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JAKE XIA: So today, we'll be talking about portfolio management as one of the application lectures. So I will share some of my research work from a practitioner's point of view, rather than from pure theoretical or math point of view. So as we mentioned at the beginning of the term, we've been teaching this class since 2012.

And in 2013, this class was recorded for MIT OpenCourseWare. The link, I put it here. It's the lecture from 2013. Some of you may have watched that lecture. It actually turned out to be one of the top most viewed courses. Our course is one of the top five, actually, on the OpenCourseWare. So I want to promise you, in today's lecture, I will give you a lot of new content. So you will make showing up, your presence here, worthwhile. So we can also make it interactive.

So this is the outline of what I will be talking about today. So first, I'll talk about what really all this portfolio construction is about, coming from a very basic level. Then we'll talk about what is the endowment model. How do we manage the endowment fund? Then I'll talk about the portfolio theory and illustrate it in some really special and simple cases for you to understand the math.

Then I will really switch gears and talk about the limitations of this theory. The theory was proposed in the 1950s. I mean, of course, a lot of people have worked in the field, tried to overcome the shortcomings of these theories. So I myself have been spending a lot of time thinking about these problems. So I will talk about my own research work, in terms of how to improve the portfolio construction theory-- for example, how to improve the risk measurement using volatility and Sharpe ratio and how to model crowding behavior. And also, you probably heard the power law, how this behavior actually originated from the crowding behavior.

Now, I will summarize. So before we start the lecture, I want to do an exercise with all of you. I'm going to hand over a piece of paper to all of you. Please pass it down. Make sure everyone gets one piece of paper. Now, what I want you to do is to write down your own investment portfolio.

Just hypothetically, think about you have $10,000 to manage. And put down investments you'd like to have in the portfolio. Put down the percentage of each investment. And I also want you not to think too hard and also too long on this. Just use your intuition. Imagine you're now a fund manager sitting at a pension fund, or endowment fund, or hedge fund. I'm just asking you to pick a few investments. And then just hand it back to me after you finish. Then we will talk about, from where you started, how to do the portfolio management.

So just spend a couple of minutes. Don't over spend your time. So while I'm talking, so you can write down your choices. So when you received that blank sheet of paper, you must be thinking, what are the criteria for me to do this? What are the considerations? So this is exactly how I want to lead you into how to think about these problems.

First of all, you need to think about, what's the objective of doing this exercise? So always say, oh, I want to get a great return for the investment. Sure. So you have a return objective. But is that what is the great return? Is that 10%, 50%, or 100%? What is the time horizon we are talking about?

And because the investment is highly uncertain, what's your loss tolerance? You think about it. You have a $10,000. How much can you afford to lose? So those are the questions. Then when you pick those investments, you should also ask yourself, what's your own ability of picking a winner? Why do you think you have an edge in predicting that market or investment?

Then of course, how many investments you should have in the portfolio? Is that 1, 5, or 50, or 100? How diversified or how well-mixed that investment portfolio should be? Then ultimately, the most important question is, how do you your size each of the investments?

It comes all down to sizing. So portfolio construction, I will show you later, is really about, when you have a list of investments you like, how do you size them accordingly in relative amounts to each other, to the whole portfolio? So now, please hand it back because I want you to listen to me, not just spending the time on writing down your portfolio.

STUDENT: Do we put our names on it?

JAKE XIA: It's your choice. Name or no name is fine. Yeah, I'm not going to call your names. But I will probably use some of your answers as examples to talk about investments. So as we are collecting these portfolio choices, I'm also going to talk about, in your decision making process-- so when you choose investments, you probably were thinking about what markets, first of all, you will be selecting your investments from. And what are the instruments?

Are we mostly finished? So by the way, for you, we are doing an exercise, where we ask you to put down your investment portfolio. Just put down the percentage, whatever you can do. So anyway--

STUDENT: Here.

JAKE XIA: You're done? OK. For the rest of you, if you want to take a longer time, that's fine. I'm just going to use some of the examples. So the decision making process I was talking about-- you need to decide which market to pick your investment. Then you need to think about what data to collect, what kind of signals you can derive, and what models you need to build, what factors you need to take into consideration, what predictions you need to make, then assemble them into strategies. Then we're talking about sizing and allocating your capital portfolio construction, then optimize it, then at the end really to manage the risk.

So now, you can see what we really come down to is how to compare different investments. To understand the relationship between different investments is to understand the return and the risk profiles of all the investments. So what is risk? So in this chart we are talking about here, I'm using volatility for the time being, which represents uncertainty of the outcome.

Later on, I will come back and tell you volatility is not really the best measure for risk. But let's save that for later. But before we talk about the investment choices, let me look at some of what you have written down. So one portfolio here has half in S&P straddle-- so this is actually option strategy-- and then 30% in VIX related ETFs, and 20% in dollar index futures. So it sounds like someone has done an internship somewhere knowing option strategies.

