Portfolio Management

Application Lecture for MIT Class 18.642

Dr. Jake Xia

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MIT Open Courseware link from 2013 lecture:
https://www.youtube.com/watch?app=desktop&v=7AjFKAVVbpA

Outline

1. What is Portfolio Construction about?
2. Endowment Model
3. Portfolio Theory – Illustrated in special cases

4. Limitations of "Modern Portfolio Theory"
5. New Measure to Replace Sharpe Ratio and for Sizing
6. Modeling Crowd Behavior
7. Power Law Originated from Crowd Interaction

8. What we learned today

**Sections 4-7 highlight my research work*

Introduction of Portfolio Construction

Please write down your own investment portfolio on a piece of paper:

Imaging you have $10,000, how would you invest? Put down % for each investment.

Considerations

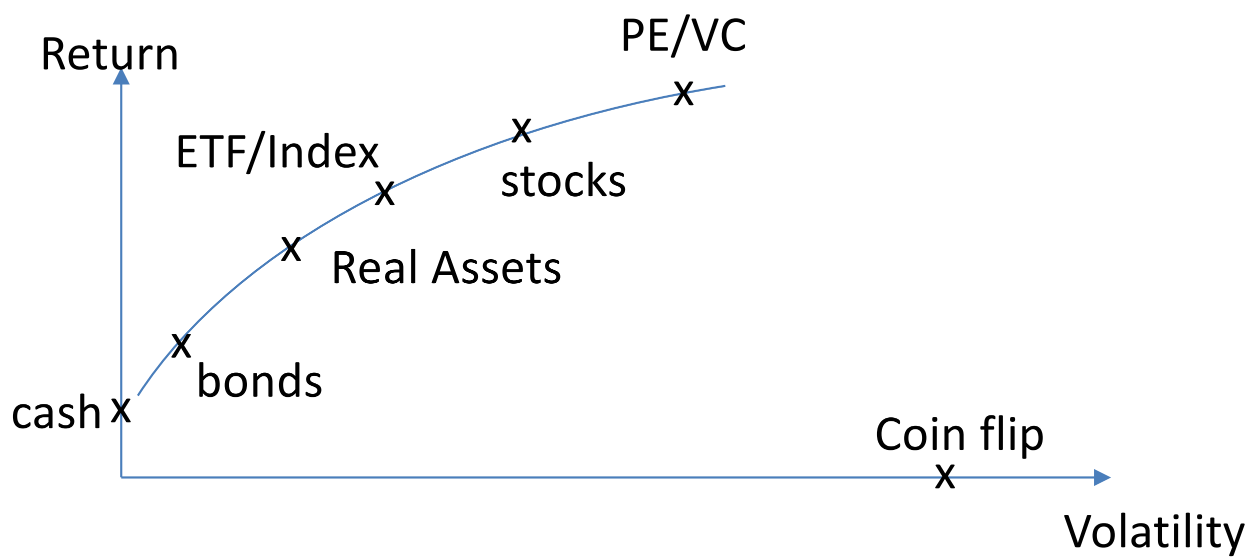
- Return objectives
- Loss tolerance
- What's your edge?
- How many investments?
- How to size each investment?

Layers of Decisions

- Markets, instruments
- Data, signals, models, factors, predictions, strategies
- Sizing, capital allocation, portfolio construction, optimization, risk management

How to compare different investments?

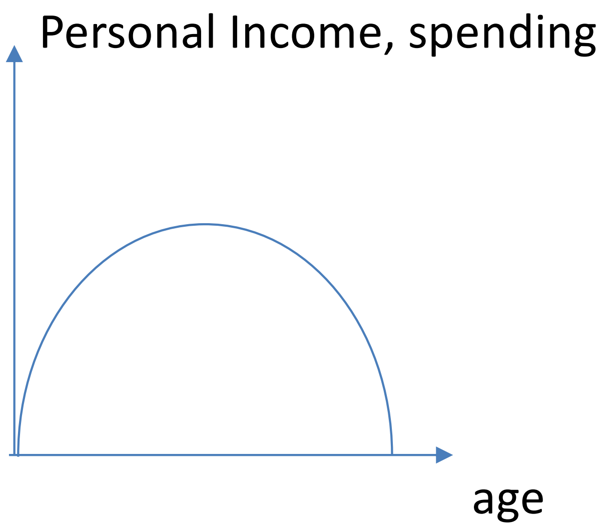
Understand the relationship between return and risk for all assets. Let’s first use volatility (uncertainty) as risk measurement.

