18.642: Topics in Mathematics with Applications in Finance

TR 2.30-4, 32-124

Instructors: Peter J Kempthorne, Jake Xia, Vasily Strela

Web Page: https://canvas.mit.edu/courses/27302

Office hours: T 4:00-5:30, 2-242 or by Appointment

18.642: Key Facts

- The purpose of the class is to expose undergraduate and graduate students to the mathematical concepts and techniques and communication used in the financial industry.
- Mathematics lectures will be mixed with lectures illustrating the corresponding application in the financial industry.
- MIT mathematicians will teach the mathematics part while industry professionals will give the lectures on applications in finance.
- Prerequisites: 18.03 (or approval of instructor), 18.05 or 18.440, 18.06
- Prior knowledge of economics or finance is not required but may be helpful for some lectures.
- The class will have 8 assignments covering math lecture topics and preparation of a mid-semester group project (a lecture note and in-class presentation). Instead of a final exam, there is a final paper on a math finance topic of the student's choice.
- The grade will be 40% based on the homework, 20% based on group presentations, 10% based on participation, and 30% based on the final paper.
- The class is 12 credits.

18.642: Approximate Schedule of Lectures

9/5		Introduction. Financial terms and concepts. Bond Math	Peter, Jake, Vasily
9/10	Math	Linear algebra	Peter
9/12	Applications	Quantitative Equity Investing	Jeff Shen, BlackRock (Zoom)
9/17	Math	Probability	Peter
9/19	Applications	Principal Component Analysis in Finance	Stefan Andreev, Two Sigma
9/24	Math	Stochastic processes I	Peter
9/26	Applications	Linear Rates Products and Models	Andrew Gunstensen, Mizuho
10/1	Math	Regression Analysis	Peter
10/3	Math	Regression Analysis: Hypothesis Testing for Parameters and Submodels	Peter
10/8	Applications	Compressing Derivative Portfolios	James Shepherd, Quantile Technologies (LSEG)
10/10	Math	Time Series	Peter
10/17	Math	Stochastic processes II	Peter
10/22	Applications	Portfolio Management	Jake
10/24	Application	Building the first federally (CFTC) regulated exchange dedicated to trading on events	Tarek Mansour and Luana Lopes Lara, Kalshi.com
10/29	Presentations	Group Project Presentations	Student Presentations
10/31	Presentations	Group Project Presentations	Student Presentations
11/5	Applications	The Spectrum of Systematic Trading Strategies in Liquid Instruments	Ross Garon, Millennium
11/7	Applications	Applying data science and artificial intelligence to managing biomedical portfolios	Andrew Lo, MIT
11/12	Math	Volatility Modeling	Peter
11/14	Applications	Black-Scholes formula, Risk neutral valuation	Vasily
11/19	Math	Stochastic calculus	Peter
11/21	Math	Stochastic differential equations	Peter
11/26	Applications	Introduction to Machine Learning	John Hull, University of Toronto
12/3	Math	Portfolio Theory	Peter
12/5	Math	Factor Models	Peter
12/10	Presentations	Final paper presentations. Conclusions	Student Presentations. Peter, Jake, Vasily

Popular Themes for the Final Paper

1. Portfolio construction and optimization
2. AI techniques for trading and forecasting
3. Risk Neutral valuation and hedging
4. Financial time series analysis and trading strategies
5. Crypto markets and strategies
6. Stress testing and regulatory modeling

Sample Titles of Final Papers

- The Analysis of COVID-19 Impact on Gold Volatility
- Yield Curve Parameterization for Fixed Income Investing
- Financial Time Series Forecasting With LSTM Networks
- Risk Neutral Measure in Asset Pricing
- Chaos Theory in Financial Markets
- Valuation of Stock Options Using Fractional Brownian Motion
- Principal Component Analysis on Foreign Exchange Rates
- The Reinforcement Learning for Options Decision Making
- Stress Testing with the CLASS Model
- A Megafund Approach to Pediatric Oncology Drug Development
- Deep Learning Techniques on Profitable Stocks Recommendation
- Pricing Ethereum using Autoregressive Moving Averages
- Comparison of Efficient Frontiers and Market Portfolios for Modern Portfolio Theory and Post-Modern Portfolio Theory
- Bayesian Deep Learning for Volatility Estimation
- Does ESG Impact a Company's Success?
- Assessing the Random Walk Hypothesis