So this portfolio has about eight investments. The largest one is cash, 20%, and 20% in QQQ, which is NASDAQ ETF. Let me see. This one is 70% bonds, 30% stocks. This one is quite interesting because, when we normally talk about 60/40 or 70/30 portfolios, the bigger piece tends to be the equity piece. But this one probably has more risk neutral or risk parity type of construction. So we'll get to that.

This has many single name stocks, Microsoft being on the top. And this is a 50% S&P, 50% Vanguard Pell fund. Probably that's what is written. And this 100% S&P. Yeah, very much representing the average market. And so this one probably has five stocks, 20% cash. Yeah, 50% US Treasury bonds, 25% S&P. So fairly similar.

So now, you've got the idea. And Peter and I used to review, over the years, year to year change of the investment selection. Only three years ago, in 2021, most of the class chose some cryptocurrencies. And Dogecoin's being the top one being selected by everyone. I haven't seen any crypto investments today. So that just tells you the index investing ETF seems to be the theme of this year.

I don't blame you. I think it's a pretty good idea. S&P, if you don't have particular edge you feel you like to have, Investing in index is a pretty good choice. So on this graph, I'm showing you here return versus volatility. So we already touched upon-- a lot of people chose cash. Cash, basically, you can view it as 0 volatility, as some minimum return.

The Fed just cut interest rate. It's still over 4%, close to 5% And they may cut more. But compared to the interest rate where it has been in the last 10 years, this is actually not a bad place to park your capital. Bonds have slightly higher return, a little more uncertainty of return.

Stocks definitely have more, but the index has less volatility. And the index, return wise, it may not be lagging the broad stock market. I mean, as you know, the index now is heavily weighted towards large cap tech stocks. And lately, those large cap tech stocks have been doing really well. So if you're long QQQ or S&P, you're essentially long the Magnificent Seven stocks. And so the ETF investment choice here is not a bad choice or index investment.

Further up on the right is the private equity and venture capital choice, which I haven't seen anyone put it down. So you may be more thinking about the public markets. But private equity and venture capital are basically investing in private companies not publicly listed on stock exchanges.

So those companies tend to be illiquid. You cannot get your money out easily. You have to stay with them for a very long time. But a lot of times, you have more influence on the company's operation and venture. For example, you invest in start-up companies. Those companies can grow very fast, can also fail very fast. So the outcome, the uncertainty range, is very big. But overall, over the years they tend to produce higher returns to justify the uncertainty or risk premium you are dealing with.

So coming back to the return objective and loss tolerance, we should think about when you write down your portfolio. So you try to maximize your return for sure. Does anyone here disagree with you should maximize your return? Probably not. At the same time, you should minimize your risk. In this case, minimize your uncertainty. So you want to maximize the ratio of return over volatility. That's one of the objectives.

But you may have constraints as well. There are certain investments you may not want to participate in. For example, many universities declared we don't want to invest in fossil fuels for the environmental concerns. And in the past, like tobacco companies or some kind of weapon companies, people have, in principle, reasons to go against them. And so there are constraints of what your investment universe should be.

Then there's liquidity, how much you need to use the money. The $10,000 you had in the portfolio, you may not want to use it all in the investment or illiquid investment all the time. So you may want to spend 5% a year, for example. So you need to take into account your spending needs. And also on the down side, how much you can afford to lose is really a very important question.

So if your $10,000 becomes $5,000, will you feel really bad? You would just say, I'm going to stop investing altogether. I cannot afford to lose. Maybe that's the tuition I saved for next year, I put aside. I just want to earn some return. I don't want to lose it. So then your risk tolerance is very low. It's very different.

So for many asset owners, inflation is also an important concern because wealth represents relative purchasing power. So how much wealth you have is not really just the dollar amount you have in your bank account. It's how much you can buy. So when the price of goods and services are going up, your purchasing power is going down. So that's inflation.

And it's also relative to the next group of people or person, how much they can buy. So for universities, that means you want to hire a professor. If there are competing offers, how much more you can offer to the professor to get them to come to your university.

So there's definitely absolute return angle for the requirement of the portfolio. And there's also a lot of people nowadays talk about beating a benchmark, beating an index. Many of you have chosen S&P as an index. But if you can produce more return on top of S&P return, that's called the alpha, relative return. That's also often being used as a measurement of investment skills. So you don't just put in a beta or broad market investment. You actually can extract more return.

So asset liability matching, cash flow is very important. So 2008 taught many asset owners a big lesson. When you don't have the right asset liability match, when your portfolio has a drawdown, your liability side will basically dominate on your balance sheet. You need to cut your spending budget, stop your building projects, and lay off people. So those are painful lessons. You need to have the cash flow well planned.

So time horizon, I mentioned, is very important. When I gave you that piece of paper, I didn't say time horizon on purpose. I want you to think about it. You should ask the question-- this portfolio is for how long? Is that for one day, or for a year, or for 10 years?

So in the investment world, people always compare returns, like year to year, sometimes even quarter to quarter. Peers try to say, oh, I outperformed-- A outperformed B. So therefore, A is better than B. So this is actually, a lot of times, unhealthy exercise because, for long term investors, you should really focus on long term return.

