships.

So we showed that every dominant strategy equilibrium is a Nash equilibrium, specifically a pure strategy Nash equilibrium. And we showed that every pure strategy Nash equilibrium is rationalizable. But there's actually one more inequality-- or inclusion we can put here. And here I'll put mixed-strategy Nash equilibrium with positive probability.

So let me define what I did here. And the proof of this inclusion is quite similar to the proof that we did in class last Thursday. So I'd encourage you to go through the argument and see if you can extend it to this case. So here what I mean is this is the set of all strategy profiles that are played with positive probability in some mixed-strategy Nash equilibrium.

So just to make sure we're on the same page, why is it the case that every pure strategy Nash equilibrium is an element of this set? Why is every pure strategy Nash equilibrium in this set of positive probability mixed-strategy Nash equilibrium profiles? Yeah.

AUDIENCE: I don't remember [INAUDIBLE].

IAN BALL: That's fine, yeah.

AUDIENCE: [INAUDIBLE] strategy [INAUDIBLE].

IAN BALL: Yeah, exactly. Every pure strategy and mixed strategy, so that's the first step. And then the second step is to say, well, if something is a pure strategy Nash equilibrium, it's also a mixed-strategy Nash equilibrium where everyone just plays their component of that strategy profile with probability 1.

So since that's also a mixed-strategy Nash equilibrium, is this strategy profile played with positive probability in some mixed-strategy Nash equilibrium? Yes, in fact, it's played with probability 1 in a particular mixed-strategy Nash equilibrium. So it's clear that this is a subset of this. The argument that this is a subset of this is maybe a little more involved. But as I said, it's quite similar to the proof we did last week.

OK, so that's the theory up to the point that we've done so far. And let me now move to the main application for today, which is imperfect competition.

And to understand imperfect competition, I think it's helpful to think of it as falling between two extreme cases. So on the one hand, we have perfect competition. And then at the other extreme, we have monopoly. And then imperfect competition falls somewhere in between. So I'll put it here.

Now, formally defining these concepts can be a little tricky, but let me give some ideas about what we mean. And again, people use these terms in slightly different meanings, in slightly different ways, in different contexts. But perfect competition, we normally think of that as a setting where there's many small firms.

Monopoly is a situation where we just have one firm. They're the only firm offering a particular good or product. So here, we have one firm.

Usually, that one firm would be large since they're the only firm. And then imperfect competition lies somewhere in between. We normally think that there are maybe a few large firms. This is sometimes also called oligopoly. So a few maybe large firms.

And the implication of this is that, in perfect competition, when we have many small firms, each firm is what's called a price taker. What do I mean by that? I'm some small firm in a market. I can't influence what the market price is for the product that I'm selling.

So the classic example of this would be maybe commodities markets, like wheat or corn. So if I'm a small farmer and I'm choosing how much corn to produce or to sell in the market this year, my decision about how much my particular farm produces is unlikely to affect the global market price for corn because there's many, many firms and many, many farms in the world that are producing corn.

And another important aspect of this is that these are homogeneous products, and that's another common attribute of perfect competition. If we're all producing corn, but my corn is special and better than everyone else's corn. Now, I might not be such a small player anymore.

Then in the case of imperfect competition, where we have a few large firms, we normally think of what we say is that these firms have market power. What does market power mean? It's really the opposite of being a price taker. Though, we can think of this kind of as a continuum.

With a price taker, I have no control over the market price for some item. If I have market power, then my production decisions can have some effect on what the market price will be. So when we have a few large firms, if someone floods the market with a lot of their goods, then the market price of that good may fall.

That's an example of market power. So if there's a few countries that produce oil in the world and one of them, say Saudi Arabia, produces much, much more oil this year, then that's probably going to have some effect on the market price for oil. It's probably going to bring down the market price for oil.

When we have a few large firms, maybe an example of this, there's different degrees of imperfect competition. But maybe I think cell phone companies are a good example. In the US, you maybe have two or three large cell phone carriers that have their large networks. But we don't have a lot of small firms, and there's some barriers to entry.

Another example of market power could be due to location. So it may be that even if there's a lot of firms producing a good, there may only be a small number of firms producing in a particular area. And this is going to be really relevant when transportation costs are high.

So for instance, maybe there's a lot of coffee shops in the world, but right around MIT, there may only be a few. And if you take your coffee over long distances, it's going to get cold. So you're not really considering a coffee shop in New York even if New York, maybe has better coffee than some places in Boston.

Another source of market power could be product differentiation. And this is what I was getting at with the corn example. Corn is the opposite case where we don't see much differentiation.

But let's think about Apple. There's a lot of different companies that produce phones, but you might find Apple iPhones to be a different product than a Samsung phone or a Google phone or a phone produced by another firm. So even if there are many companies that are producing a smartphone, if these companies produce slightly differentiated products, then Apple has a bit of flexibility in what price it charges. If an iPhone is a bit more expensive than a Samsung phone, a lot of people will still choose that phone because they like Apple more. And the same with Samsung, there may be some Samsung loyalists out there.

And then monopoly is the case, the extreme case where we have a single firm. So they still have market power. Maybe they have extreme market power.

It's always hard to find an example of a pure monopoly, but can you think of any? Are there any cases that you've come across where you think there's really only one firm that's selling a particular good or offering a certain service or product? Yeah.

AUDIENCE: Subway.

IAN BALL: The subway is a great example, yeah. So the subway, say the MBTA in Boston. There's only one game in town.

You can't just, as a private company say, I'm going to start digging up the streets and building my own subway system. Now, of course, what counts as a monopoly, it always depends on what we consider a close substitute. So it is true that, instead of taking the subway, you could take a Bluebike, or you could take a Lyft. That's some kind of a substitute. But in terms of getting on a train that goes quickly underground and doesn't have any contact with traffic, that really is something that only the MBTA, only a government monopoly is able to provide.

What are some other examples? Yeah.

AUDIENCE: This is more of a monopsony, but if you're in a coal mining town and there's only one mining company.

IAN BALL: Right, that's a great example. Yeah, maybe monopsony, maybe I won't put it up here just to avoid confusion with monopsony. But that's the case where you're the only buyer in town. You're the consumer of labor.

Maybe, in that context, if there's a labor market in that town, if there's a labor union, that might be a monopoly in that town as well. But that's a great example of a monopsony. Yeah, in the back.

AUDIENCE: Normally, like private companies are contracted for power or something by the government.

