Fall 2025
14.12 Game Theory
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# Problem Set 2

**Problem 1** (Dominance and best responses). Consider a two-player strategic-form game with strategy sets $S_1 = \{T, B\}$ and $S_2 = \{L, R\}$. For each of the four properties below, either fill in the payoffs so that the game satisfies the stated property, or show that it is impossible to do so.

(i) For player 1, strategy $T$ weakly dominates $B$, but $B$ is a best response to some belief about player 2's strategy.

(ii) For player 1, strategy $T$ weakly dominates $B$, but $B$ is a best response to some belief that puts strictly positive probability on each of player 2's strategies.

(iii) For player 2, strategy $L$ is not strictly dominated by any pure strategy, but it is strictly dominated by a mixed strategy of player 2.

(iv) For player 2, strategy $L$ does not weakly dominate strategy $R$, and strategy $R$ does not weakly dominate strategy $L$.

**Problem 2** (CEO compensation). Exercise 3.7.

**Problem 3** (Beauty contest). Consider the beauty contest game from Section 4.2, in which each player $i$ chooses a real number $x_i$ from $[0, 100]$. Assume that there are only two players and that the payoff functions of the two players are given by

$$u_1(x_1, x_2) = -\frac{1}{2}(x_1 - \theta_1)^2 - \frac{1}{2}(x_1 - x_2)^2,$$
$$u_2(x_1, x_2) = -\frac{1}{2}(x_2 - \theta_2)^2 - \frac{1}{2}(x_2 - x_1)^2,$$

for some known numbers $\theta_1, \theta_2 \in [0, 100]$. Compute the set of rationalizable strategies for each player.

**Problem 4** (Insider trading)**.** A leading chip-maker has a problem: it can only produce 7-nanometer chips while its competitors sell superior 4-nanometer chips. It is trying to develop 4-nanometer chip capability, but there is a 50% chance that the production of such chips will be delayed, in which case it will lose some of its market share.

There are two players, named Executive and Investor. Executive works at the chip-maker and knows whether the production will be delayed. She has one unit of stock in the chip-maker that she is considering selling at some fixed price $p$. Investor does not know whether the production will be delayed, but he knows that Executive knows it. He is considering buying the stock from Executive (at price $p$) if she sells it.

The timeline is as follows. First, Nature chooses between (production is) On Track and (production will be) Delayed, each with probability $1/2$. Then, observing Nature's move, Executive decides whether to Sell or Keep the stock. If Executive decides to Sell the stock, then Investor decides whether to Buy the stock or Pass. If Executive Sells and Investor Buys, then the stock will be traded; there will be no trade otherwise. The payoffs of Executive and Investor are as in the following table:

| | On Track | Delayed |
|---|---|---|
| Trade | $p, v_I - p$ | $p, \delta v_I - p$ |
| No Trade | $v_E, 0$ | $\delta v_E, 0$ |

where the first entry is the payoff of Executive; $\delta \in (0, 1)$; $v_I > v_E > 0$; and $\min\left\{v_E, \frac{1+\delta}{2} v_I\right\} > p > \delta v_I$.

(a) Write this as an extensive-form game.

(b) Write this as a normal-form game.

(c) Check if any player has a dominant strategy.

(d) Compute the set of rationalizable strategies for each player.

(e) Compute the set of Nash equilibria (in pure strategies), and briefly discuss your finding.

(f) Assuming $v_E > \frac{1+\delta}{2} v_I$, can you find a price $p^*$ for which there is a Nash equilibrium in which the players may trade? (The price $p^*$ need not satisfy the inequality imposed on $p$ in the problem statement.)