But inevitably, in the real world, people look at short term returns. So you cannot ignore it. But that's the peer pressure for a lot of investment managers. That's the career risk because their board may think, in the short term they are not good. They try to replace them.

So coming back more on why investing, so on the left, on your left, if you look at the personal income and spending as one person's age, as age as the function, so your income level kind of peaks somewhere in your mid-career. So let's call it around the age of 50. So you're earning the most. You're probably also spending the most.

But you as a student, you probably don't earn that much. You don't spend that much either. I mean, same is true when you're over the peak point. You're getting into the retirement stage. So you don't earn that much. You don't spend that much. So you need to plan accordingly of your personal portfolio. So that's related to the question of time horizon.

And also, the marginal benefit of having more wealth and money starts to diminish when you have had certain minimum amount. So that's the bottom curve. Your benefits start to plateau. And so also, do you want to take more risk or not at that point? But for organization or endowment on the right, it's a little different because it's a perpetual portfolio. It's a very long time horizon. And the spending keeps going up as inflation goes up. So you cannot basically be complacent and say, I just put all the portfolio into some conservative investments. You have to keep producing those kind of returns.

So the endowment is setup for university-- typically, they spend 5% of the portfolio. And on top of that, there's an expected 3% inflation. So the nominal return target is 8%. So as you know, it's probably not always easy to make 8% consistently year over year, especially when interest rate is low. You don't have a bond. There's safety bond that you can put in.

Also, endowment tends to have a long time horizon, definitely more than 10 years. That's what the people always say. Universities have hundreds of years of history and looking to have another many more hundreds of years. And on the university's spending budget, operating budget, now, roughly about 40% of the budget is coming from endowment spending.

So you can see universities are now more and more relying on endowment returns. And that's because the research grants from federal agencies and the industries are coming down. So this has to be made up from the investment portfolio. And so this is just a list of the strategies endowments tend to invest in. I mentioned cash, government bonds, corporate bonds, and the credit products, meaning the issuer, unlike the US government, it has the risk of defaulting. So therefore, it will pay you higher interest.

So there are hedge funds betting on the macro markets, betting on the Fed or stock market, the general direction. There are also quantitative funds doing the trend following. People usually call it CTA, commodity trading advisors, or stat arb, which is to capture the relative value between a pair of stocks or actually many pairs of stocks. So it's statistical arbitrage, they call it, which is really to find statistical patterns to generate returns. So this is heavily computer driven strategies. And many big quantitative shops are still doing a lot of these.

So there are fundamental equity hedge funds. They buy stocks and also sell short stocks. And they are also the platforms you probably heard-- the big platforms, they're called multi strat or multi PM, portfolio manager, platforms. They hire hundreds of different teams trading on the same platform in a very different wide range of strategies, but achieve a high degree of diversification. And they can level up the portfolio quite a bit.

And so I mentioned private equity earlier. There are also real asset investments-- real estate, natural resources, farmland, timberland, those type of investments-- and the new assets, such as crypto, or intellectual properties, or legal claims, and so on. So basically, anything you can think of can generate a return. It can also be considered.

At the first lecture, I already highlighted some of the features of the endowment model. So endowment model nowadays tends to focus on hiring external managers and doing both public and private investing. And we hire both generalists, who look at everything, and also specialists in certain domains, such as biotech, AI, and so on.

We focus on both absolute return and also relative return to the benchmark. So we are mostly active managers. So we have some passive beta index investments. But it's a minority of the investment. It's also not always permanent. So whenever we can find an active strategy or manager to beat a benchmark or index, we will always go for the active investment.

Yeah, so as you can see, this investment process is centered around manager selection, selecting the good investing fund managers. And Harvard, actually in the old days, used to have a lot of internal fund managers. But that model has changed from seven years ago to focus more on external managers.

So now, I'm going to shift gears a little bit to talk about portfolio construction problems, so shifting towards the math part of the discussion a little bit. So the classic portfolio construction problem is stated as such-- assume you know your return objectives. You have a loss tolerance. And you can predict each investment's return volatility, which is the standard deviation, and also the correlation matrix of these investments.

The question is what percentage you should put in each of the investments. So that's the portfolio construction problem. And the objective of that, as we mentioned earlier, is to maximize the return for the whole portfolio, while minimizing the portfolio risk. In this classic problem, it's to minimize the uncertainty or volatility of the whole portfolio.

So as you can see, the portfolio construction problem really is a sizing problem of investments at all different levels. You can relate that to asset allocation problems. So what is the asset allocation problem? So you group those investments into different asset classes. So as I showed you earlier, the list of these strategies, you can also think of them as lists of different asset classes.

So from top down, the discussion needs to be had is how much you want to be in each category or each asset class. And sometimes, people will say, well, that's still way too many asset classes. If you have 10 to 15 asset classes, that makes the optimization problem very hard. So people tend to reduce them to big risk factors, sometimes, say, 3 to 5 risk factors, usually equity, bonds, plus one of the other things could be inflation.