IAN BALL: Yeah, so often, utilities, and I put cell phones up here. It's a close call. But some utilities, especially in a local market, if you have an apartment here in Boston and you try to find your internet provider or your phone provider-- I guess, people don't have landline phones anymore.

But your internet provider, you may only have one effective option. So that's another good example. Yeah.

AUDIENCE: I'm not sure if that is imperfect competition or monopoly, but pharmaceutical companies and drugs they make.

IAN BALL: Great, so I think, again, there's sometimes blurry lines between imperfect competition, monopoly. But I think a great example of this would be a patent. So if I'm a private company and I produce a drug, I spend a lot of money developing this drug, I'm then given a certain period of time by the government where I'm the exclusive supplier of that drug.

Now, again, whether a monopolist or not might depend on, well, is there a very close substitute for that drug that's offered by another company? But if I really come along with a completely new drug, I have a government monopoly. The government says, no one else is allowed to produce the drug that you're producing for some period of time, and the goal of that is to give firms the incentive to invest in these drugs because developing them is quite expensive. Any final examples? These are all good.

I think another big one is sports. So the NFL, the MLB, there was actually a Supreme Court ruling that gave the MLB an official exemption from anti-monopoly laws. So the NFL, we take it for granted that there's a draft for the NFL.

Why is there not a draft when you guys are trying to get jobs at tech companies after you graduate? Well, they wouldn't all participate in a draft. They're competing against each other.

So the reason why we can have a draft is that it's legal for these firms to band together and enter this collective agreement where they all engage in a draft and they agree to follow the rules of this draft. If Google and Facebook tried to have a draft for MIT students after you graduated and have first-round picks and second-round picks, that would violate antitrust laws. But the NFL and the MLB, et cetera have gotten a special government exemption, and that allows them to do various things, bargain with broadcasters in a collective way, have things like a draft, have restrictions on the building of new stadiums and the development of new teams.

OK, so this is a bit of actual economics, maybe more applied economics for us today. But now let's move into some of the classical theoretical models that can explain this. Maybe one more, I guess, I had in my notes that I didn't say would be the US postal Service.

Again, it's a bit of a partial monopoly. There's very specific services that no one else can provide, but it does compete with, say, UPS and FedEx for certain other services. So some of these things are a bit-- are not so clear cut. And of course, when firms go to court, this is exactly what they argue about.

So firms love to argue that there are actually a lot of close substitutes for their products, and they're not really a monopolist in this small market because really they're operating in this much broader market. So firms always want to argue that the market they're in is not some narrowly defined market where they're the only player but, in fact, a much broader market where there's other forms of competition. So if I were the subway and I wanted to say I'm not a monopolist, I would say, no, I'm in the transportation market, and there's tons of forms of transportation whereas if I wanted to argue that they were monopolist, I might say-- I might define transportation a lot more narrowly.

So now that we have that background, let's get into a few classical game-theoretic models that try to understand the interaction of firms in imperfect competition and in monopoly. And the first ingredient for these models is going to be defining what demand is. So again, if you take an econ class, you will have seen this.

But demand captures at a given price, how many units of a product am I willing to purchase? And naturally, your demand for a product is going to depend on the price. Generally, people will buy more of something if the price of that good is lower.

But let's go through a really simple example to try to make this more concrete. Let's think about-- let's say it can be the demand for apples. It doesn't really matter.

And let's say, in this economy, there's only three consumers. We'll call them consumer 1, consumer 2, and consumer 3. And apples, I would say, are not an infinitely divisible good. I mean, you could sell half an Apple, but that's really not done.

We're going to think of these as-- you're only going to be able to buy 1, 2, or 3 apples. You can't buy 1.75, or you can't buy a lottery over apples. Though, maybe you could at some point.

So let's look at consumer 1, and let's say that their valuation for the first apple they get is $1. That's how much it's worth to them to get an apple. But they only want 1 Apple. Once they get that first apple, the value they get from a second Apple, the marginal value of an additional Apple drops to 0 because they're just not that hungry. And apples are perishable, so they're not-- it's going to go bad by next week when they want another apple.

Then we have consumer two. They are similar to consumer 1 in that they only value 1 Apple, but they value that first Apple a bit more. So for consumer 2, let's say that first Apple is worth $2 to them. But again, they only need 1 Apple, so after that, their value drops to 0.

And then consumer three really loves apples. And for them, the first Apple is worth 3, and they actually would even be willing to get a second apple. Once they have the first, the second one isn't quite as valuable, but it does have positive value.

So that's going to be $1 for the second apple and then 0 after that. And after that, it's going to keep being 0. They don't all of a sudden want the 10th apple.

So once we understand the individual consumers' demand, we can put it together to create a market demand, but let's just make sure we understand the individual's demand. So let's say I'm consumer 3, and the price of apples is $4. How many apples will I demand?

None, right? Because even this first apple is not worth it to me, so if the price is $4, I'm not going to purchase any apples. But if the price is $2, then I'm going to buy exactly 1 apple because the first Apple is worth 3 to me, so it's worth it for me to buy that first apple at $2.

But now, once I've already bought 1 Apple, the additional value I get from a second Apple is only $1, but I'd have to pay $2 to get it, and therefore it's not worth it for me. So using each of these individuals, we can compute each of these individual's demand. So this is going to yield an individual demand.

But if I'm a firm and I'm considering what price to charge for my apples, I don't really care about individual demand. I don't care if James is buying my apple. I care about how many people in the whole market are going to buy my apples. So what's relevant to the firm is not the individual demand, but what's called the market demand. And that's what we're going to try to capture here.

So what is the market demand for apples in this very simple market that only has three consumers? Well, what we're going to do is we're going to need to plot the demand for apples as a function of the price. And there's a long convention in economics that we always put price on the vertical axis.

So if you think of quantity as a function of price, that may feel kind of backwards to you from the way it's done in math, but this is just a historical convention. Some people argue that there's good reason for it. But we'll just take it as a convention, and we're always going to write things this way. So we have price and quantity.

And on the vertical axis, let me make some marks. By drawing won't be perfect, but let's say this is $1, $2, $3, $4, $5. So what we want to do is, for each price, we want to compute what is the total quantity demanded at that price by the market?

So if the price is 5, what is the total market demand for apples?

AUDIENCE: 0.

IAN BALL: 0, right? No one's willing to buy at 5. And in fact, no one's willing to buy at 4 either.

So if we start up here, demand is just 0. I'll try to make it shaded so you can see it. It's 0, 0, 0.

