Fall 2025
14.12 Game Theory
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Problem Set 8

Problem 1 (First-price auction with uniform distributions)**.** Suppose that there are $n \geq 2$ bidders. The bidders' valuations are independently and uniformly drawn from [0, 1]. Consider a first-price auction. Compute a symmetric Bayesian Nash equilibrium in linear strategies. How does each bidder's equilibrium bidding function vary with the number n of bidders? Interpret.

Problem 2 (First-price auction with non-uniform distributions)**.** Exercise 20.7.

Problem 3 (Average-price auction)**.** Exercise 20.10. (Hint: Look for a symmetric Bayesian Nash equilibrium with linear bidding strategies.)

Problem 4 (Auctions with reserve prices)**.** Suppose that there are 2 bidders. Their valuations are independently and uniformly drawn from [0, 1]. In class, we solved for a Bayesian Nash equilibrium of the first-price auction and of the second-price auction. In both cases, we found that the auctioneer's expected revenue was 1/3.

Now suppose that the auctioneer uses a *irst-price auction with reserve price* r, where $0 \leq r \leq 1$. In order to participate in the auction, a bidder must bid at least r. If neither bidder participates, then the good is not allocated and no one pays anything. If at least one bidder participates, then the auction proceeds as a usual first-price auction: the highest bidder wins the good and pays her own bid (with ties broken with a fair coin flip).

1. Compute a Nash equilibrium of this auction.
2. Compute the auctioneer's expected revenue. What is the auctioneer's expected revenue in the special cases $r = 0$ and $r = 1$? Discuss.

3. Find the reserve price r that maximizes the auctioneer's expected revenue (from part 2). What is the associated maximal expected revenue? Discuss.