It could be currency. It could be credit and other dimensions. But the factor analysis can also expand into many detailed levels. You can have 50 factors or even hundreds of factors. But it's just different ways of grouping the investments to make the optimization problem a little easier.

So then what is the risk management problem of a portfolio? So risk management problem, in many ways, is actually the same problem as portfolio construction. It's the sizing problem. But in addition, you need to think about how to avoid concentration. Concentration, of course, is also a sizing problem and also illiquidity. And what's unacceptable losses and unwanted risks To constrain the portfolio?

So I'm going to spend a few minutes on the portfolio theory, which is illustrated in two assets. Asset 1 has a return, R1; volatility, sigma 1; and will have a weight in the portfolio of w1, similarly for asset 2. So the sum of the two weights should be equal to 1, w1 plus w2.

And the portfolio return, which is denoted here as R sub p, is the weighted return of R1 and R2. And the variance-- or in this here, it's the sigma p squared, the volatility squared, can be expressed in terms of the weights and the volatility of each of the two investments, as well as the correlation, rho, between the two investments. So this is a very common equation you probably have seen many times in any probability or statistics classes.

So let's look at some special cases. So when the correlation, rho, is positive 1, meaning the two assets are perfectly correlated, one moves up, the other moves up at the same time, one goes down, the other one goes down at the same time. So that's called perfect correlation.

So if rho equals to 1 and we can simplify the sigma p, which is the portfolio volatility, it becomes the weighted sum of the two volatilities-- w1 times sigma 1 plus w2 times sigma 2. Or you can also express the w1 in terms of 1 minus w2. So therefore, solving for w2 in terms of sigma p and the sigma 1 and sigma 2.

So then you can derive the overall return of the portfolio, R sub p, in terms of w2, because w2 above is a function, is a linear function of sigma p. So you know, in this case, the return of the portfolio, Rp, is a linear function of sigma p. So on this chart, you can see that straight line when rho equals to 1. That's the special case.

When rho equals to minus 1, that means they are perfectly negatively correlated. So one goes up, the other one goes down. So when that is the case, you will actually easily derive that this sigma p is actually-- you have two branches of solutions. So it depends on the value of w1 sigma 1 compared to w2 sigma 2. So that's the two separate lines on the left. That's the other solution. In the middle is the other values of the correlation between negative 1 and plus 1, including 0 correlation, rho equal to 0.

So what if the first asset has 0 volatility, sigma 1 equals to 0? So then from the equation earlier, we can derive sigma p, the portfolio volatility is simply w2 times sigma 2 because sigma 1 is 0. And therefore, w2, the weighting on the asset 2, is sigma p divided by sigma 2. That's a straightforward result.

And the portfolio return, R sub p, now can be written as R1, which is the first asset with 0 volatility, which you can think of as a cash right. We mentioned earlier, cash has zero volatility. So this cash or riskless asset has return R1. Its Rp is still a linear function of sigma p. But it's a straight line, in this case. You can see the slope of that line is determined by the difference of R2 minus R1 and divided by sigma 2.

So by the way, this is so-called capital allocation line in the portfolio theory. So we can look at, let's say, a constructed portfolio. You already have a sigma p, which has many assets in it, and also Rp. By adding one riskless asset, or cash, which returns Rf. Rf is basically like a risk free return. You can draw a tangent line over the sigma P point. You pick the one point on that efficient frontier.

I should explain efficient frontier, meaning on that return versus volatility chart, because you have different weightings-- so let's come back to this. Let's look at this curve here. So for different w1 and w2's, you can have different resulting portfolio return and the portfolio volatility. So this line, the upper part of this is called the efficient frontier because by lowering your risk and moving your return up, that's your objective. You are improving the portfolio. So any point on this line is a potential solution of that portfolio construction.

So when we add the risk free asset on this Rf point, we can basically combine that risk free asset with the existing portfolio to further improve the risk-- the efficient frontier to move up to that straight line. So that slope, Rp minus RF divided by sigma p is so-called Sharpe ratio. In simple terms, think about your return over volatility. So that's the Sharpe ratio definition.

Sometimes, you also hear the term alpha and beta. That's compared with a benchmark. So in this case, you can write down the portfolio return minus the risk-free asset in terms of alpha plus beta times benchmark minus the risk-free return. And the beta is a function of correlation of the portfolio to the benchmark and the ratio of the volatility.

What if you want to level up the portfolio and borrow more, meaning make the weight of the first risk-free asset to be minus 100% and make the other asset be 200%, so still sum up to 1? So that means you're borrowing money in cash. You're putting the proceeds into the risky asset. So you're moving the volatility up to this point by investing more into the second asset.

So that's also possible. That's a typical exercise people do in risk parity portfolios. By the way, if you don't follow any of those, that's fine. The reason I want to talk about this is really to give you some flavors of what the portfolio theory is about because then I'm going to tell you, in practice, a lot of these need to be improved.

So the example of portfolio with two assets that we talked about is the 60/40 equity bonds. Some of you wrote down 70 bonds, 30% stocks. That's very much similar to the risk parity portfolio, which they try to make equal risk contribution coming from the bonds and equity by borrowing more to invest in the bonds because bonds typically have lower volatility than equity. So you need to have more to make them have equal risk contribution.