But then something happens at 3. When the price drops down to 3, well, now this person, player 3, is willing to buy exactly 1 apple. So the market demand is going to jump to 1 when the price hits 3. And I should put some marks here.

So here, maybe I'll put it at a line like this, it's going to jump up to 1 apple when the price hits 3. And then it's going to stay. If the price is, say, 250, well, that's going to make the third consumer pretty happy. If the price is now only 250, he's actually going to make a surplus from this first purchase, but it's not going to entice him to buy another apple. And it's also not going to entice either of these players to buy an apple.

So the demand is going to be constant until we hit a price of 2. And then at a price of 2, how many apples will be demanded? Now 2 because the third player is still going to demand an Apple, but now the second player is going to demand one apple as well.

So now we're going to have this step again up to 2. OK, I think you can see the pattern, but let's try to understand what happens when we get to a price of one. Now what is demand going to be when the price gets to be 1 or maybe a little bit below 1?

Now it's 4. And notice what happens. When the price hits 1, the first player is now willing to buy 1 apple, but that's not the only thing that happens. Now the third player is willing to buy a second apple instead of only 1 apple.

+ now demand increases for two reasons. We have new consumers entering the market choosing to buy who didn't previously buy. And we also have this apple lover increasing the number of apples that he buys from 1 apple to 2 apples. So now it's going to jump up to 4, and then it's going to stay like this. So we have some jumps in the demand curve, and it looks like this.

So that's our demand curve. It looks kind of jagged, and the reason is that we're only considering a market with three people. But if I'm a supermarket selling apples, I'm, hopefully, selling to more than just three people. So in reality, we're going to be aggregating a lot of these individuals.

And the more individuals we aggregate as we change the scale of our graph, it's going to get smoother and smoother. So in our idealized model, with maybe many consumers, we're going to visualize this as a continuous function. Now, you might argue, well, it's still really discrete. Prices are discrete, people are discrete.

But to make the math a bit cleaner, I think not much is lost, and a lot is actually gained in terms of tractability if we make it continuous. So we might think of a demand function that looks something like this just as an example. So again, the interpretation is the same-- as the price goes down, the total number of apples demanded increases.

And again, it increases for different reasons. It can increase because some consumers start buying who didn't previously buy, and other consumers increase the number that they actually buy. So we're going to call this the demand function, and we think of this as a function Q of P.

So I always have to turn my head when I think about this. We have price on the vertical axis. We have demand, the total quantity demanded on the horizontal axis. And Q is a function of P.

Now, it's also sometimes helpful to think of it the other way, to think if I wanted a certain number of apples to me demanded, what is the price that I would have to set in order to ensure that was the demand? So instead of thinking of the function from prices to quantities, we can think of it the other way-- from quantities to prices. And we sometimes call this P of Q.

And this is often called the inverse demand function because, mathematically, this is just the inverse. If we think of Q of P as just a function, then P of Q is literally the inverse of that function.

So in problem sets and when we set things up, sometimes we'll describe demand in terms of the demand function, the demand as a function of price. And sometimes, we'll write down the inverse demand, which is the price as a function of quantity. These are just different mathematical representations of exactly the same object, which is a correspondence between prices and quantities. Any questions on this?

So now that we've set up demand, we'll start with the monopoly problem. The monopoly problem is a bit easier. We don't really have to do game theory. And you've probably seen this in other econ courses. And then we'll move to competition where we really have to take into account interaction between the firms.

So now let's look at the monopoly problem. So what we imagine is I'm a monopolist. I'm the only person selling some good. Say it's apples, but usually, that's not a good example of monopoly. And what we're going to assume is that the monopolist has some constant marginal cost c, which is greater than or equal to 0.

What do I mean by constant marginal cost? What I mean is if the monopolist wants to produce Q units of a good, then the cost of those Q units is just c times Q.

So here, Q is just a number, c is just a number. We might think that, at some point, it becomes really costly to produce an additional unit. So often, we think that costs in general, at some scales, become convex eventually, that if you have to really increase production and start hiring people to work overtime and plant the apples too close to each other in the farmland and all these things, costs become convex. That's probably realistic in a lot of settings, but let's stick to constant marginal cost for now over a certain range of production.

That's probably pretty realistic. And now we have to specify the demand. So let's understand here the difference that these are assumptions about the firm, which in this case is a monopolist.

And now, when we write down things about demand, we're not making assumptions about the firm anymore. We're making assumptions about the population of consumers who are buying that firm's product. So what we're going to assume, just to make the math pretty easy, is we'll assume that our demand is Q of P equals-- well, we want to just say 1 minus P, so that as the price goes up, the demand goes down. That makes sense. People buy less when the price is higher.

But the problem is, well, if the price gets really high, then this demand would be negative. And that's a problem. We don't think that demand is ever negative. So we're going to write the maximum of 1 minus P And 0 so that, once the price gets high enough, specifically when the price gets above 1, demand is just going to be 0. No one's willing to buy at that price, but demand won't actually become negative.

Now, when we solve things, we often kind of cheat a little bit, and we forget about the max. And we hope that, at the end, the prices we consider won't be big enough that this matters. So what we'll often do is we'll reason as if the max isn't here and hope that at the end, our answer involves a price that's less than 1. And if it does, that's not going to be a problem.

But if we found a price greater than 1, then we'd have some issues. So that's just something to keep in mind when you're working with these kinds of demand functions. And let's remind us that this is an assumption, well, not just about one consumer because we have many, many consumers, so it's assumption about the population of consumers.

Sometimes hear about demand-side and supply-side economics. These are assumptions about the supply side, the supplier of the good, and these are assumptions about the demand side, the consumers who consume the good. So now let's draw this, and I want to be a little more precise.

Not only is c greater than or equal to 0, but let's actually say it's between 0 and 1. It's going to make things a bit easier. It could be exactly 0, but let's say it's strictly positive, and it's between 0 and 1.

So now let's draw this demand. So as usual, I'm going to have quantity on the horizontal axis and price on the vertical axis. And let me mark 1 and 1.

Of course, this just makes the numbers easy, but if you want to think of this depending on the product, maybe this is $0.01 or $1 or $100 or $1,000, but we'll just call it 1. And similarly, maybe this is tons of apples or bushels of apples. It could be various things, but we'll just use these simple units here.

And what does this demand look like? Well, if the price is greater than 1, then no one's willing to buy at all. So the demand looks like that until we get to 1.

If the price is exactly 1, then demand is 0. And on the other hand, if the price is 0, then demand is 1. So we know that there's-- here's another point on the demand curve.

