Counterparty Risk Optimization

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2024-10-08

Contents

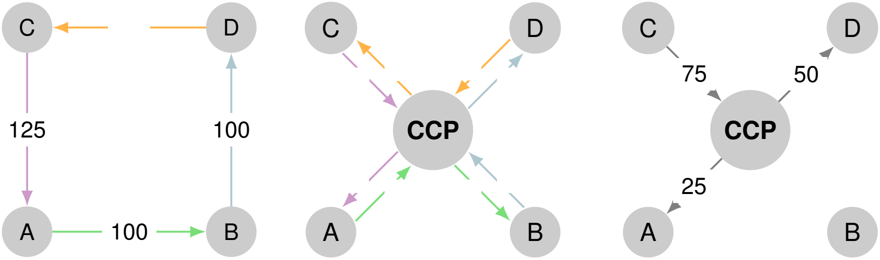
Derivative Risks

- **market risk**: arises from market movements
- **credit risk**: arises from debtors non-payment of an obligation
- **operational and legal risk** arises from people, systems and events
- **liquidity risk** arises when transaction cannot be executed at market price or from an inability to fund outgoing cashflows
- **counterparty risk** arises from the potential exposure of the market value of transactions when a counterpart defaults

Mitigation of Counterparty Risk

- **netting** : net together offsetting payments
- **hedging** : create new trades to offset the risk
- **collateralisation**: collection of collateral or margin payments
- **other contractural clauses** such as resetting mark-to-market (mtm) value or break clauses
- **central clearing counterparties (CCPs)**

Central Clearing Counterparties (CCPs)



bilateral trading   novation to ccp   ccp nets positions

The CCP facilitates offsetting positions to be netted against eachother

intuition: total counterparty risk $\sim \sum_{counterparties} \|position\|$

Risk Measures

Risk should measure

- *size* of the potential loss
- *probability* of that loss occurring

A function $\rho : X \rightarrow \mathbf{R}$ is *coherent* if it satisfies the following properties

- **normalised** $\rho(0) = 0$
- **homogeneity** $\rho(\alpha X) = \alpha\rho(X)$
- **translation invariance** if A is a deterministic portfolio with guaranteed return α then $\rho(X + A) = \rho(X) + \alpha$
- **monotonicity** if $X_1 \leq X_2$ in all scenarios then $\rho(X_1) \leq \rho(X_2)$
- **sub-additivity** $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

Value at Risk and Expected Shortfall

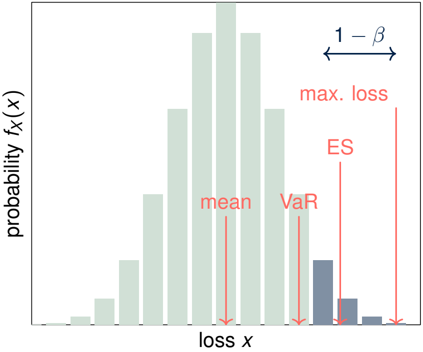
Value at Risk (VaR) is the maximum expected loss at a specific confidence level(β) over some time horizon

$$\int_{-\infty}^{VaR_\beta} f_X(x)\mathrm{d}x = \beta$$

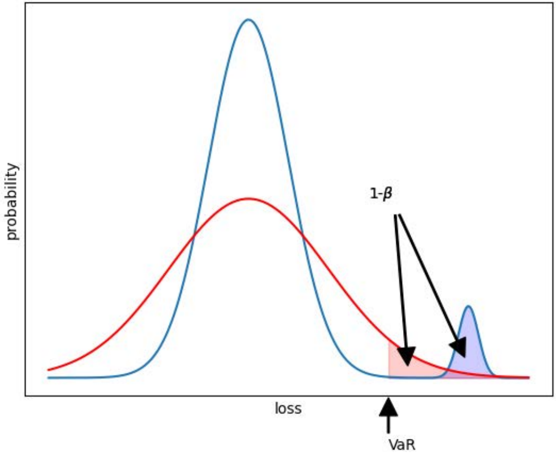
Expected Shortfall (ES) is the expected loss conditional on having exceeded VaR

$$ES = \frac{1}{1-\beta}\int_{VaR_\beta}^{\infty} xf(x)\mathrm{d}x$$

$$ES \geq VaR$$



fat tails



both portfolios have the same VaR but the blue portfolio has a higher probability of a much bigger loss

Sub-additivity of VaR and ES

A		B	
Loss	Probability	Loss	Probability
100	4%	100	4%
0	96%	0	96%
$VaR_{0.95} = 0$		$VaR_{0.95} = 0$	
$ES_{0.95} = 80$		$ES_{0.95} = 80$	

A+B	
Loss	Probability
200	0.16%
100	7.68%
0	92.16%
$VaR_{0.95} = 100$	
$ES_{0.95} = 103.2$	

- VaR is not sub-additive
- ES is sub-additive

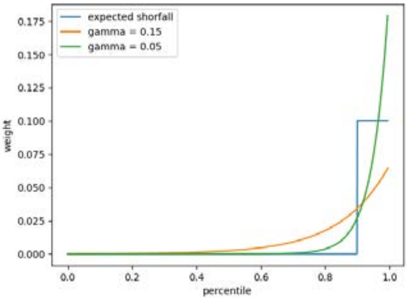
(at least in this case)

easy to find contrived examples where VaR is not sub-additive

hard to *prove* that Expected Shortfall is sub-additive

Spectral Risk Measures

A risk measure can be characterised according to the weights it assigns to percentiles of the loss distribution