Introduction to Financial Markets and Investment Strategies

Jake Xia

MIT Course 18.642

September 5, 2024

Markets:

Local Exchanges, OTC, ECN

Products:

Money, FX Currencies

Stocks, Equity Indices, ETFs
IPOs, Primary/Secondary

Loans, Bonds, Credit Products,
Interest Rates
Commodities
Real Estate, Mortgages, ABS

Options, Futures, Forwards, Swaps,
Derivatives, Structured Products

Participant Types:
Commercial Banks: deposits, lending

Investment Banks:
- Equity
- Fixed Income
- IBD, Corporate Finance, Raise Capital

Asset Management (E&F, Pension, etc)
Wealth Management (family offices)

Hedge Funds
Retail Investors
Central Banks
Corporates

Objectives:

Maximize gain while minimize loss

Investors—have money, need return
Borrowers—need money, have risks

Trade between lender and borrower is the main driving force.

Example: LP vs GP

Trader Types:

1. Hedger: reduce risk

2. Market Maker: earn bid and offer

3. Proprietary Trader and Fund Portfolio Manager: seek return by taking risk

Financial Mathematics:

1. Pricing Models:

    relative value, arbitrage-free.

2. Risk Management:

    sizing, portfolio construction, capital allocation, leverage, liquidity, counterparty, human risk aversion, greed/fear.

3. Trading Strategies:

    "Holy Grail" strategy, "perpetual money machine", Robo-Trader.

Do people always learn from their experiences by historical pattern extrapolation?

Efficient Market Theory and behavioral finance.

Zero-sum game?

Investment Strategies
--- some important choices to make

Internal (direct)

vs

External (Fund of funds)

Public (equity, bonds, FX, commodity)

vs

Private assets (PE, VC, RE, NR)

Passive (beta benchmarks)

vs

Active (alpha)

Systematic (quants)

vs

Discretionary (fundamental)

Deterministic

vs

Statistical

Trend Following

vs

Mean Reversion

Short Term (fast, liquid)

vs

Long Term (slow, illiquid)

Value: V(0)>>P

vs

Growth: V(T)>>V(0)

History (repeating patterns)

vs

Future (extrapolation)

Understand your own objectives and constraints.

Pick an area of what you are the best at, vs the rest of the market.

Find the suitable strategy for you.

Math is a powerful tool. That is your edge!

Discounting and Bond Math

Vasily Strela
MIT Course 18.642
September 5, 2024

Time Value of Money

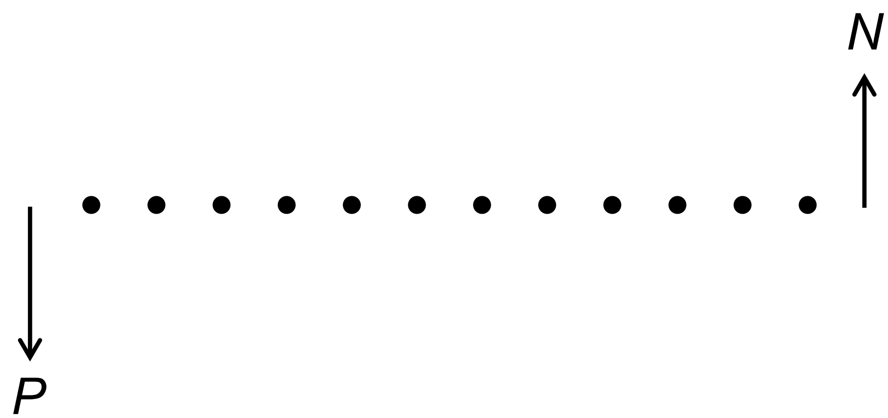
- Assume constant interest rate r
- **Compounding (interest earns interest)**
  - In 1 year *$1* investment grows to $1+r$. In two years to $(1+r)^2$. In n years to $(1+r)^n$.
  - If reinvested m times a year *$ 1* investment grows to $(1+r/m)^{mn}$ in n years.
  - If compounded continuously *$ 1* investment grows to e^{rt} in time t.
  - Read more at *https://en.wikipedia.org/wiki/Compound_interest*