And because of the form of our demand, this is what's called a linear demand. So the demand curve is just going to be a straight line that connects our point here and our point here. So maybe I'll draw it something like this. If my axes were straight, this should be, I guess, an isosceles triangle. But maybe I haven't done it quite right.

OK, and just so we understand, at any price P here, this value is Q of P, let's understand. So at this price here, I walk horizontally to the value of the function Q of P. I come down, and that's the value Q of P here, which is, in this case, is exactly 1 minus P.

So now I need to represent one more thing on the graph. So far, the graph is only representing the demand side of the market. Now, I also want to represent the firm's costs. So let me put c here, and I'll draw a horizontal line here. So this is going to represent the firm's cost of production.

And now let's understand, if the firm were to set a price of P, what would their profits be? So let's try to write this down. I want to understand what is pi of P?

So this is-- suppose I'm the monopolist. I choose to set a price of p. What are my profits going to be? If I set this price of p?

Well, let's break it down into two parts. The first is my profit per unit that I sell. So for each unit that I sell, I sell it at a price of p, but I have to pay a cost of c to produce it. So my profit per unit is p minus c.

But then the question is I'm getting this profit per unit. But how many units do I actually sell? So it's going to be p minus c times Q of p.

And now we see the basic trade-off that the monopolist faces. What happens when the monopolist raises their price? They increase their profit per unit.

If I increase the price of apples, each apple I sell is great for me. I'm making a lot of money on each apple I sell. But when I raise that price, I'm not going to sell as many apples.

So the number of units that I sell goes down. And the basic trade-off for the monopolist is to balance these two effects, the per-unit profits against the total number of units that are sold. And let's write this, for prices that are between 0 and 1, this is going to take the form like this.

So in general, Q of p is the maximum of 1 minus p and 0. But as long as p is not too big, then this is just going to reduce to 1 minus p. And I can substitute this in here.

Now we can nicely see this graphically. Graphically, if I look at this price p, well, this is going to be the amount that I sell. And p minus c is my profit per unit. So what I see is I can actually shade in my profit as this square or rectangle.

So the shaded rectangle equals profit. Let's make sure we understand it. I could actually look at the entire rectangle all the way down to the horizontal axis. That would represent revenue because that entire rectangle, its area is its height, which is the price, times its width, which is the demand.

So the entire rectangle tells me the price times the amount that I sell. That's revenue. That's the amount of money I take in. But I'm not counting this little part of the rectangle in my profit because this is the cost I have to pay to produce that amount.

So remember, profit is profit which equals revenue minus cost, revenue meaning the total amount of money that I take in and cost being the amount that I have to pay in order to produce the amount that I sell. So I don't want to make the graph too ugly, but this would be the cost. I have to pay a cost of c per unit, and then the width here is the number of units that I sell.

So what is the monopolist trying to do? They're trying to choose this price p to maximize their profit. That is, choose this price p to make this rectangle-- to make the area of this rectangle as large as possible.

Now, let's just understand some extreme cases, some things they wouldn't want to do. What if they set the price to be 1? What would their profit be?

0, right? If they set the price to 1, no one's going to buy. What if they set the price to c? Then their profit is also 0. A lot of people are going to buy, but if they're charging exactly what their cost is, they're not making any money.

And in fact, if they set a price below c, they'd actually be losing money because their revenue would be smaller than their cost. So it's pretty clear that the price should be somewhere between 1 and c. And let's actually figure out what that is. So let's now find the optimal monopoly price. So maybe I'll call it the monopoly problem.

Well, what is the monopolist doing? They're maximizing over the price p of pi of p, which equals p minus c times 1 minus p. Now, again, you could say I'm not-- this is not quite correct because, technically, for some prices they might choose, this is not a reflection of demand.

But this is the correct formula for the reasonable range of prices between 0 and 1. And if I check at the end that the price I get is between 0 and 1, then everything's going to be OK. But recognize that I'm not literally claiming that if my price is 10, then my profit is going to be negative because if I set a price of 10, in fact, my profit will be 0 because no one's going to buy the good.

Well, how do we maximize a single variable function? We take derivatives and set them to 0. You can check that this is going to be a concave function. So if we set the derivative to 0, we'll actually find the maximizer.

And so the derivative pi prime of p, well, we're just going to use the product rule. So we take the derivative of the first term times the second. The derivative of the first term is 1.

So we just multiply that by the second term, and we get 1 minus p. Then we have the first term p minus c times the derivative of the second term, which is negative 1. So we get negative 1 here.

So we get 1 minus p plus minus 1 times p minus c. Let's try to simplify this. Well, we're actually going to get 1 plus c. minus 2p.

So we want to set this to 0 and solve for 0 equals this. We're going to pull the 2p to the other side and divide by p. And what we can see is that the monopoly price p star is going to be 1 plus c over 2.

So in fact, it will be the price. What is 1 plus c over 2, geometrically? It's the midpoint between 1 and c. And if you draw this correctly, the optimal price 1 plus c over 2 will, actually, make this rectangle a square.

I won't try to show it here because I think my picture isn't exactly to scale, but what you'll find is that this will be a square. And this is actually a nice geometric fact. So you could do all the calculus here. I mean, it's not too hard in this case, but it sometimes gets messier.

One way to think about this is, well, p minus c plus 1 minus p, that turns out to be constant. So what you're actually being asked to do here is to find a rectangle of maximal area given a fixed perimeter. So this is just a reflection of the geometric fact that if I fix a perimeter, and I ask you to find the rectangle of largest area, given that perimeter, it's going to be a square. So that's actually why we get a square here. But that's just a fact to keep in mind.

So now we found p star. Let's now find Q star. So Q of p star is going to be what? Well, q of p is just 1 minus p. So we're going to get 1 minus 1 plus c over 2.

And if we simplify that, we can think of this as 2 over 2. So we're going to get 1 minus c over 2. So what this says is, it's going to be optimal for the monopolist to set a price of 1 plus c over 2, and when they set that price, the number of units they're going to sell is 1 minus c over 2.

Now, the first thing we should observe, which I think makes sense, is that the price the monopolist finds it optimal to charge is increasing in their production costs. And this makes sense. So whenever you solve these problems, I think it's good to go through the intuition. Does it make sense that, when something is more expensive to produce, I charge a higher price? Yeah, that certainly makes sense.

And in fact, it's often helpful to decompose this in a very special way. So we can actually write this as c plus 1 minus c over 2. Let's check that that agrees.