- VaR assigns 100% to β percentile
- ES assigns equal weight to all percentiles > β
- exponential spectral risk $W = e^{-(1-q)/\gamma}$

a risk measure is sub-additive if *weight* is a non decreasing function of *percentile*

Time Horizon for VaR and ES

- $\Delta P_i \sim N(\mu, \sigma) \Rightarrow$

$$VaR = \mu + \sigma N^{-1}(\beta)$$

$$ES = \mu + \sigma \frac{e^{-Y^2/2}}{\sqrt{2\pi}(1-\beta}$$

where Y is the β percentile point of a standard normal distribution $N(0,1)$

- T-day VaR = 1-day $VaR * \sqrt{T}$
- T-day ES = 1-day $ES * \sqrt{T}$
- If the correlation between ΔP_i and ΔP_{i-1} is ρ, then variance of $\Delta P_{i-1} + \Delta P_i$ is

$$\sigma^2 + \sigma^2 + 2\rho\sigma^2 = 2(1+\rho)\sigma^2$$

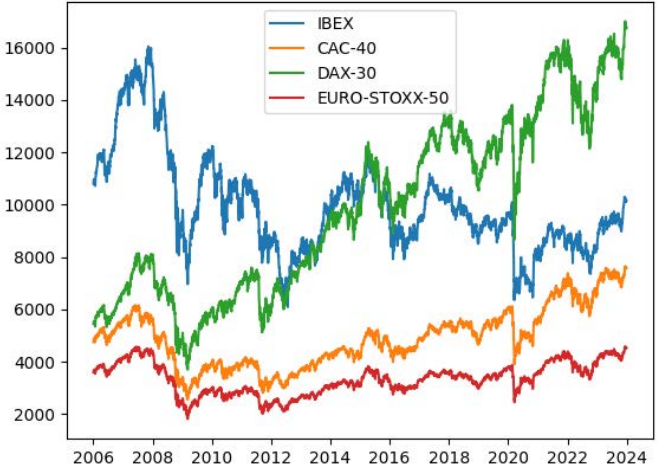
	T=1	T=2	T=5	T=10	T=50	T=250
$\rho = 0$	1.00	1.41	2.24	3.16	7.07	15.81
$\rho = 0.05$	1.00	1.45	2.33	3.31	7.43	16.62
$\rho = 0.1$	1.00	1.48	2.42	3.46	7.80	17.47
$\rho = 0.2$	1.00	1.55	2.62	3.79	8.62	19.35

effect of autocorrelation on ratio of T-Day VaR (ES) to One-Day VaR (ES)

	t	t+1	t+2	t+3
t	1.00	0.12	-0.01	-0.02
t+1	0.12	1.00	0.12	-0.01
t+2	-0.01	0.12	1.00	0.12
t+3	-0.02	-0.01	0.12	1.00

observed auto-correlations for EURO-STOXX

Historical Simulation



Historical Simulation (II)

v_i^j = value of j^{th} market variable on day i

δ_j = portfolio weight for j^{th} market variable

$\Delta P_i = \sum_j \delta_j \frac{v_i^j - v_{i-1}^j}{v_{i-1}^j}$ = change in value of the portfolio on day i

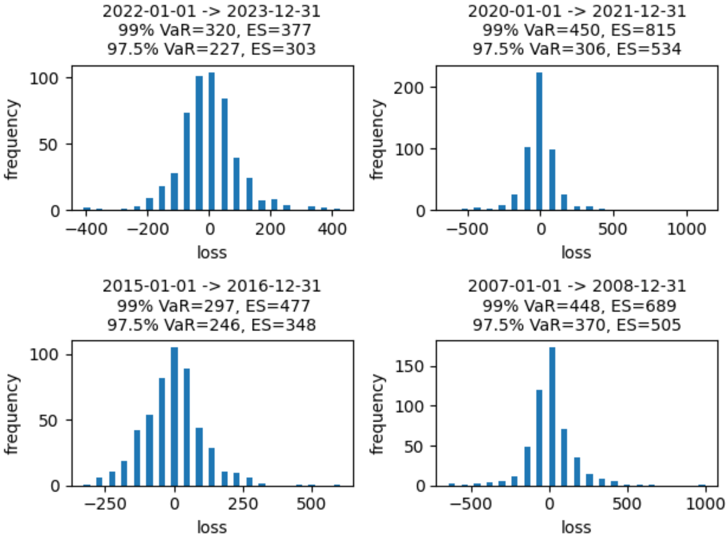
	CAC	DAX	IBEX	EUROSTOXX
2006-01-02	4757.54	5451.57	10814.60	3605.95
2006-01-03	4803.23	5496.46	10839.30	3638.42
2006-01-04	4838.52	5523.67	10898.90	3652.46
2006-01-05	4849.67	5526.41	10929.60	3661.65
2006-01-06	4867.15	5537.58	10929.60	3666.99
...	...	...	...	...

v_i^j

CAC	DAX	IBEX	EUROSTOXX
3000.00	4000.00	2000.00	1000.00

δ_j

Historical Simulation (III)



Historical Simulation (IV)