So that's the idea. Again, today, I'm not going into the math. In the previous years, sometimes I spent more time on that. But I feel I want to talk about other things a bit more today. But before we move away, I want to talk about the importance of rebalancing. Because we had this 2013 OCW video, just two weeks ago, I think somebody made a clip out of that class, sped it up to 1.5 times, and made it a 3-minute video of the only free lunch of diversification, which is exactly this example.

So I'm going to go through it with you quickly. Just think about you have two assets, assets in a two-year period. In year 1, the first one doubles. And then in year 2, it halves. So I think it's a little easier to show on the blackboard. But the other one does the opposite.

So they both started here. The first asset doubles, then halves. So this is year 1. This is year 2. The second asset halves first, then doubles. So those were the R1 and R2's. So if you start as equal weight of the two, and after year 1, you can calculate. The portfolio is up 25%.

So this is just simple math. The first asset 1 is 50% weight. Asset 2 is also 50% weight. One doubles, one halves. So total, you made 25% But if you don't do anything at this point, asset 1 will be dominating the portfolio. Asset 2 will be just a small portion. Then because asset 2 then doubles, you're coming back to the same point. After two years, your portfolio's return is exactly 0%, even though you had a diversified portfolio with perfectly negatively correlated assets.

But if you rebalance at this point to equal weight, then your portfolio in year 2 will make another 25% return in year 2. So you're making compounded return of 25% two years in a row. So some of you may point out that isn't this just betting on mean reversion? And if you think it's a trend trade, you shouldn't do this. That's right.

The question you should ask yourself, what is the basis of your sizing or weighting of the two assets? So you started with an equal weighting, meaning that you think the two assets have the same likelihood of performing and the same amount of risk. That's why you size them equally. And of course, you also know they are negatively correlated.

So at the end of year 1, do you maintain that view or not? If you think it's still the same projection of risk and return, then you should maintain the equal weight. You should not change the weighting unless, at the year 1, your projection has totally changed. You think asset 1 will continue to outperform asset 2. Yes, in that case, you can change to a non-equal weighting.

So that's the point. If you believe they have the same probability distribution, then there's no reason for you not to rebalance. So people say there's a free lunch. The only free lunch is diversification. But you have to rebalance to eat it. So that's on that.

So then I'm going to shift the gear to talk about the limitations of so-called modern portfolio theory, in short MPT. I mentioned this was mostly developed in the 1950s. And Harry Markowitz pioneered in this area. He actually got a Nobel Prize in the, I think, early '90s for his work. So all I have just described to you so far, the mathematical part, has a lot to do with his work.

But you can tell already, a lot of these depend on your confidence in the capital market assumptions, which is the projections of return volatility and the correlation matrix of these investments. And the mean variance optimization problem often has unlimited number of solutions. And you have to put artificial constraints to bound the solutions. They are very sensitive to small change of these capital market assumptions. So in practice, it's very hard to use it. Probably not many people use it.

So with those questions, I myself have always been curious about this and have done various research work in this. So today, I'm going to highlight a few. The first one is more related to the sizing question, using a different approach. I call it a gain-loss ratio. The second part, I will talk a bit about the effect of crowding behavior and how to model it. And lastly, I will talk about the power law distribution coming out of these crowd interactions, why these are all relevant to portfolio construction.

So issue number 1 for the MPT, volatility is a poor measure of risk. Why do I say that? Because volatility is just the standard deviation or the range of uncertainties. So if I am long, out of the money call option-- think about it. You have one investment. And Steven, you worked on options. You probably can get this easily.

If I'm not out of the money option, do I wish the volatility to go up or down? Do I want my portfolio to have high vol or low vol? I want my portfolio-- yes, yeah?

STUDENT: The higher vol.

JAKE XIA: Yes, right. The higher vol, the better. So on the other hand, if I'm short on the money option, I probably want to have lower vol because I don't want to get exercised. So it really depends on your gain situation, your payout, or your potential loss. And the Sharpe ratio is derived from volatility. So basically, it suffers from the same issue.

So over the years, there are methods, like the Sortino ratio, that try to differentiate the upside range and the downside range. I think it's a good measurement. But it doesn't give you a direct link to sizing. The ultimate question we need to answer is sizing.

So the new way to compare different investments I'm basically using, or I came up with, is to use expected gain and expected loss. I call the G and L. Here are both use positive numbers. So when I say loss, these are absolute numbers. So we are looking for the best skill, which is getting much bigger expected gain, much bigger than the expected loss. Or written in different forms, in the G minus L, which is the return, divided by GA plus L.

And you change the format a little bit. It becomes 1 minus 2L divided by G plus L. This is really exactly the sizing percentage from Kelly's criteria in the binary betting. So this number is bounded from minus 1 to plus 1. So it can be directly translated into sizing. And so let me give you a few examples to help you understand this.