This is 1/2 plus 1/2 half of c. And this is c minus c over 2, so we still have 1/2 c. And then we have a 1/2. So the algebra works out.

And it's helpful to decompose it this way because c is the cost of production. So this term has a name. Has anyone heard what we might call this term here?

This is often called the markup. And you might talk about it. You go to a store, say, wow, that's a really high markup. If you go to a restaurant, I could buy this chicken at a grocery store for this price.

But this restaurant is charging a lot more. That's the markup. It's the difference between the price you're paying and the cost that they're actually paying to produce it. So this is called the markup.

So we see that monopolists charge a markup. They don't charge a price equal to their cost of production. They charge a price that's strictly higher than their cost of production. And that's because they're taking advantage of or exploiting their monopoly power.

Great. So now let's move to the actual game theory and think about what happens if we have multiple firms. So now what if there are multiple firms?

Well, we have to think, how are these firms going to compete? It's not so clear what they're doing. And we're actually going to study two classical models of how firms compete. So one model is what's called quantity competition.

And here, I think the model you should have in mind is you're a farmer. You think of two farmers. They're choosing how many apples to bring to the market. And once they get to the market, the price that they can get at the market depends on how many apples people bring to the market. So here, in quantity competition, firms choose quantities.

And then the market chooses the price. Maybe sets price. So this is the oil example I gave earlier. If the US-- say, I gave Saudi Arabia before. Let's say the US, if the US increases its oil or its natural gas production and floods the market, that's going to decrease the market price of oil.

So the US isn't just saying this is the price of oil I'm charging. They produce more oil, and then they have to take the price that they're given. So that's the model of quantity competition.

The other model is maybe the more traditional model that you think of price competition, where, here, firms choose prices. You have two stores side by side. They each post a sign that says, this is how much it costs to buy this item at our store. So here, firms choose prices.

And then those prices determine how much consumers actually buy from each store. So then you could say the market sets demand. Or I'll say quantities-- maybe I'll say the market chooses demand-- or chooses quantity.

So I see two stores. They each have a sign that says how much they're charging. And I choose which store to go to.

So I think these are two very natural ways of thinking about competition. We're going to find in the next few minutes that these actually give very different answers. So I think there's the math, which allows us to analyze these two models.

And then I think the job of the economist is to try to figure out which of these models is more appropriate in a particular situation. Because they both seem reasonable, and I gave a few examples of markets that might fit one better than the other, but whenever we have two models that both seem reasonable, they give very different results, we really have to think hard about which is the right model to capture the situation we have in mind.

So it turns out this one came first. This is a model by Cournot. It's easy to remember these dates because they mirror each other. This was in 1838.

And then, in this model, the price competition is by Bertrand, and this is actually 1883. I mean, the dates don't matter, but I just think it's fun that the dates are like that.

And Cournot came along, wrote down this model. He was motivated by, actually, spring water, these two firms that were sitting on a spring and were, I don't know, bottling or I don't know how water was sold in the 1800s, putting water into buckets and selling it. And then Bertrand wrote an angry article a few years later saying, this is a crazy way to model competition. Firms don't choose quantities. They choose prices.

And today, modern industrial organization economists still think about different ways of modeling competition and what's the right way to think about it. There's no right way, but in different situations, which one is better. OK, So let's go through these formal models. So we'll start with Cournot. We'll go chronologically.

So this is sometimes known as quantity competition. Sometimes called Cournot competition. Those are synonymous. So maybe here I'll call it Cournot competition.

So what's the model? We have two firms, and we'll call those firms 1 and 2. And each firm has the same constant marginal cost of production. So we'll say constant marginal cost c at both firms. And let's assume c is between 0 and 1, just as we did before.

And now we have market demand. We're actually going to look at the inverse demand-- because it's a bit more convenient in this setting-- P of q equals 1 minus q. But notice here that this inverse demand is actually exactly the same demand we had before. Because if p of Q equals 1 minus q, well, how do I-- let's solve for Q. That means Q is equal to 1 minus p. So this is exactly the inverse demand that is associated with this demand over here.

So these are just two different ways of writing it. And again with the understanding that if quantity gets too big, then the price is just 0. So if I want it to be really formal, I would say the maximum of 1 minus q and 0.

But this is a crucial point. This Q here is the market quantity. So the price at the market depends on how much water is brought to market. But the amount of water or apples brought to market depends on the entire market, not just an individual firm.

So the crucial thing here is that, each firm i, chooses a quantity qi. So firm 1 chooses their quantity, q1. And firm 2 chooses their quantity, q2.

And notice this is lowercase. So we're generally going to use lowercase to denote the quantity chosen by a particular firm. But the individual choices of the firms, when added together, determine what the market demand is. So in this case, the market quantity Q is going to be the sum of the amount produced by each firm. So this is going to be q1 plus q2.

And the final assumption-- maybe I'll write it explicitly is that firms maximize profits. Expected profit-- or I guess there's no [INAUDIBLE] say profit. Now, remember, this is an important assumption. Going back to what we said before, when we set up game theory, we said what people maximize is utility, and utility is not always the same as money.

So another way of saying firms maximize profit is that firms are risk neutral. Their von Neumann-Morgenstern utility function as a function of money is just linear. Is this a reasonable assumption? In some contexts, yes. In other contexts, no.

So maybe in other contexts, we'd really want to think of a firm that's risk averse over their profits. But in this model, we're assuming their risk neutral. So all they're trying to do is maximize profits.

OK, so now let's write down what the profits are. So let's look at firm 1. It's easier to reason from one firm's perspective. Though, it's always going to be symmetric.

I'm going to write pi 1 of q1, q2. So let's just understand pi is often used for profits just because it's the Greek pi, like p for profit. And we already used p for prices, so we use pi for profits. What I want to understand is I'm firm 1. What is the profit that I get if I produce a quantity of q1 and the other firm produces a quantity of q2?

Well, let's understand this. Well, just as before, we can decompose this into two parts. We have to keep track of my profit per unit, and then the number of units I have.

The number of units I have is easy. Let's write that down. That's q1-- q1, the number of units I sell.

But then what is going to be my profit per unit? Thoughts here? So what do I want to multiply q1 by?

AUDIENCE: 1 minus q1 plus q2.

IAN BALL: Yes, there's a parentheses in there. Yeah, exactly. So let me write it-- let me first write it as this. And then I'll break it down in the way you suggested.

So q1 is the amount I'm producing. What is my profit per unit? Well, my profit is just the price that I'm able to sell it for minus the cost of production.