	2022-01-01	2020-01-01	2015-01-01	2007-01-01
12	226.69	305.96	246.47	369.99
11	229.28	309.30	254.17	372.76
10	235.26	344.38	257.59	380.05
9	237.44	351.35	260.55	382.57
8	239.70	369.55	268.87	392.87
7	239.89	393.51	286.93	400.31
6	316.83	397.21	291.90	421.37
5	320.18	450.41	296.98	448.43
4	341.13	596.02	335.56	489.14
3	362.01	712.32	452.86	592.39
2	370.56	800.91	508.75	654.86
1	436.24	1151.84	612.80	1017.71

biggest 12 losses from 500 in each time period

Model Building

$$\Delta P = \sum_{i=1}^{n} \delta_i \Delta x_i \quad \delta_i = \Delta P / \Delta x_i$$

$$\Delta x_i \sim N(0, \sigma_i) \Rightarrow \Delta P \; N(0, \sigma_P)$$

$$\sigma_P = \sqrt{\sum_i \sum_j \rho_{ij} \delta_i \delta_j \sigma_i \sigma_j}$$

$$\sigma_P^2 = \delta^{\mathbf{T}} \mathcal{C} \delta$$

$$VaR(T, \beta) = \sqrt{T} N^{-1}(\beta) \sigma_P$$

$$ES(T, \beta) = \sqrt{T} \frac{e^{-N^{-1}(\beta)}}{\sqrt{2\pi}(1-\beta)} \sigma_P$$

	CAC	DAX	IBEX	EUROSTOXX
CAC	0.000102	0.000097	0.000080	0.000094
DAX	0.000097	0.000108	0.000082	0.000096
IBEX	0.000080	0.000082	0.000089	0.000076
EUROSTOXX	0.000094	0.000096	0.000076	0.000096

variance covariance $\mathcal{C}$ matrix for indices based on period 2022-01-01 to 2023-12-31

$VaR(1, 99\%) = 225 \;\; ES(1, 99\%) = 257$

Probability of exceeding VaR on m or more days from a sample of n is

$$\sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^k$$

$n = 500, m = 12, p = 0.01 \Rightarrow P = 5.2\%$

Fundamental Review of the Trading Book

- **Basel I** based on VaR(10 days, 99% confidence)
- **Basel II.5** based on Stress VaR
- **FRTB** based on ES(variable horizon, 97.5% confidence)
  - $99\% \mathit{VaR} = \mu + 2.326\sigma$
  - $97.5\% \mathit{ES} = \mu + 2.338\sigma$

Expected Shortfall as as Optimization Problem

- $X \in \mathbb{R}^n$ is the set of available portfolios
- $Y \in \mathbb{R}^m$ is the set of possible market changes
- $f(x, y) \in \mathbb{R}$ is the loss associated with a given portfolio $x \in X$ and given market change $y \in Y$
- $\rho(y)$ is the density of the probability distribution of y
- $\psi(x, \alpha) = \int_{f(x,y) \leq \alpha} \rho(y) dy$ is the probability of $f(x, y)$ not exceeding α

with above definitions, we can rewrite

- $VaR_\beta(x) = min\{\alpha \in \mathbb{R} \text{ s.t. } \psi(x, \alpha) \geq \beta\}$
- $ES_\beta(x) = \frac{1}{1-\beta} \int_{f(x,y) \geq VaR_\beta(x)} f(x, y)\rho(y) dy$

Expected Shortfall as as Optimization Problem II

$$F_\beta(x,\alpha) = \alpha + \frac{1}{1-\beta}\int_{y\in\mathbb{R}^m} [f(x,y)-\alpha]^+ \rho(y)dy$$

where

$$[t]^+ = max(t,0)$$

- As a function of α, F is convex and continuously differentiable
- $ES_\beta = \min_{\alpha\in\mathbb{R}} F_\beta(x,\alpha)$

and the portfolio $x \in X$ with the minimum ES can be found by solving the optimization problem

$$\min_{(x,\alpha)\in X x \mathbb{R}} F_\beta(x,\alpha)$$

Expected Shortfall as as Optimization Problem III

$$F_\beta(x,\alpha) = \alpha + \frac{1}{1-\beta}\int_{y\in\mathbb{R}^m}[f(x,y)-\alpha]^+\,\rho(y)dy$$

$$\frac{\partial}{\partial\alpha}F_\beta(x,\alpha) = 1 + \frac{1}{1-\beta}(\psi(x,\alpha)-1) = \frac{1}{1-\beta}(\psi(x,\alpha)-\beta)$$

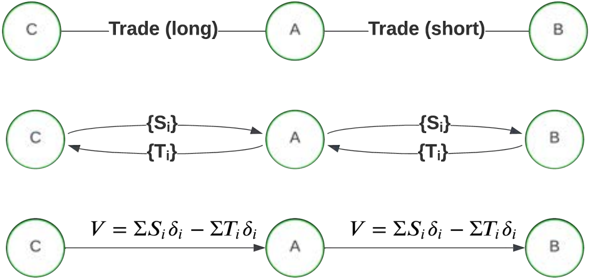
evaluating F_β at the point where $\psi(x,\alpha)=\beta)$. The integral equals

$$\int_{f\geq\alpha_\beta}[f(x,y)-\alpha_\beta(x)]\,\rho(y)dy = \int_{f\geq\alpha_\beta} f(x,y)\rho(y)dy - \alpha_\beta(x)\int_{f\geq\alpha_\beta}\rho(y)dy$$
$$= (1-\beta)ES_\beta - \alpha_\beta(1-\psi(x,\alpha_\beta))$$