So if you are flipping a coin, you know the head has a certain probability and the tail has a certain probability. And when it's a head, it has a payout you will get or lose. And when it's tail, you have a payout or lose. So expected gain is the probability of the head times the payout, if you bet on head. Expected loss is the probability of the tail times the payout on that side.

So in that discrete example, it's fairly easy to understand. But in the continuous example, it's the terminal distribution. Let's say you have a probability density function describing all the outcomes. Then you need to use that probability density function times the payout integrated over all the losing parts and integrated over all the gaining parts to get the expected loss and the expected gain.

So it doesn't have to be always separated by 0. If you have a target return, you integrate over, say, from minus infinity to the targeted return, that's your expected loss. That is what needs to be included in L. So when you do this, your optimization problem is no longer return versus volatility. But in the vertical axis is gain. In the horizontal axis, L is loss.

So you need to basically maximize G while minimizing L. So this is all relevant because, as I just explained earlier, minimizing volatility is not always for your benefit. When you are long option, you need to have higher volatility. But in this case, when you specify expected loss, that's what you need to budget and control your investment.

So in the investment game we gave you at the beginning, I hope you all have submitted your choices in September. And some of you rebalanced in October. I wanted you to track the daily gain and the loss in dollar amount. And at the end of the game, which is mid-November, I want you to summarize all the daily gains and also summarize the daily losses to derive the G and L.

So some of you said, in order to win this game, why don't I just pick the highest volatility stock to have the best potential to produce highest return? Yes. But your higher volatility investment will suffer on the denominator, which the G plus L-- this ratio will indicate the quality of your investment, even though your G minus L, which is the return, may be good. But the denominator will normalize it to make it basically comparable to other investments. So the optimization problem-- yes, please.

STUDENT: So in practice, what do you use to estimate the expected gain?

[SNEEZE]

JAKE XIA: Yeah, bless you.

STUDENT: [INAUDIBLE] estimate the distribution of the pricing?

JAKE XIA: Yes. But no more difficult than getting the return and standard deviation. Yeah, so it's the same amount of work. It's just that you're separating the numbers. You're calculating half distribution instead of the whole distribution. So it is. So especially when you do the optimization problem, as I'm about to explain, these lines-- basically, for example, the 45-degree line indicates G minus L equal to 0, so 0 return line.

So you don't want to pick any investments below that 45-degree line. You want the lines that have higher slope, indicating higher G over L ratio. But when you combine them, combine, let's say, two assets into one, calculating the combined portfolio G and L, which is actually path dependent, which actually should be because, when you have two investments, the final outcome of the expected gain and the loss indeed will be path dependent. That means you probably need to do a lot of Monte Carlo simulation to derive that. Yeah, but it's no more work amount than the traditional distribution calculation.

Yeah. OK, let's move on to modern portfolio theory issue number 2. How do we predict the future based on the past? How confident can we be in the capital market assumptions-- return, volatility, and the correlation? So because we human thinking process-- our thinking process is always extracting patterns from the data. So we have observations. I'm just outlining the typical thinking process.

You have observation. You have measurable data. Try to quantify them. Then you try to extract useful information or recognize patterns. Then you try to build the models, having inputs, outputs and conditions. Then you predict output from the inputs and the conditions. Then you repeat your observation. Then you iterate, try to calibrate your model parameters.

When you are very good at it, you can actually control conditions and inputs to get the desired outcome. So that's true for any subject-- physics, engineering, finance. But finance involves human behaviors, which is much more complicated. So the historical patterns may not be repeatable. And also, the crowding behavior and also adaptive behavior of human players will make the process much more complicated. So I want to show you a video. Let's see if this works.

So you are watching is a video of birds or bats in a totally is self-organized way to form a group behavior. What they do is they watch their neighbors. They adjust their actions. And the group as a whole displays certain patterns. The financial market is very similar to this, except there's a central observation you can actually see, which is the stock price. You can also look at your neighbors, too. But nowadays, it's much easier to look at the central observation.

In the past, I've also shown the London Bridge video. How many of you have heard the Millennium Bridge of London? Yes? OK. It opened, the Millennium Bridge, around 2000. On the first day, when people walked on it, it starts to sway. And then people tried to keep their balance. They become synchronized in steps. That made the sway even bigger.

So market does the same thing. When panic feeds into panic or greed feeds into greed, market can form bubbles and the crashing. And these kind of behaviors are not easily predictable. But it's part of crowding behavior. So how to understand and model this kind of behavior?

So we can look at it as a feedback loop. So S here is the agent's action. O is the observation. A is kind of an amplifier or agent's action fed through into the observation. As you can see, the change of the observation is the sum of the agent's action times the power they have, which is A. Then agent's action, the change of agent's action, dS, is influenced by external force, as well as the feedback loop of the observation plus some random noise. So that's the basic framework of crowd modeling.

When the agents behave in normal conditions, they are more rational. So they are less reactive to the observations. Or you can call it the feedback loop. Parameter B is actually taking a lower value. But when observation changes in a more volatile fashion, the agents tend to react more. They get into a reactive state. So more agents in the crowd get into reactive state. The crowd actually becomes more synchronized.

