Name: ________________________ MIT ID: ________________________

Fall 2025
14.12 Game Theory
Ian Ball

14.12 Midterm Exam

October 28, 2025

You have **80 minutes** to complete the exam.

This exam is **closed book**. You may not use any electronic devices or written material brought into the exam.

Some students are taking the exam at different times. **You may not discuss the exam until solutions are posted online.**

The exam has **five** questions worth a total of **100 points**:

- Problem 1: 10 points
- Problem 2: 15 points
- Problem 3: 25 points
- Problem 4: 25 points
- Problem 5: 25 points

Please write all answers in the spaces provided. For full credit, you must justify all of your answers.

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Problem 1 (10 points)**.** Consider a strategic-form game with n players, labeled $i = 1, \ldots, n$. Each player i has a finite strategy set S_i. Each player i has a utility function

$$u_i \colon S_1 \times \cdots \times S_n \to \mathbf{R}.$$

Using formal mathematical expressions, complete the following definition.

A pure strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a *Nash equilibrium* if ...

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Problem 2 (15 points)**.** Consider the following strategic-form game between two players:

Player 2

Player 1		L	C	R
	T	$1, 2$	$0, -1$	$0, 0$
	M	$0, 1$	$3, -2$	$0, -2$
	B	$0, 0$	$0, -1$	$2, 1$

(In each cell, player 1's utility is listed first.)

1. Compute the set of rationalizable strategies for each player.

2. Find all pure-strategy Nash equilibria.

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Problem 3 (25 points)**.** Three players are bargaining over how to split \$1 between them. Formally, a *splitting* of the dollar is a vector

$$x = (x_1, x_2, x_3),$$

where x_1, x_2, x_3 are nonnegative numbers satisfying $x_1 + x_2 + x_3 = 1$. Here, x_i denotes player i's share of the dollar. The players are risk-neutral over money. They discount future payoffs using common discount factor δ, where $0 < \delta < 1$. Specifically, if the splitting x is implemented in period t, then player i's utility is $\delta^t x_i$. The bargaining protocol is as follows.

- In period 0, player 1 proposes a splitting of the dollar. The other two players observe the proposed splitting and simultaneously choose whether to *accept* or *reject*. If both accept, the splitting is implemented and the game ends. Otherwise, play proceeds to the next period.
- In period 1, a fair three-sided die is thrown to determine which player is selected to propose. (So each player has probability 1/3 of being selected.) The selected player proposes a splitting of the dollar. The other two players observe the proposed splitting and simultaneously choose whether to *accept* or *reject*. If both accept, the splitting is implemented. Otherwise, no agreement is reached, and each player gets payoff 0.

The following questions are about this bargaining game.

1. Consider a history in period 1 in which player 2 has been selected to propose. In the subgame starting at this history (which is a decision node for player 2), find a strategy (in this subgame) that survives backward induction. [Hint: when a player is indifferent between accepting and rejecting a proposal, it is convenient to select *accept*.]

2. Now consider period 0. Building on your answer from part 1, find a strategy in the full game that survives backward induction. You do not need to fully write out the strategy, but you must specify the outcome of the strategy and each player's resulting payoff.

3. What happens to the payoffs you computed in part 2 as δ approaches 1 and as δ approaches 0? Provide intuition for each of these findings.

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4. Suppose that the bargaining protocol is changed so that in each period, the splitting is implemented if and only if *at least* one player accepts. Repeat parts 1 and 2 under this assumption.

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Problem 4 (25 points). Consider the following game of Cournot quantity competition with costly entry. There are two firms, denoted firm 1 and firm 2. If a firm enters the market, its marginal cost of production is zero. But each firm must pay a fixed cost to enter the market. This fixed cost is F_1 for firm 1 and F_2 for firm 2. The costs F_1 and F_2 are known. Assume that F_1 and F_2 are nonnegative.

In the first stage, the firms simultaneously choose whether to enter the market. In the second stage, the entry decision of each firm is observed. Each firm i that has entered chooses simultaneously a quantity q_i to produce. If total production is Q, then the market price is

$$P(Q) = \max\{2 - Q, 0\}.$$

Each firm's payoff is its revenue from sales (if it enters and then produces) minus its entry cost (if it enters).

1. For which values of the parameters F_1 and F_2 does there exist a subgame perfect Nash equilibrium in which both firms enter? Justify your answer.[1]

[1] Once you solve for an equilibrium of a given stage-2 subgame, you do not need to check that the subgame has no other equilibria.

2. If $F_1 < F_2$, can there be a subgame perfect Nash equilibrium in which only firm 2 enters? If yes, find some values of F_1 and F_2 with this property. If not, show that this is impossible for all parameters F_1 and F_2 satisfying $F_1 < F_2$.

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3. Suppose $F_1 = F_2 = 0$. Construct a Nash equilibrium (not necessarily subgame perfect) in which only firm 1 enters.

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Problem 5 (25 points)**.** Consider the following stage game G:

Player 2

Player 1		A	B	C
	A	$2, 2$	$0, 1$	$-3, 3$
	B	$1, 0$	$1, 1$	$1, 0$
	C	$3, -3$	$0, 1$	$0, 0$

(In each cell, player 1's utility is listed first.)

The first part of this question is about the stage game.

1. Compute all pure-strategy Nash equilibria of the stage game.

Now consider the infinitely repeated game $G(\delta)$. That is, the game G is played in every period $t = 0, 1, 2, \ldots$. In each period, all past actions are observed. Both players discount payoffs using the discount factor δ, where $0 < \delta < 1$.

For each of the following strategy profiles, determine all discount factors δ (with $0 < \delta < 1$), if any, for which the strategy profile is a subgame perfect Nash equilibrium of $G(\delta)$. Justify your answer.

2. In period 0, each player plays A. Thereafter, each player plays A if every player has *always* played A so far. Otherwise, each player plays B.

3. In period 0, each player plays B. Thereafter, each player plays B if every player has *always* played B so far. Otherwise, each player plays C.