So

$$min_{\alpha\in\mathbb{R}}F_\beta(x,\alpha) = \alpha_\beta + \frac{1}{1-\beta}[(1-\beta)ES_\beta - \alpha_\beta(x)(1-\beta)] = ES_\beta(x)$$

Counterparty Risk and Margin for Derivatives



To mitigate the risk that a counterparty may not pay S_{next} or T_{next} at the next cashflow payment date, *variation margin* $VM = V$ is collected on a daily basis

If a counterparty defaults then they stop posting VM. During the time taken to rehedge or unwind that position, the value V moves due to market movements. This potential loss is covered by collecting *initial margin* (IM)

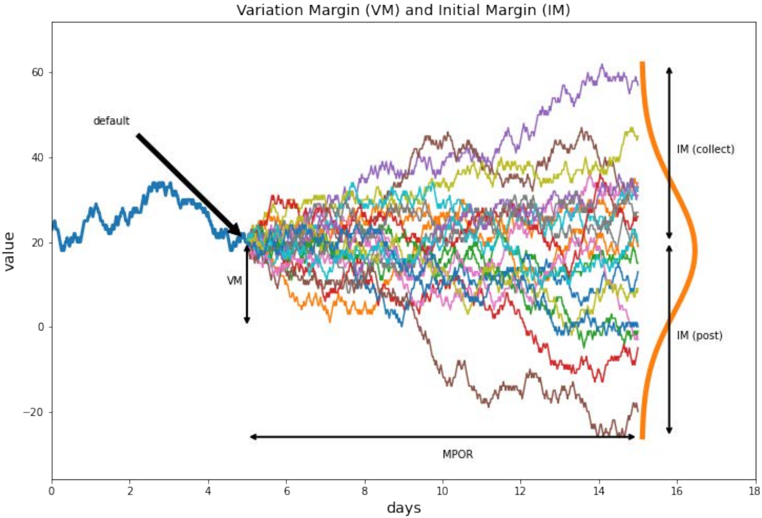
Margin

Margin is a form of collateral posted between counterparts to cover potential losses from derivatives

- *Variation Margin* (VM) covers current present value
- *Initial Margin* (IM) covers potential valuation changes due to market movements between when a counterpart default and when the party can exit the position, at time $t = \delta t$

δt is called the *Margin Period of Risk* (MPOR)
Margin is *posted* by the counterparty to the party which *collects* the margin

Margin


Variation Margin (VM) and Initial Margin (IM)
default
VM
IM (collect)
IM (post)
MPOR
value
days
60
40
20
0
−20
0
2
4
6
8
10
12
14
16
18

ISDA SIMM

Standard Initial Margin Model (SIMM)

- covers non centrally cleared (i.e. *bilateral*) derivative contracts
- has been phased in since 2016 as a result of Basel III rules
- is an exposure based proxy for 10-day VaR at 99% confidence i.e $\delta t = \frac{14}{365}, \beta = 0.99$

Assume we have a portfolio facing some counterparty with value $P(t)$ and that counterparty defaults at time $t = 0$.

- variation margin covers the value at $t = 0$ i.e. $VM(0) = P(0)$
- assume change in P only due to market perturbations
- ignore path dependence, cashflows occurring in $(0, \delta t)$, discounting effects

$$P(\Delta t) \quad \approx P(\mathcal{M}(0) + \Delta\mathcal{M}) \quad \approx P(0) + \delta\Delta\mathcal{M} + \frac{1}{2}\gamma(\Delta\mathcal{M})^2$$

where $\mathcal{M}(t)$ is the state of the market at t and $\Delta\mathcal{M}$ is some perturbation in the market between $t = 0$ and $t = \Delta(t)$

ISDA SIMM II

Assume market risk factor movements are jointly Gaussian with zero mean

$$\Delta\mathcal{M} \sim \mathcal{N}(0, \mathcal{C})$$

where $\mathcal{C}_{ij} = \sigma_i \rho_{ij} \sigma_j$ is the covariance matrix.
$P(\Delta t)$ is Gaussian with mean,μ and variance σ^2

$$\mu = P(0) + \frac{1}{2} Tr(\gamma\mathcal{C})$$
$$\sigma^2 = \delta^T \mathcal{C} \delta + \frac{1}{2} Tr((\gamma\mathcal{C})^2)$$

trying to find an amount *IM* such that

$$\mathbb{P}(P(\Delta t) - P(0) < IM) = 0.99$$

which implies

$$IM \approx 2.326\sqrt{10}\sqrt{\delta^T \mathcal{C} \delta} + \frac{1}{2}\sqrt{10}\, Tr(\gamma\mathcal{C}) + \lambda\sqrt{\frac{1}{2} Tr((\gamma\mathcal{C})^2)}$$