So if we look at the order parameter, which I defined as the ratio of the sum of all the actions together divided by the sum of the absolute number of the actions, if they are all synchronized, that number should be 1. If they are not synchronized at all, that number can be very close to 0.

So what the first chart on the left shows is the order parameter as a function of number of agents in the crowd in the reactive state. So then when more agents become reactive to the observation, the order parameter can become very high. The crowd gets synchronized.

And on the right, you can see the order parameter is also a function of the random noise term in the previous page. The higher the noise, the less likely the agents-- the crowd will be synchronized. So this type of behavior, by the way, is not new in physics. In some of the circuit engineering systems, when you inject a noise, you can also reduce the order of parameter synchronization. But in social science, this actually is also a similar behavior.

So I further simulated the bubble b process. So when you have an external force showing on the upper left, it's kind of ramping up, then coming down, then oscillate. The number of agents in reactive state starts to peak up, then also drop when the external shock diminishes.

The observation, as a result, also follows, but then comes down. But if the external shock, on the other hand, maintains at a higher level forever, eventually, all the members of the crowd, all the agents, get into the reactive state. So the whole crowd becomes synchronized. The observation basically shoots through the roof. So it becomes a very unstable system. So that's the bubble. At that point, when the system becomes unstable, your bubble will burst.

So that's the part of the crowding behavior. So the purpose of showing that is, when we look at portfolio management, this type of bubble forming and bursting process is a very important aspect of our understanding beyond the fundamental analysis. This is all about behavioral finance.

It has nothing to do with the fundamental of the economy or the fundamental of a company. But then you probably also have heard power law distribution. So what is power law distribution? It's the winner take all, the 80-20 rule, or the Matthews effect, many different terms describing essentially a social phenomenon that there are a small number or percentage of the agents taking a majority share of something in the group.

So let's think about wealth distribution. So 20% of the people probably have 80% of the wealth. And this is also true. You look at the top 20%. Within that 20%, the top 20% has 80% of the wealth. Then within that, it's still true. This is so-called scale free or scale invariance. This is the unique property of power law distribution.

So in mathematical terms, the probability of b times x-- x is the random variable. b is the scaling factor-- equal to some function of g of b times the probability of x. So this you can prove mathematically. If and only if this p of x is a power law distribution, that scale-free property maintains.

So an example of power law distribution, I mentioned the wealth distribution. Venture capital funds returns-- the top tier funds tend to get the majority of the profit of the startup companies. There's a book published two years ago. It's called *Power Law,* by Sebastian Mallaby. You should check it out. It's very interesting. It's not mathematical. But it talks about how the top tier VC firms winner take all phenomenon.

City sizes, same thing. Big cities get bigger. Why? In the network theory, because all the nodes want to connect to a more popular node to attract more traffic. So the network space, internet connections, web pages or social media posts, this type of power law is very common.

But in nature, we've observed a lot of normal distributions, such as height. So a lot of people are probably centered around call it between 5 to around 6 feet, maybe just under 6 feet. This is the majority of the population. It's not like most people are very short, but there are a few giants, really, just much taller. So that's power law. Power law does not exist in the height distribution.

So now, we commented on many social phenomena obey power law distribution. But why? This is the question that always makes me very curious. So what's the cause of such skewed wealth distribution? So eventually, I realized this actually has a lot to do with the crowd interaction.

So the same mechanism we looked at earlier, the agents have the feedback loop. That's the reason of the cause of this type of power law. In nature, you don't have that kind of a feedback mechanism. Once you are born, you grow to a certain height. Your height doesn't change. But in social phenomena, that keeps changing. So agents have different powers influencing the market in financial market. The rich can get richer. So we need to model each agent differently. So we call it heterogeneous agents.

So for a random variable x, the normal distribution we're all familiar with is the e to the x minus the mean divided by 2 sigma squared. But for power law, people, a lot of times, get confused about the terminologies. The Pareto law talks about the cumulative density function of the probability. And Zipf's law talks about the rank of that outcome. So basically, the random variable x versus the rank of X, which is more displayed in this chart.

But they are all related. When we normally look at the power law in a probability density function, that behaves like this versus the normal distribution. So even distribution, of course, is a delta function. So the parameters of these three different ways of representing power law, they are all linked by that formula. So power law probability density function is C over x to the power of a. a is related to the cumulative density function, power k, as well as the Zipf's law of parameter b.

So if we look at the feedback mechanism, we can derive that the agents with more power, in this case, which is represented by a, varying over time, can gain more power when they bet in the right direction, when the observation shows their betting is in the right direction. So that's how rich get richer. So that's how concentration of power is formed. Super agents will be born, in this case.

The system may or may not become stable when that happens. And I mean, this actually can be expanded into a discussion of entropy. What is order? What is stable? But this is beyond the scope of this lecture. Just to come back, this is all relevant to portfolio management. So I want to summarize, in this page, of what we learned today.

So portfolio management is about how to size each investment. You need to clarify your objectives. You need to know your loss tolerance. Diversification helps. But you need to rebalance. Volatility is not risk. Sharpe ratio doesn't capture skill nor provide sizing. We should use the new ratio I presented to you.

