Fall 2025
14.12 Game Theory
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Problem Set 3

Problem 1 (Nash equilibria in extensive form games)**.** Exercise 5.6.

Problem 2 (Tariffs)**.** Exercise 6.1.

Problem 3 (Nash equilibria in zero-sum games)**.** In class, we showed that in a finite, two-player, zero-sum game, a mixed strategy profile is a Nash equilibrium if and only if each player's strategy in that profile is a security strategy. To prove the forward implication, it suffices to show that for any profile (σ_1, σ_2), if either σ_1 or σ_2 is *not* a security strategy, then (σ_1, σ_2) is not a Nash equilibrium.

In class, we showed that if σ_1 is not a security strategy for Player 1, then (σ_1, σ_2) is not a Nash equilibrium. Complete the symmetric argument for the case that σ_2 is not a security strategy for Player 2. That is, show that for any mixed strategy profile (σ_1, σ_2), if σ_2 is *not* a security strategy for Player 2, then (σ_1, σ_2) is *not* a Nash equilibrium.

Problem 4 (von Neumann poker)**.** John von Neumann introduced the following stylized game of poker, with an *ante* of \$1 and a fixed bet size of \$$B$, where $B > 0$. There are two players. First, each player must put \$1 into the pot. Then, the hands are dealt. Player 1 is privately dealt her "hand" $X \in [0, 1]$. Player 2 is privately dealt his "hand" $Y \in [0, 1]$. The hands are independent and uniformly distributed.[1]

Next, Player 1 chooses whether to check or bet. If Player 1 checks, then the game proceeds to a showdown: the player with the higher hand wins the pot (so the winner

[1] So, the probability that X or Y lies in any interval in $[0, 1]$ is simply the *length* of that interval.

gains \$1 and the loser loses \$1).[2] If Player 1 bets, then Player 2 chooses whether to fold or call. If Player 2 folds, then Player 1 wins the pot (so Player 1 gains \$1 and Player 2 loses \$1). If Player 2 calls, then he must also place $\$B$ into the pot, and then the game proceeds to a showdown: the player with the higher hand wins the pot (so the winner gains $\$(1+B)$ and the loser loses $\$(1+B)$). Assume that each player's utility satisfies $u(\$z) = z$ for all $z \in \mathbf{R}$.

Represented in strategic form, this is a zero-sum game. But even this simple variant of poker is very complicated, and each player has infinitely many strategies. The game has a Nash equilibrium of the following form. For some parameters $0 < a < c < b < 1$ (which you will solve for below), Player 1 bets if $X \leq a$ or $X \geq b$, and otherwise, Player 1 checks. Player 2 calls if $Y \geq c$ and folds otherwise. In this problem, you will analyze this equilibrium.

(a) Suppose that Player 2 follows the conjectured strategy (parameterized by c). Compute Player 1's expected payoff from betting and from checking, for each hand X.

(b) Suppose that Player 1 follows the conjectured strategy (parameterized by a and b). Compute Player 2's expected payoff from calling and from folding, for each hand Y.

(c) Using your calculations above, solve for a, b, and c (in terms of B) such that this strategy profile is a Nash equilibrium. [The key step is setting up the right system of equations; you are welcome to use a computer to solve the system.]

(d) By the result from class (extended to infinite games), each player's strategy in this Nash equilibrium is a security strategy. Discuss the qualitative features of these strategies.

(e) Player 1's expected payoff in this Nash equilibrium equals the *value* of the game. Compute this value (which depends on B). Find the value of B that maximizes the value. Interpret.

[2] In the case of a tie, the pot is split, but this occurs with probability zero.

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