ISDA SIMM Practical Matters

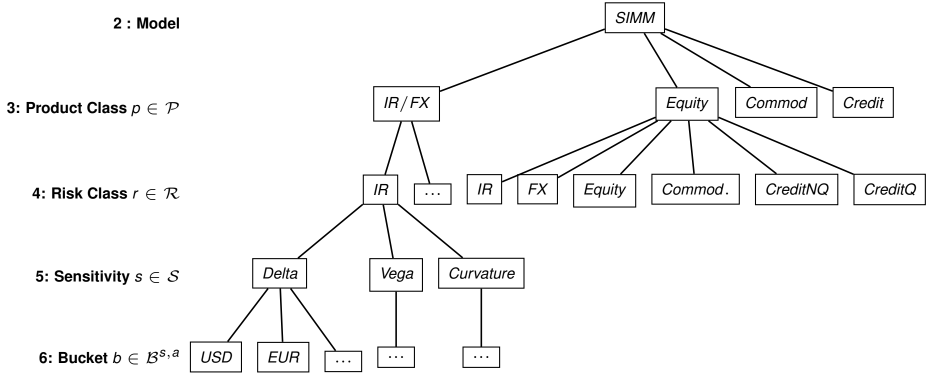
Problems with the formulation so far

- too many risk factors in $\mathcal{M}$. ie $\mathcal{C}, \delta, \gamma$ are too big
- γ is hard to compute

more simplifying assumptions

- $\gamma \propto \frac{\mathcal{V}}{T\sigma_N}$ where $\mathcal{V}$ is the vega of the portfolio
- risk factors (components of $\mathcal{M}$) can be grouped into *buckets* and *risk classes* (eg IR, FX, Equity)
- the portfolio can be separated into product classes with no netting benefit between them
- correlations and other parameters are recalibrated annually
- additional terms address *thresholds* and *concentration risk*.
- some variations of scope between regulators. The posted IM is the maximum SIMM for any regulator

SIMM Tree



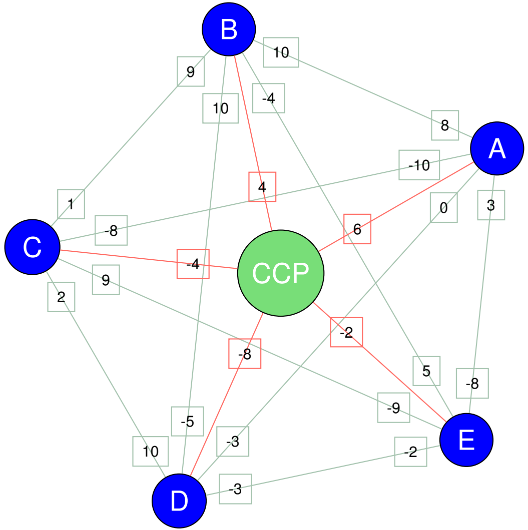
2 : Model
3: Product Class $p \in \mathcal{P}$
4: Risk Class $r \in \mathcal{R}$
5: Sensitivity $s \in \mathcal{S}$
6: Bucket $b \in \mathcal{B}^{s,a}$
SIMM
IR / FX
Equity
Commod
Credit
IR
...
IR
FX
Equity
Commod.
CreditNQ
CreditQ
Delta
Vega
Curvature
USD
EUR
...
...
...

Large Positions

Various approaches are used to address (concentrated) positions

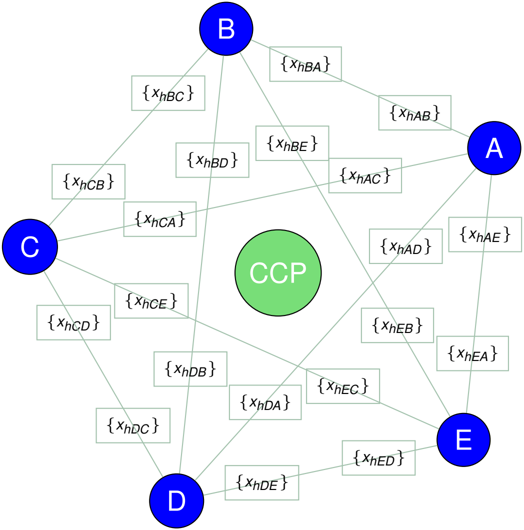
- adjust the Margin Period of Risk $\Delta T = \Delta T_0 \max(1, \frac{N}{N_0})$. For $N > N_0$, $IM \sim N^{\frac{3}{2}}$
- Addons $W(\delta)\ \delta \mathcal{C} \delta^T$ where W is some monotonically increasing function, eg piecewise linear

Multilateral Margin Optimization



network of banks with cleared and bilateral positions

Multilateral Margin Optimization (II)



goal of multilateral optimization : find hedge trades to reduce the total initial margin in the system

The Margin Minimization Problem

$\mathcal{H}$ is a set of hedges, $\mathcal{M}$ is a set of market risk factors, $\mathcal{P}$ is a set of parties, $\mathcal{D}$ is some risk metric

$$\text{minimize} \sum_{p\in\mathcal{P}} \left(\sum_{q\in\mathcal{P},q\neq p} SIMM_{p,q} + \sum_{ccp\in\mathcal{CCP}} IM_{p,ccp} \right)$$

$$\text{where } SIMM_{p,q} = SIMM_{p,q}(\mathcal{D}_{p,q}),\; IM_{p,ccp} = IM_{p,ccp}(\mathcal{D}_{p,q})$$

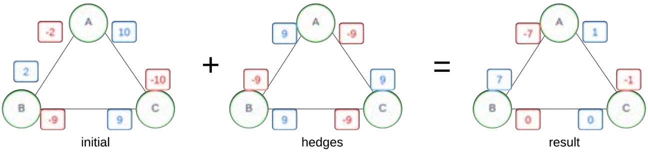
$$\text{such that } \mathcal{D}_{p,q,m} = \mathcal{D}^0_{p,q,m} + \sum_{h\in\mathcal{H}} x_{h,p,q}\mathcal{D}_{h,p,q,m} \quad \forall m \in \mathcal{M}$$

$$x_{h,p,q} = -x_{h,q,p} \quad \forall h \in \mathcal{H}, \forall p, q \neq p \in \mathcal{P}$$

$$\sum_{q\in\mathcal{P}} x_{h,p,q} = 0 \quad \forall h \in \mathcal{H}, p \in \mathcal{P}$$

$$\mathcal{T}^-_{p,q,m} \leq \sum_{h} x_{h,p,q}\mathcal{D}_{h,p,q,m} \leq \mathcal{T}^+_{p,q,m} \; \forall m \in \mathcal{M}, \forall p, q \neq p \in \mathcal{P}$$