But the tricky thing here is, what is the price? Well, the price is determined by the market demand. And therefore, it's p of the market quantity-- sorry, not market demand, market quantity-- which is the sum of q1 plus q2, the market supply.

And this is what makes this a game theory problem. Because how will I do as firm 1 depends not only on what I produce and what I bring to market, but also on what my opponent brings to market, q2. Why does it matter how much my opponent produces? Because how much my opponent produces affects the price at which I sell my goods. So now we have a strategic interaction between the two firms.

And let's write this out once more. We have q1 times, well, p of q1 plus q2 is just 1 minus-- in parentheses q1 plus q2. So that's 1 minus q1 minus q2 minus c.

So now it doesn't really make sense to say, what is my optimal quantity? With the monopolist, we just said what price should I charge? That was easy.

Now the optimal quantity for me depends upon the quantity that the other firm produces. So instead of just talking about an optimal quantity, we have to talk about a best response. Now we have a game.

Let's understand this as a game. A strategy for firm 1 is a quantity. A strategy for firm 2 is also a quantity. And the utility of firm 1 is given like this. And we can symmetrically write down the utility for firm 2.

So now we have a game, and we can talk about best responses. So a best response by firm 1 to a quantity q2 is equal to what? Well? We need to maximize this expression.

And this kind of expression comes up a lot. So I want to-- you can do the algebra, but I think it's nice to know a little trick. So if you're ever maximizing something like this where a is positive-- so I'm choosing x, to maximize x times a minus x, you can just derive this with calculus.

But what you'll find is x equals a over 2. That's always the optimum. And this kind of form comes up again and again. So it's just a nice thing to know right away for exams and quizzes.

Again, it's the fact that a rectangle of constant perimeter, its area is maximized by a square. If I take x equals a over 2, then both of these terms are equal to a over 2. That's the geometry of it. You can do calculus to solve it.

But now we can immediately find what my best response is. We just have to be careful to figure out what the a is. So we have to think, OK, I'm firm 1. I'm choosing q1.

So from my perspective, what part of this do I control? And what part of this is a constant? Yeah.

AUDIENCE: q1 is controlled [INAUDIBLE].

IAN BALL: Right, so q1 is controlled. So the a here is 1 minus q2 minus c. We can think of this as the constant a minus q1 where a is 1 minus q2 minus c. So if I just do this out, I see that my best response is 1 minus q2 minus c divided by 2.

OK, and again, let's ask if this makes sense. So as before, we see that the amount I produce is decreasing in the cost. So if the cost of production is higher, I produce less.

Does this make sense? Can someone give intuition for this? Why do I want to produce less when the cost of production is higher? Why don't I just keep producing more? Thoughts?

Well, my amount of production affects the market price. So if the cost is really high, it's only worth it for me to produce if the market price is high, which means I can't keep producing a lot. Otherwise, I'd pull the market price down too low.

So again, a higher cost means I produce less simply because I require a higher price to make it worth it. And then it's also decreasing in q2. So the more that my competitor produces, the less that I produce. Does that make sense?

Well, if my-- yeah. Go ahead.

AUDIENCE: They are producing more than the price is going down.

IAN BALL: Exactly right. So the more they produce, the lower the price I'm going to face for a given amount of my production. And therefore, it's optimal for me to produce less.

So now we've computed the best responses. I've only done it one way, but we can see what it's going to be for the other firm. BR2 of q1 is just going to be symmetric. It's going to be 1 minus q1 minus c over 2.

So if I'm firm 2 and firm 1 is choosing a quantity of q1, this is the optimal quantity for firm 2 to produce. And now I want to solve for an equilibrium, which means I need-- maybe I'll just write it like this.

Maybe I'll put these in here. So what is my equilibrium condition? In equilibrium, each firm is playing a best response to the other firm's quantity.

So I need q1 star, what firm 1 is choosing, to be a best response to what q2, firm 2, is choosing. And I need the amount of produced by firm 2, q2 star, to be a best response for firm 2 to the amount produced by firm 1. So I now have a system of equations.

But let's understand, my system of equations is a linear system of equations. And it has two equations and two unknowns. And in general, we can solve those systems pretty easily. We could go through the algebra, but I'll spare you the algebra. And what we'll get is q1 star equals q2 star equals 1 minus c over 3.

And let's ask, what is the market price going to be in this case? Well, the market price is going to be 1 minus q1 minus q2. But each of these is 1 minus c over 3, but I subtract that twice. So I get 1 minus 2/3 1 minus c.

And again, it's nice to pull c out. So we can do a little trick. This is c. Let's add c and subtract c. And what we get is c plus 1 minus c over 3.

So one thing we can make a comparison with our monopoly case. In our monopoly case where we had only one firm, the markup was 1 minus c1 over 2. Here, when we have two firms competing in Cournot, the markup is 1 minus c over 3.

So we see that we have a smaller markup, which maybe makes sense. And we can actually generalize this. So let's understand it intuitively.

So let's look at the general case with n firms. And this we can actually just guess by the form that we have here. So I'm not going to go through the derivation, but I'm just going to write down the answer.

And it's pretty easy to check using the same argument. So we have n firms. How much is each firm going to produce?

Maybe I'll write q1 star equals qn star. So it turns out with n firms, we're going to have a symmetric equilibrium, every firm is going to produce the same amount, and based on our pattern, with two firms, they each produce 1 minus c over 3. And notice in the monopoly case, we got-- did we solve for q? 1 minus c over 2.

Now, in the monopoly case, they were choosing prices. But actually, with a monopolist, whether you choose price or quantity doesn't make a difference. So with one firm, we got 1 minus c over 2. With two firms, we got 1 minus c over 3.

It's a pretty clear pattern there. It's always 1 minus c divided by the number of firms plus 1. So this is 1 minus c over n plus 1.

So I think this makes sense. Let's again go through our intuition. Each firm produces less when the cost is higher. That makes sense. Each firm also produces less when the number of other firms goes up. So the more firms there are, the less each firm produces.

And the reason is that the residual demand that they can capture is lower. The other firms are already capturing a lot of the market. Or another way of saying it is, the other firms are already dragging down the price. So I don't want to bring so much more.

It's also good, though, to look at the total market quantity. So Q star which equals q1 star up to qn star is-- let's sum up the amount produced by each firm. So we can look at the total amount produced by the market.