- **Discounting**
  - Assuming annual compounding, *$1* received in n years is worth $1/(1+r)^n$ now

- **Present value (PV)** of *$1* received at future time t is called **discount factor** for time t.

Zero Coupon (Discount) Bond

- Borrow *$P* now. Pay **notional** payment *N* at time T (**maturity**).

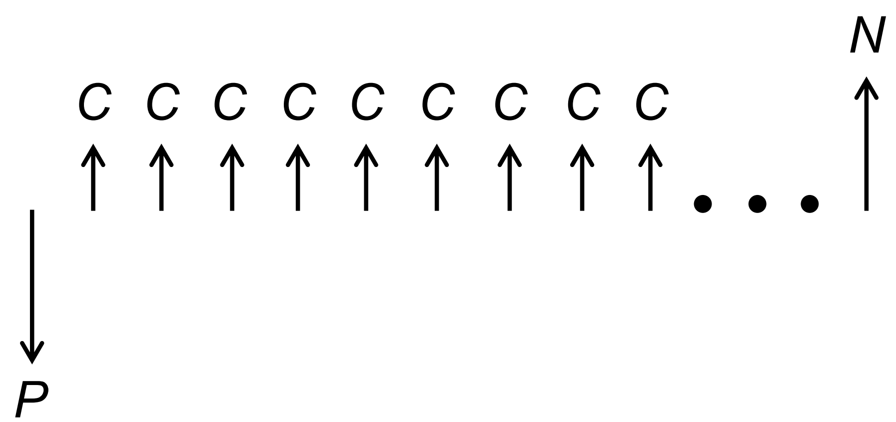


- Price *P* of a zero coupon bond maturing in *n* years is

$$P = \frac{N}{(1+r)^n}$$

Price of a Coupon Bond

- Borrow *$P* now. Pay periodic **coupons** *C* and notional payment *N* at the end.