Toy Model of Margin Minimization



Moving a fixed amount of some derivative around a 3-party system is guaranteed to satisfy

- party - counterparty symmetry: $x_{h,p,q} = -x_{h,q,p}$
- cashflow flatness: $\sum_q x_{h,p,q} = 0$

margin $\sum_{p,q} |x^0_{hpq} - x_{hpq}|$ is minimised when x is the median of x^0

Numerical Solutions of Constrained Optimization Problems

$$\text{minimize} \quad f_0(x) \text{ subject to } f_i(x) \leq b_i, \quad i = 1 \dots m$$

- solving convex optimization problems is much easier than general non-linear optimization problems
- convexity is equivalent to sub-additivity $f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y)$
- Minimizing ES is a piecewise linear problem
- *SIMM* is *nearly* convex
- Concentration addons are *potentially* convex
- the state of technology for solving convex optimisation problems is very mature
- but the ease of solution is highly dependent on problem specifics

Modelling Languages and Solvers

modelling languages *vastly* simplify the task of setting up an optimization problem

solvers perform the actual numerical steps

several commercial and open source modelling languages and solvers are available

```
import gurobipy as gp
from gurobipy import GRB
m = gp.Model()
x1 = m.addVar(vtype=GRB.INTEGER, name="x1", lb=0)
x2 = m.addVar(vtype=GRB.INTEGER, name="x2", lb=0)
m.setObjective(5*x1 + 8*x2, GRB.MAXIMIZE)
m.addConstr(x1 + x2 <= 5, "c1")
m.addConstr(3*x1 + 7*x2 <= 25, "c2")
m.optimize()
```

Solver output for a simple model

```
Gurobi Optimizer version 9.5.2 build v9.5.2rc0 (linux64)
Thread count: 24 physical cores, 48 logical processors, using up to 24 threads
Optimize a model with 2 rows, 2 columns and 4 nonzeros
Model fingerprint: 0x48b07709
Variable types: 0 continuous, 2 integer (0 binary)
Coefficient statistics:
  Matrix range     [1e+00, 7e+00]
  Objective range  [5e+00, 8e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [5e+00, 2e+01]
Found heuristic solution: objective 25.0000000
Presolve time: 0.00s
Presolved: 2 rows, 2 columns, 4 nonzeros
Variable types: 0 continuous, 2 integer (0 binary)
Found heuristic solution: objective 28.0000000

Root relaxation: objective 3.100000e+01, 2 iterations, 0.00 seconds (0.00 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0     0               0      31.0000000   31.00000  0.00%     -    0s

Explored 1 nodes (2 simplex iterations) in 0.01 seconds (0.00 work units)
Thread count was 24 (of 48 available processors)

Solution count 3: 31 28 25

Optimal solution found (tolerance 1.00e-04)
Best objective 3.100000000000e+01, best bound 3.100000000000e+01, gap 0.0000%
```

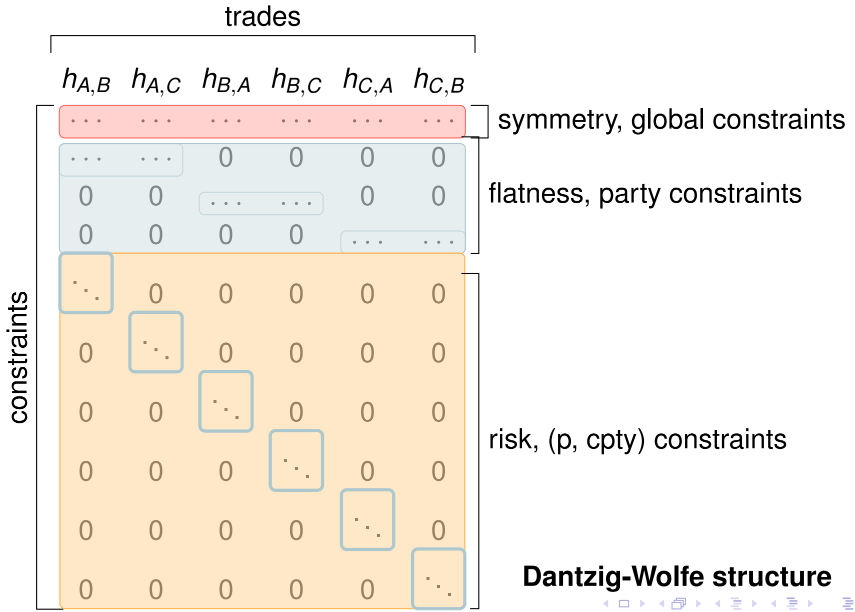
FX Delta Margin

- 25 parties
- 30 currencies
- cleared and uncleared Non Deliverable Forwards
- 1 maturity (short dated)
- $\sim 18,000$ variables
- $\sim 40,000$ constraints
  - pairwise pointwise risk (linear inequality)
  - pairwise IM increase (convex inequality)
  - notional efficiency (convex inequality)
  - symmetry (linear equality)
  - flatness (linear equality)
  - lot sizes (round notionals to nearest million) (integer)
- 1 convex objective : SIMM + Cleared IM