And what we're going to get is n over n plus 1 times 1 over c. So what we see is that this is decreasing in n, but this is increasing in n. So when there's more firms, each firm produces less. But the total amount produced by all firms is actually increasing.

And as n gets really large, what is this going to approach? Just 1 minus c because n over n plus 1 is going to converge to 1. So we just get 1 minus c. So I'll say, and this converges to 1 minus c as n goes to infinity.

And what about the market price? Remember, it doesn't make sense to talk about the firm's price because they're all selling at the same price. So there's a firm quantity and a market quantity, but there's only a market price. There's no individual firm price in this model.

Again, we can see the pattern. This is going to be c plus 1 minus c over n plus 1. And this is decreasing in it.

And again, I think this makes sense. The more firms there are, the stiffer the competition between the firms, and the lower the price is. And in fact, what happens to the markup as n gets really big? The markup goes to 0.

So what we see is that the market price in this case converges to c as n goes to infinity. And this exactly captures perfect competition. So when there are many, many firms competing with each other, they no longer have the market power to charge a markup. And in the limit, as we have many, many firms, every single firm-- well, every firm is going to produce a quantity such that the market price is exactly equal to cost of c.

And can anyone interpret 1 minus c? Why do we get 1 minus c as the quantity in this case? It turns out this is what we might call the efficient quantity.

What do I mean by efficient? Well, if it costs c to produce a good, for whom is it efficient to buy the good? It's efficient for people to buy the good if they value it more than the cost of production.

But our demand curve exactly tells us that the number of people who value the product, or more precisely, the number of units of the product that have marginal utility above c is exactly 1 minus c. So what happens when we have many, many firms is we converge to an efficient outcome. The market price is exactly c, and the amount that's produced and consumed is exactly the amount that it's efficient to produce and consume.

People who value each unit at more than c are going to buy it, and people who don't value it at more than c don't buy it. So we see this is an initial illustration that perfect competition leads us to efficiency. And here, perfect competition meaning the limit as the number of identical firms grows very, very large.

Great, any questions about this? Yeah.

AUDIENCE: Compared to some other quantity other than 1 minus c, why are those other quantities inefficient?

IAN BALL: Great, so let's look at the-- do we have a demand curve somewhere? I forget if I've erased something. I may have erased it.

So let me just draw a little curve here. So one way to interpret, this comes down to how we interpret demand Q of P. So what Q of P means, let's imagine just to make the argument simpler that everyone only consumes one unit of the good.

So when demand's going up, as price drops, it's because more people are choosing to buy, not because a single person is buying multiple units. The argument goes through, but that'll just make it simpler. So what Q of P is, it's the number of people who are willing to buy at price p.

But let's think this through a little bit differently. Who's willing to buy at price p? It's people whose value for the good is at least p.

So another way of thinking of demand Q of P is-- actually, it's the number of people-- it's the number with value at least p. It's the number of people who say, for them, the apple is worth at least p. But that means that Q of c, which in our example is exactly 1 minus c, it's the demand at c, is exactly the number of people whose value is at least cost.

So this captures-- if we look at everyone in the population, some people value the good at more than it costs to produce. Some people valued at less than it cost to produce. The efficient allocation is for people to get the good exactly if their value for the good exceeds the cost of producing it. And in this case, that turns out to be 1 minus c. It's the number of people whose value for the good is more than 1 minus c.

Any other questions? Great, OK, so now that we've done Cournot in the last few minutes, let's move to Bertrand competition. So maybe come down. No, I guess I'm going to get stuck with these boards. I'll come all the way over here.

So now let's look at Bertrand competition. So almost everything's the same as before. We still have two firms.

They still have constant marginal cost of c, and demand still takes the same firm-- the same form. So firms, I'll just write it quickly, firms 1 and 2. Constant marginal cost c.

And the market demand. And this time it's more convenient to write it this way, the demand Q of P equals the max of 1 minus p and 0. And c where c is between 0 and 1.

But now the difference with Bertrand is that we imagine that, instead of firms choosing quantities, they choose prices. So each firm i chooses a price pi. And now we have to try to understand, what are consumers going to do?

If firm 1 charges a price of p1 and firm 2 chooses a price of p2, what would you do as a consumer? Which firm would you buy from? The firm with the lower price.

So our assumption is going to be that, whenever the firms charge different prices, all consumers buy from the lower-priced firm. And how many units do they buy? Well, they buy from the market demand at that lower price. So let's see if we can write down a formula.

So mathematically, so we understand, this is a story. This is the motivating story. But mathematically, this is just a game.

And we have to specify the strategies and the payoffs. We've already specified the strategies because what each firm chooses is a price. So to finish specifying this mathematically as a game, we need to say, what is each firm's payoff as a function of the price she charges and the price that the other firm charges?

So let's do it. It's symmetric, so let's do it from the perspective of firm 1. What I want to understand is pi 1 of p1, p2.

So as before, we're going to use pi to denote profit. I'm going to write pi 1 to say this is firm 1's profit. But now from 1's profit is not a function of the quantities we choose, but it's a function of the prices that we choose.

And this is going to take a different form depending on the relative values of p1 and p2. So let me break this up into cases. There's going to be the case where p1 is less than p2. There's going to be the one where-- case where p1 equals p2 and the case where p1 is greater than p2.

All right, so let's try to fill this in. I think this is the easiest case here. So if I charge more than my opponent, what is my profit going to be?

Just 0, right? I'm not going to sell anything. So I get 0 here.

Now let's do this case. If I charge less than my opponent, well, now let's first-- let's break it up into two pieces as we always do. We need to think about, what is my profit per unit?

And then what is the number of units I'm going to sell? So my profit per unit is going to be p1 minus c. I'm selling at a price of p1, and my cost is c.

And notice how this is dual to what we did before. Before, I directly chose quantity, but the price depended on what the other person did. Here, I directly choose price, but the quantity depends on what the other person does and also what I do.

So what is going to go in here? What is going to be the quantity I sell at price p1? Well, everyone, if they buy, is going to buy from me.

So the amount that I sell is going to be the market demand at a price of p1. But we already said the market demand at a price of p is just 1 minus p. So we're going to get 1 minus p1, again, assuming p1 is less than 1, but we're going to be a little sloppy here.

And now we see the same central trade off but in the other way. If I raise my price, I get more per unit, but the number of units I sell goes down. What about this case? This is a little tricky.

We have to make an assumption. What happens if both firms charge exactly the same price? Yeah.

AUDIENCE: You both get half of the market.