- Assuming annual coupons, price *P* of a bond is

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^n} + \frac{N}{(1+r)^n} =$$

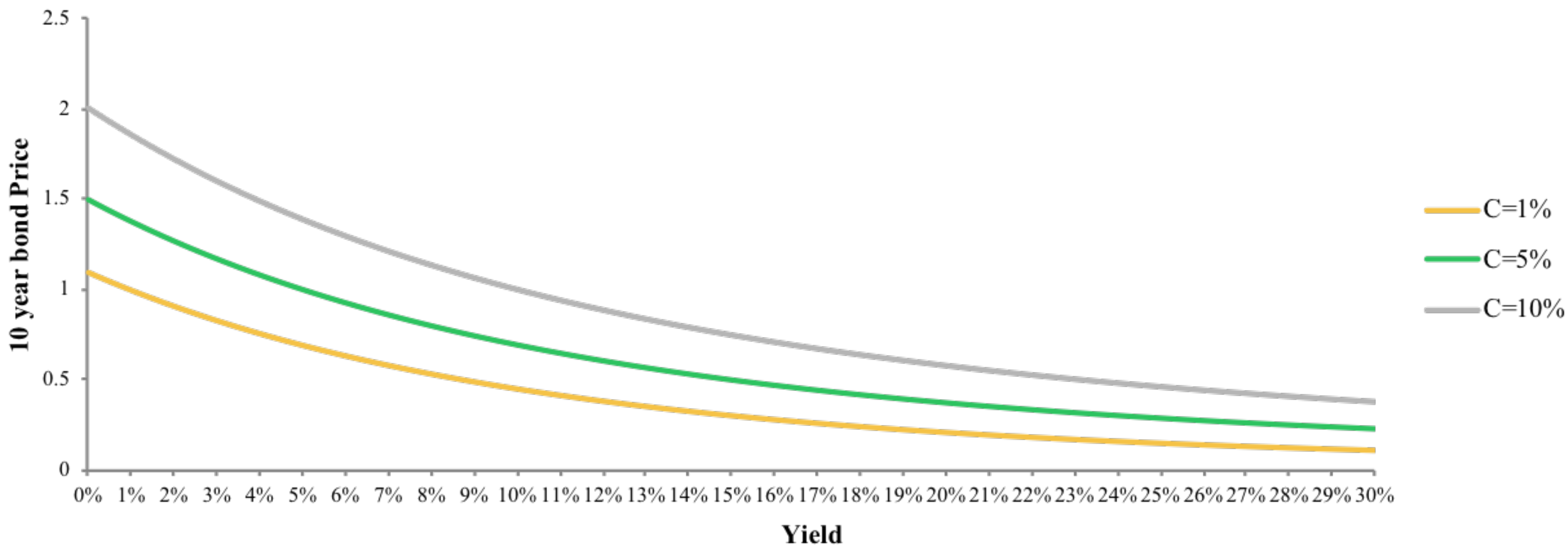
$$C\,\frac{(1+r)^n - 1}{r(1+r)^n} + \frac{N}{(1+r)^n}$$

Yield of a Bond

- **Yield** is a discount rate needed to obtain current price of a bond
- Yield of a par bond is equal to coupon
- Yield of a *n* year zero coupon bond is