FX Delta Margin - Numerical Complexity

- inner-most step is Newton step
- inverting $\nabla^2 f$ is the limiting step
- matrix inversion $\sim n^3$ where $n \sim 40,000$ (10^{13} FLOPS)
- in fact we can do *far* better than this
  - positive semi definite $\implies$ Choleski Factorization
  - sparsity ($\ll 0.1\%$ of matrix is non-zero)
  - structure
    - small number of constraints bind multiple trades
    - vast majority ($36,000$) of constraints are risk constraints
    - each NDF trade only has exposure to 1 risk factor, implying a (block) diagonal (or banded) matrix

Block Matrix Structure



Dantzig-Wolfe structure

Nearly Parallel Risk

2 identical trades with exposure, $3E$ and E to 2 risk factors. Scaling everything by $3E$ can lead to unstable solutions

- $\begin{bmatrix} 1 & 1 \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{3} \end{bmatrix}$ has infinite number of solutions $y_1 = 1 - x_1$
- $\begin{bmatrix} 1 & 1 \\ 0.3333 & 0.333 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.333 \end{bmatrix}$ has a single solution $(0, 1)$
- $\begin{bmatrix} 1 & 1 \\ 0.3333 & 0.333 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.3333 \end{bmatrix}$ has a single solution $(1, 0)$
- $\begin{bmatrix} 1 & 1 \\ 0.3333 & 0.333 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.3 \end{bmatrix}$ has a single solution $(-110, 111)$ (infeasible if $x_i \geq 0$)

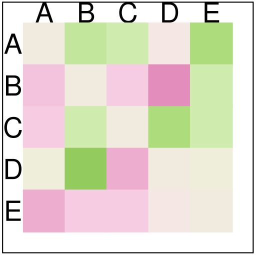
FX Delta Margin

Step I : solve the continuous problem

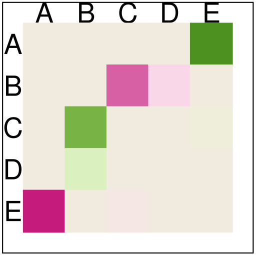
```
Threads: 48 physical cores, 96 logical processors,32 threads
Optimize a model with 114127 rows, 104378 columns
and 618202 nonzeros
Model fingerprint: 0xc3d79de8
Model has 661 quadratic constraints
Coefficient statistics:
Matrix range      [4e-04, 1e+01]
QMatrix range     [3e-01, 1e+00]
Objective range   [1e-02, 1e+00]
Bounds range      [1e-05, 2e+04]
RHS range         [1e-05, 6e+03]
Presolve removed 62099 rows and 55441 columns
Presolve time: 0.38s
Presolved: 53358 rows, 49577 columns, 473486 nonzeros
Barrier solved model in 58 iterations and 11.21 seconds
```

FX Delta Margin (5 parties)

5 party system, 1 risk factor, no constraints



initial risk

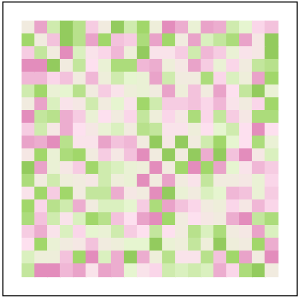


optimised risk

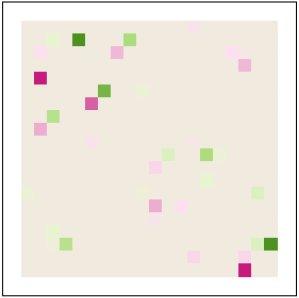
risk on each party is netted. Parties have *either* short or long positions, never both

FX Delta Margin (20 parties)

20 party system, 1 risk factor, no constraints



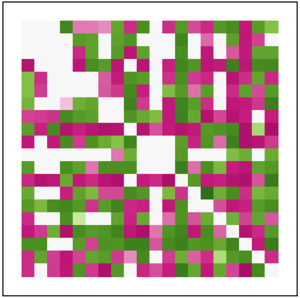
initial risk



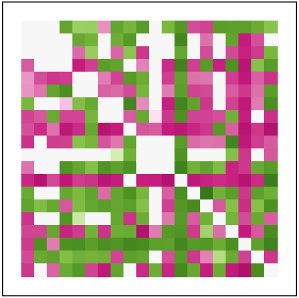
optimised risk

FX Delta Margin (Reality)

A real system with 20 parties and real constraints



initial risk



optimised risk

IR Margin Optimization

- 15 parties
- 3 currencies (USD, EUR, GBP)
- swaptions and cleared swaps
- multiple expiries and maturities
- 5000 variables, 10,000 constraints
- each trade has exposure to multiple risk factors (eg swap expiring in 12 years is exposed to 10Y Swap Rate and 20Y Swap Rate)

Other considerations

- feasibility (*does the solution satisfy all constraints*)
- optimality (*does it find a good IM saving*)
- nice
  - round numbers
  - small numbers of trades
  - trade packages
  - centrally cleared vs bilateral
- fair