IAN BALL: Right, that's the standard assumption that if consumers are indifferent, maybe they're randomize, maybe they, whatever, for whatever reason, it's going to be half/half. So it's going to be p1 minus c times 1 minus p1 over 2. So we're both charging the same price p1. The market demand is 1 minus p1, but I get half of that, and my opponent gets half of that.

So now, mathematically, we're done. We've described the game. Now what we want to understand is the best response. So now let me understand the best response of firm 1 to a price of p2.

And there's going to be a few cases we have to be careful about. So let's break this down. The first case is if p2 is less than c.

Next case is p2 equals c. Next case is c is less than p2. And this is going to be a final case at the end, which I'll get to. But let's go through it step by step.

What if my opponent charges a price less than c? Strictly less than c. What price would I like to charge? Yeah.

AUDIENCE: Well, at that point, you should just not because if you compete with them and actually get customers, you're not going to be making any money.

IAN BALL: Great, so the only way I'm going to be able to sell to anyone is if I charge a price that's at most p2, but that price is already below c, so I'm definitely going to lose money. So the best I can do is get a payoff of 0. But what price will ensure that I get a payoff of 0?

Well, 0 actually would not be a very good price to set. I need to charge. Well, let's see, there's a lot of things I could do. I can actually charge. It's a little tricky, but actually, anything above p2 is going to work.

So maybe I'll write it down as a set. I mean, we're being a little careful here. If my opponent charges p2, you're exactly right that my goal is to sell nothing because the only way I can sell something is if I'm losing money.

How do I ensure that I sell nothing? Well, I don't even want to charge p2 because then I'll get half the market. And then I'll lose money on that half. So it has to be strictly above p2, so anything strictly above P2 up to infinity. So I'm using round parentheses to denote strict inequalities.

Now, in particular, I mean, one thing that's weird about this is s might be willing to charge a price below cost, which seems really strange, but I'm willing to do it as long as the other person's price is even lower, and I'm not actually going to sell. What if my opponent charges exactly c? Now what do I-- what am I willing to charge?

Again, I'm not going to be able to make any money because the best I can do is sell it at a price of c, and my cost is c. So I'm not going to make a positive profit. But I want to make sure I don't make a negative profit. So what should I do? Yeah, over here.

AUDIENCE: Anywhere from c to infinity.

IAN BALL: Exactly, and in this case, can I charge a price of c exactly or not?

AUDIENCE: Yeah.

IAN BALL: I can? And why is that different from this case here?

AUDIENCE: You charge [INAUDIBLE] probability 0 at the same time.

IAN BALL: Exactly, so here we have a closed parenthesis, and it goes like this. Notice the difference. If I charge exactly the same price as my opponent, I get half the market. If their price is strictly below c, then getting half the market means I lose money because I'm losing money on every unit I sell. Here, getting half the market gives me a profit of exactly 0 because each unit I sell is exactly at a price of c.

Now, OK, maybe to make the cases a little easier, let's say I'm just going to say c is less than a 1/3 just to-- so I don't have to deal with really complicated inequalities here. So let's just say c is less than a 1/3. It's not so important.

So now what happens if my opponent charges strictly more than c but less than or equal to 1 minus c over 2? The 1 minus c over 2 is not-- you'll see later why that comes up. No, 1 plus c over 2, sorry.

So actually, I don't need-- sorry, I don't need c less than a 1/3. Never mind, ignore that. I made a mistake, and then that caused me to make another mistake conditional on that. So now my opponent's price is strictly greater than c. What price do I want to charge?

Well, they're charging more than c, so I can certainly make positive profit. But what's my goal? What do I want to try to do? Yeah.

AUDIENCE: Charge as small under p2 as possible.

IAN BALL: I want to just undercut them. And that creates a problem here because, however much undercut them, I could undercut them by a little bit less. Let's just say I charge $1 less.

Well, I'd rather charge 1 cent less. I'd rather charge 1/2 a cent less. So it turns out, technically, it's empty. There's not a best response because whatever I do, I could do strictly better because I could-- indeed, my goal is to undercut them by the smallest amount, and I can't undercut them by 0 because then I only get half the market.

So I really want to be strictly-- it's the classic thing. What's the biggest number that's strictly less than 1? Well, it doesn't exist.

Now, as soon as I write this down, I think you should-- some warning bells should go off, and you should say, well, is all this analysis being driven by some weird things about the real numbers and limits not existing? Or is this really something about the real world? And you should ask, well, what would happen if we required prices to be on a finite grid and say prices have to be denominated in cents?

And it turns out that the conclusions will be basically the same. So whenever you have this sinking feeling that there's some weird math going on, you should ask this question, and people have. And it turns out the answer's the same, so that's why we're not too concerned.

OK, and then the final case, this is the trickiest one where p2 is greater than one plus c over 2. This is a bit tricky. But what do I want to do in this case? You might think I want to just undercut my opponent.

But it turns out that's not the case here. So let's recall in the monopoly case where this demand function, if I'm a monopolist, what is the optimal price for me to charge as a monopolist? It turns out it's exactly 1 plus c over 2. If I were a monopolist, that would be the best price to charge.

So let's say my opponent is charging a ridiculously high price. Well, then it's as if I'm a monopolist. And what I actually want to do is charge exactly the monopoly price of 1 plus c over 2.

So notice where this undercutting argument breaks down. You might think, oh, I want to just undercut them. But that is assuming that they're charging a pretty reasonable price.

If they're charging such a high price that if I just undercut them, no one's going to buy from me, then just undercutting them no longer makes sense. So if they're charging strictly more than the monopoly price, then I can just charge the monopoly price. I'm charging less than them, so I get the whole market. And in fact, I get the highest possible profit because I get exactly the monopoly profit. So here, you get the monopoly profit.

So now we can work this all out. We understand the best response of firm 1 to firm 2. We can compute firm 2's best response to firm 1. But it turns out, if we do all the math, there's a unique equilibrium in which p1 star equals P2 star equals c.

So we both charge exactly c. And we can check, it's easy to see that this is an equilibrium. Showing it's unique is a bit more work, but seeing it's an equilibrium, if my opponent charges c, I'm willing to charge c. And therefore, if we each charge c, this is an equilibrium.

Now, notice a fundamental difference between Cournot and Bertrand. With Cournot, each time we added a firm, the price got closer and closer to the efficient price of c. With Bertrand, it only takes two. As soon as we had two firms competing, this undercutting motive was so strong that we immediately got down to the efficient price of price equals marginal cost. All right, I'll end there, and I'll see everyone on Thursday.