Fall 2025
14.12 Game Theory
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Problem Set 7

Note: In the definition of Bayesian games from class, we assume that each player's payoff depends on the action profile and the state of nature, θ, but not directly on the type profile, $t = (t_1, \dots, t_n)$. In the notes, each player's payoff can also depend on the type profile. The definition in class is not more restrictive; it simply requires that the state θ is defined so as to include all parameters that affect payoffs. In this problem set, you may use either convention.

Problem 1 (Competition in drug development)**.** There are two firms who compete to develop a new weight loss pill. Simultaneously, each firm i chooses a budget $x_i \in [0, 1]$ for developing the pill. Each firm i succeeds in developing the pill with probability $\theta_i \sqrt{x_i}$, where success of one firm is stochastically independent from the success of the other firm and $\theta_i \in \{1/2, 1\}$ is the effectiveness of the mechanism that firm i uses for its pill.

Firm i gets payoff $1 - x_i$ if it develops a pill and the other firm does not; otherwise, it gets payoff $-x_i$.[1] Assume that each firm i privately knows θ_i. The two possible realizations of θ_i are equally likely, independent of the realization of θ_j (where $j \neq i$). Compute a Bayesian Nash equilibrium of this game.

Problem 2 (Group project)**.** We consider a variant of the group project game from class. There are two players, labeled $i = 1, 2$. Simultaneously, each player i chooses effort $e_i \in \{0, 1\}$. Exerting effort costs each player 1 unit (of utility). The project

[1] Each firm can get revenue 1 as a monopolist, but gets zero revenue if (a) it faces competition from the other firm or (b) its pill fails.

succeeds if and only if both players exert effort; the value of a project success depends on the state θ of the project. Formally, player i's utility is given by

$$u_i(e_1, e_2, \theta) = \theta e_1 e_2 - e_i.$$

The state θ of the project can be either $\theta_B = 1/2$ or $\theta_G = 3/2$. The state θ equals θ_G with probability π, where $0 < \pi < 1$. Player 1 observes the state of the project. Player 2 does not observe the state of the project, but she does observe a signal t_2 about the state θ (player 1 does not observe this signal). The signal t_2 can take two values, denoted b and g. The accuracy of the signal is ρ, where $\rho \geq 1/2$. That is, if $\theta = \theta_B$, then $t_2 = b$ with probability ρ and $t_2 = g$ with probability $1 - \rho$. If $\theta = \theta_G$, then $t_2 = g$ with probability ρ and $t_2 = b$ with probability $1 - \rho$. The parameters π and ρ, as well as the description above, is common knowledge.

(a) How many pure strategies does each player have?

(b) The set of Bayesian Nash equilibria of this game depends on the values of π and ρ. Which pure strategy profiles $s = (s_1, s_2)$ are Bayesian Nash equilibria for *some* values of π and ρ? Justify your answer.

(c) For each strategy profile $s = (s_1, s_2)$ in your answer to part (b), characterize the values of $(\pi, \rho) \in (0, 1) \times [1/2, 1]$ for which s is a Bayesian Nash equilibrium.

Problem 3. Exercise 14.16.

Problem 4. Exercise 14.26.