Capital market assumptions are not reliable. And the crowding behavior can push markets to extremes. You should pay attention to powerful super agents. Who are the most powerful super agents you can think about? Anybody has any idea in the market?

STUDENT: The Fed.

JAKE XIA: Yes, the Fed. Yes. So when you invest in the market, you have to pay attention because they have such a big power to change the direction of markets. And in the old days, some hedge funds have more power. But I would say, nowadays, it's definitely the governments. So you need to understand the key drivers of your portfolio.

So I want to stop the lecture here. I want to take some questions. Then I will save a couple of minutes to talk about the investment game. But before I get there, let's open up for questions.

I packed in a lot of content for today. I know. The idea is not necessary to ask you to follow in details because some of the math you probably need to derive it yourself to get a feel. But I want to give you the concepts of what the portfolio construction is all about. It's about sizing of investments. Sizing is about comparison about investments. So how to compare them, that's what it really comes down to. So for that, please, questions. Yes?

STUDENT: [INAUDIBLE] that can influence investments is taxes that you have to pay on capital gains or how long you own something can vary [INAUDIBLE] certain types of investments might be more tax sensitive than others. Is that something that is considered heavily in terms of investment supports [INAUDIBLE]?

JAKE XIA: Yeah, so that's a great question. Endowments are tax free, by the way. So don't consider that. But if you run a family office, you have to be very careful to plan your capital gains. So the family offices, or private investors, tend to invest more in long term private equity because they don't have to realize those gains. They invest less in public equity for the tax reason. So it is a very important parameter. Yeah. Any other questions? Yes?

STUDENT: What is the best risk measure? What should you measuree risk with?

JAKE XIA: So I still think we should try to understand the potential loss or expected loss-- so that's the L I discussed earlier-- because people tend to think gain and loss are symmetrical. But normally, it's not the case because you can construct your positions of very asymmetrical payouts. So you need to understand, when you buy an option, of course, your maximum loss is the option premium you paid. It's very clear.

But when you buy a stock, that may not be so clear. So you need to understand how much the stock can lose. It can move down. And what's the likely range it can move down to and those kind of things. So I think the risk measurement should be shifted or focused more on the downside protection. Understand your downside, which is really expected loss.

STUDENT: [INAUDIBLE] the question expected loss versus worst-case loss. How do you--

JAKE XIA: Yeah, those are two different things. So roughly, you can think about worst loss may be a few standard deviations away. I mean, expected loss probably is really the weighted average of outcome times the payout, that kind of thing. So you cannot construct your portfolio all based on worst case. Then you will not be doing anything. So that's why expected loss is probably a better way to measure for sizing your investments. But you need to be aware of your worst loss, yeah, in case something happens. Yeah? Yes?

STUDENT: I was going to say, going off the worst-case loss, [INAUDIBLE] implement stop losses? How effective have you found those? [INAUDIBLE] as well?

JAKE XIA: Yeah. So yeah, I think it's a very good question both of you brought up. Stop loss is an interesting concept. So stop loss typically is not set at the worst loss. It's set usually quite close, probably even closer than the expected loss. If you're trading at the hedge fund platforms, they give you a very tight stop loss for the reason that they try to artificially construct an asymmetric payout for the portfolio managers or the traders.

When you lose a little bit, they cut your position in half. Then you lose another few percentage points, cut another half, then get you out. So when you make money, they try to let you ride on the winds to create that kind of asymmetric optionality. But that forces you to pick investment that has that kind of a probability distribution.

So if you think an investment has a worst case loss which is very bad, but has a very low probability, let's just say, you need to think about if that-- almost like the firm takes that risk for you because they are responsible to cut you out. But if you think the upside is very big, then you should take that bet. You get it. Yeah.

STUDENT: [INAUDIBLE]

JAKE XIA: Yeah, right. Yeah, that's a lot of the issues with hedge funds. They get a cut on the profit, but they don't bear the downside responsibilities. So they are long a free call option. Yeah. So that's why a lot of people want to be a hedge fund manager.

So I will just touch upon the investment game. We already talked about it. When we pass November 15, you should keep track of G and L numbers and let us know how you performed. And I will try to verify. But last slide, I want to just coming back to the point, a lot of people ask how to become a hedge fund manager. Maybe I want to poll the audience here a little bit.

Is there a value, you think, from that trading or investment game to do some paper trading games? Or do you think you want more lectures on the research process, how to do data collection, signal generation, backtesting, and all the things and the portfolio optimization, as we talk about today? Or some people say, I just want to learn more about how to become a manager, how to do fundraising, how to build a team, how to negotiate the terms and how to assemble systems and a back office, legal taxes, and so on.

So I'm actually thinking maybe there's something we, as an organization, or some other people should really just help the younger potential PMs a little more to incubate them to become a better fund manager. If any one of you think that's something valid, this idea, you can talk to me, email me, or just come to me after class. Maybe there's something beyond what we talk in this class that could be useful to any one of you. So with that, thank you very much.