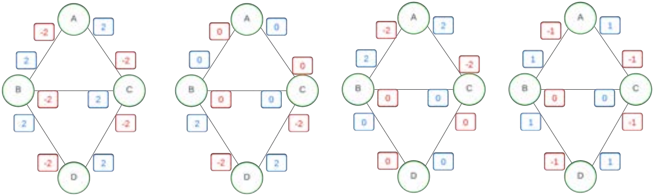
FX Delta Margin - Rounding

MIP problem so remove as many variables and constraints as possible first

```
Optimize a model with 49083 rows, 38685 columns
and 116685 nonzeros
Model fingerprint: 0xba7bdcd2
Variable types: 28935 continuous, 9750 integer (0 binary)
Presolved: 6068 rows, 5935 columns, 18230 nonzeros
Variable types: 2364 continuous, 3571 integer (30 binary)
Root relaxation: objective 3.348605e+03, 6049 iterations,
0.13 seconds

Explored 5211 nodes (149715 iterations) in 57.36 seconds
Best objective 8.262033036720e+03
```

Fairness



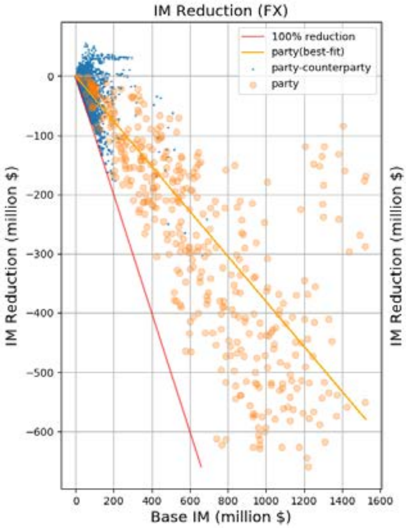
initial | scenario 1 | scenario 2 | scenario 3

Party	Scenario 1	Scenario 2	Scenario 3	Single Party Best
A	4	0	2	4
B	4	4	4	4
C	4	4	4	4
D	0	4	2	4

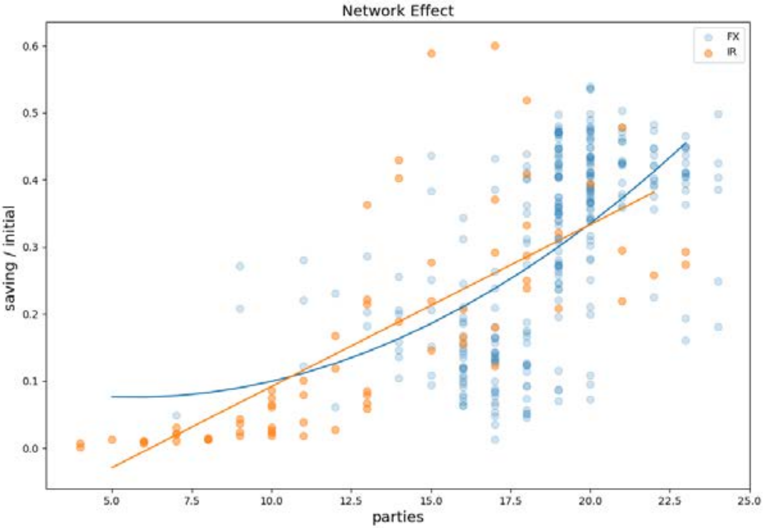
3 ways to achieve the same overall margin saving in a 4 party system

2-step algorithm: compute best possible for each party and then solve $\min \sum_p (IM_p - IM_p^{best})^2$

Observed Margin Savings



Network Effect



Summary

- Value at Risk and Expected Shortfall are risk measures that describe in a single number the riskiness of a financial portfolio
- Over the Counter Derivatives can be centrally cleared via a Central Clearing Counterparty or bilateral
- trading derivatives creates counterparty risk which can be mitigated by the collection of Variation Margin and Initial Margin
- Optimization of Initial Margin within a network of financial participants helps to reduce counterparty risk
- IM Optimization can be achieved via well understood constrained convex optimization numerical methods.
- Numerical Issues have a significant impact on the difficulty
- Interesting problems arise from making the solution Nice and Fair as well as Feasible and Optimal

Further Reading

Steven Boyd and Lieven Vandenberghe (2004) **Convex Optimization** *Cambridge University Press*
Jon Gregory (2020) **The XVA Challenge** (4th edition) *Wiley*
John Hull (2023) **Risk Management and Financial Institutions** (6th edition) emphWiley

Artzner, P., F. Delbaen, J-M. Eber and D.Heath (2022) **Coherent Measures of Risk** *Mathematical Finance* 9(3), 203 – 228
https://www.ise.ufl.edu/uryasev/files/2011/11/CVaR1_JOR.pdf **Optimization of Conditional Value at Risk**

https://www.bis.org/bcbs/publ/d317.pdf **Basel III regulatory framework for initial margin for non centrally cleared derivatives**
https://www.isda.org/a/b4ugE/ISDA-SIMM_v2.6_PUBLIC.pdf **ISDA SIMM 2.6**
https://people.math.ethz.ch/~embrecht/ftp/Seven_Proofs.pdf **Seven proofs of the subadditivity of Expected Shortfall**
https://people.orie.cornell.edu/gennady/techreports/VaRsubadd.pdf **Subadditivity Re–Examined: the Case for Value–at–Risk**
https://www.isda.org/a/KV9EE/Margin-Requirements-for-NonclearedDerivatives-April-2018-update.pdf **Margin requirements for non cleared derivatives**

18.642 Topics in Mathematics with Applications in Finance

Fall 2024