18.099 - 18.06CI.

Attention: Next week at MIT Tuesday is a virtual Monday; and Monday is a holiday.

Due on Tuesday, Feb 17 in class. First draft due on Thursday, Feb 12.

(1) Give an example of a linear space over the rationals which is not a linear space over the reals. Can you find an example of a real linear space which is not a rational linear space? Explain your answer.

(2) Let A be the linear space of real infinite sequences $(a_1, a_2, a_3, \ldots)$ over $\mathbb{R}$ with coordinatewise addition and multiplication by numbers. Which of the following are linear subspaces in A?
   (a) Sequences with only finitely many nonzero terms;
   (b) Sequences with only finitely many zero terms;
   (c) Cauchy sequences, namely $\{a_i\}_{i=1}^{\infty}$ satisfying the following condition: for every ε there exists a number $N > 0$ such that $|a_n - a_m| < \varepsilon$ for all $m, n > N$;
   (d) Sequences $\{a_i\}_{i=1}^{\infty}$ for which the series $\sum_{i=1}^{\infty} a_i^2$ converges.

   Prove your answers.