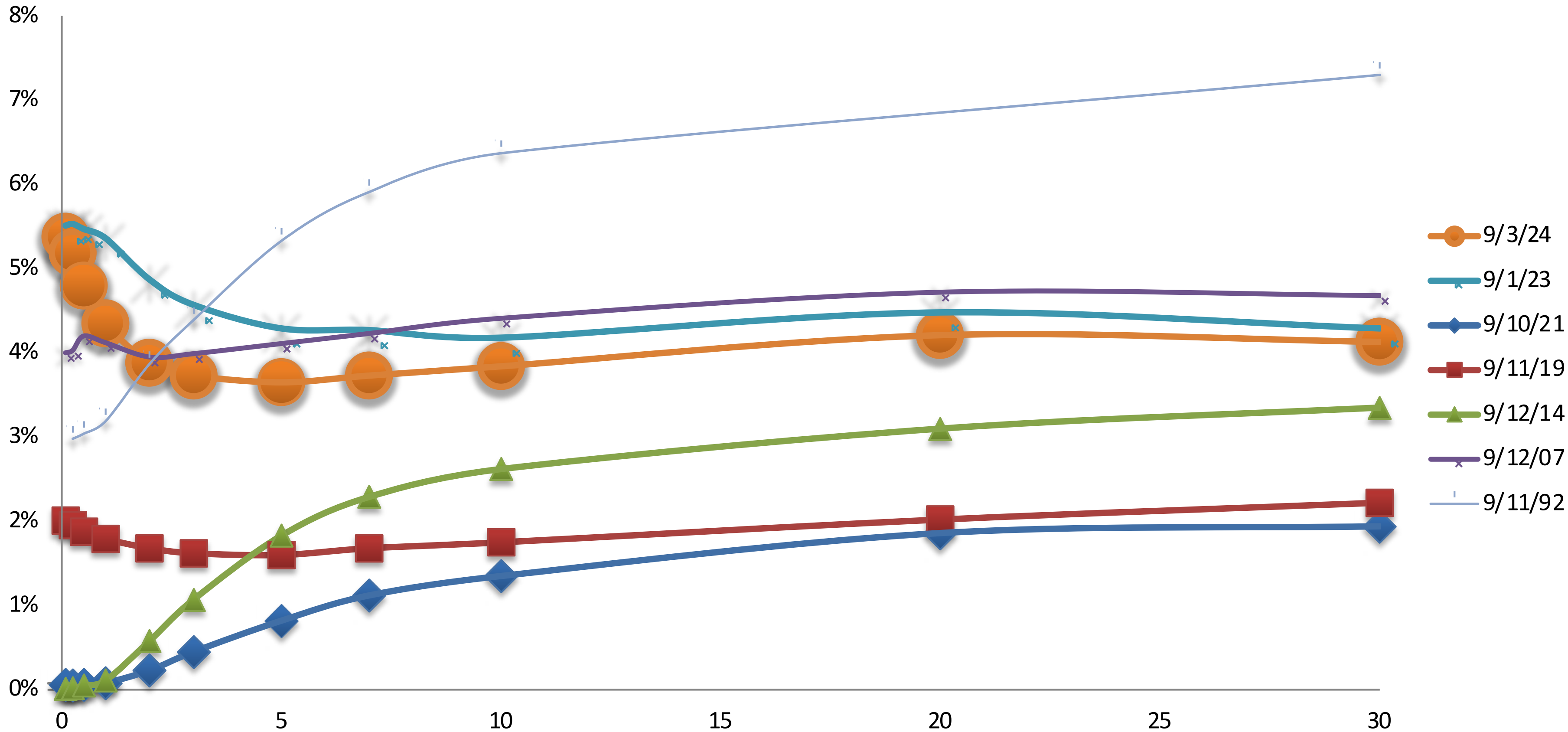
$$y = \frac{1}{\sqrt[n]{P}} - 1$$

- Price *of a bond decreases with yield and increases with coupon*



Yield Curve

- **Term Structure** of yields: Bonds of different maturities may have different yields.
- Discount factors for all times can be bootstrapped from the yield curve



Source: treasury.gov

Bond Price Sensitivities

- **Duration**: $D = -\frac{1}{P}\frac{dP}{dy}$

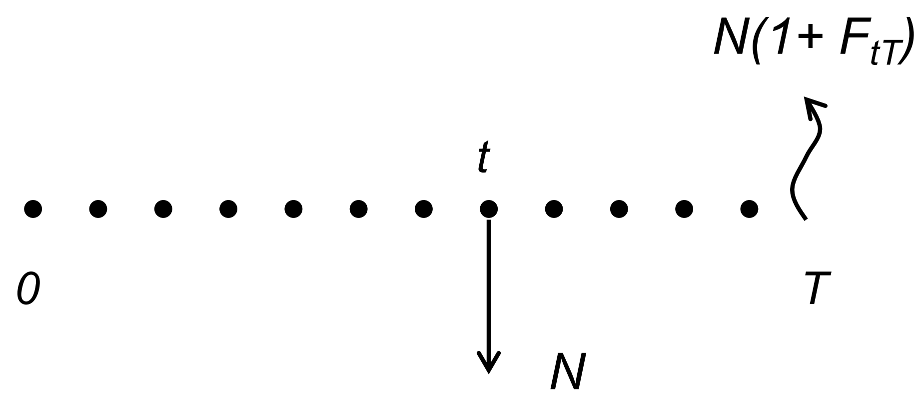
- Duration of a zero coupon bond: $D_Z = \frac{T}{1+y}$

- Duration of a "continuous" par bond: $D_p = \frac{1 - e^{-yT}}{y}$

- **Convexity**: $D = -\frac{1}{P}\frac{d^2P}{dy^2}$

Forward Rate

- **Forward Rate Agreement (FRA):** contract that determines the rate of interest F_{tT} to be paid at time T on an obligation beginning at a future start date t



$$1 = \frac{1}{\Lambda_t}(1 + F_{tT})\Lambda_T$$

$$F_{tT} = \frac{\Lambda_t}{\Lambda_T} - 1$$