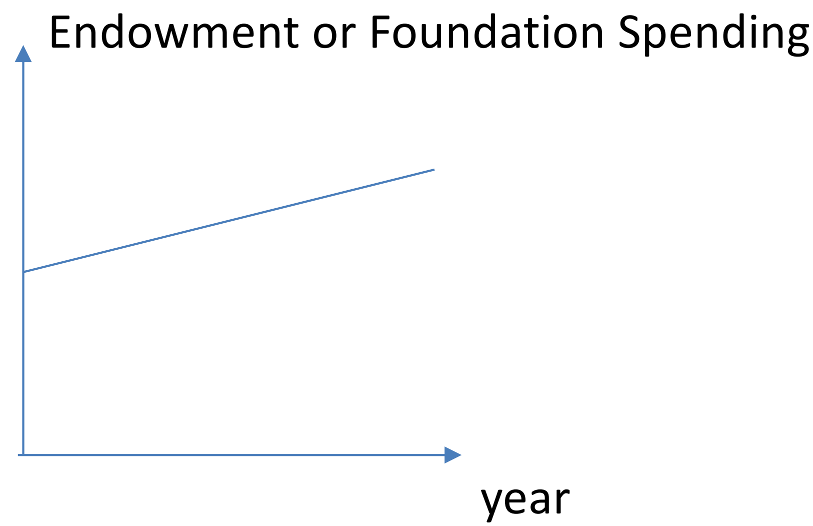


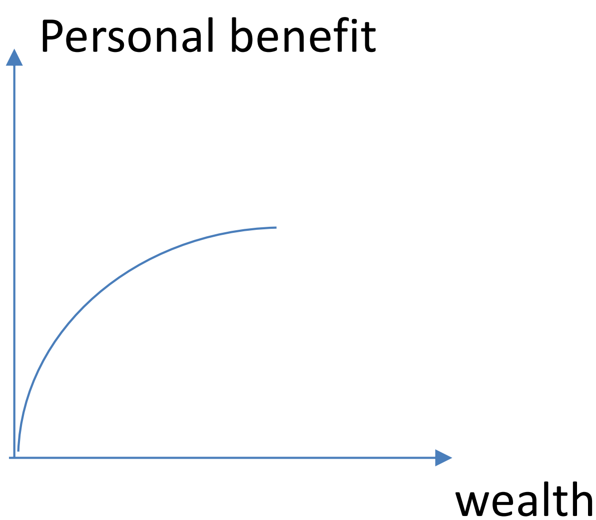
Return Objective and Loss Tolerance

- What is your objective function? Max(R/σ)
- What are the constraints?
- Liquidity, spending needs
- Risk/loss tolerance, drawdown, vol
- Inflation, relative purchasing power
- Absolute return or relative return
- Asset-Liability matching, cash flow
- Time horizon
- Peer pressure, career risk

Why invest?


Personal Income, spending
age


Endowment or Foundation Spending
year


Personal benefit
wealth

Endowment Setup

5% spending

3% inflation

8% nominal return target

Long term (>10years)

Liquidity, 30-40% of annual operating budget

Markets and Strategies

- Cash
- Government Bonds
- Corporate Bonds and Credit Products
- Macro and Relative Value Hedge Funds
- Quant Funds (CTA, Stat Arb, ML etc)
- Equity, Stock Long/Short
- Multi-Strategy Platforms
- Private Equity (Venture, Growth, Buyout)
- Real Estate and Natural Resource
- New Assets (Crypto, IP, Legal Claim etc)

Endowment Model

Internal vs external managers

Public vs private markets

Specialists vs generalists

Absolute vs relative (to benchmarks) returns

Passive vs active

Peer comparison

Equity beta, leverage, liquidity, downside protection

Asset classes, regions, strategies, sectors

Manager selection, a bottom up approach

Can you predict fund performance? Luck or skill?

The Classic Portfolio Construction Problem

Assume

- you know your return objectives
- you have a loss tolerance
- you can predict each investment's return, volatility and correlation to other strategies,

What % do you invest in each opportunity?

Objective of optimization:

- maximize portfolio return
- minimize portfolio risk

Portfolio Construction Problem

=

Investment Sizing Problem at All Levels,
e.g. Asset Allocation Problem

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Risk Management Problem:

avoid concentration, illiquidity,
unacceptable losses, unwanted risks

Portfolio Theory – Illustrated with Two Assets

Asset 1: return R_1 , vol σ_1 , weight w_1

Asset 2: return R_2 , vol σ_2 , weight w_2

$w_1 + w_2 = 1$

Portfolio return:

$R_p = w_1 R_1 + w_2 R_2$

$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2$

where ρ is the correlation between the two assets' returns.

Portfolio Theory – Illustrated in special cases

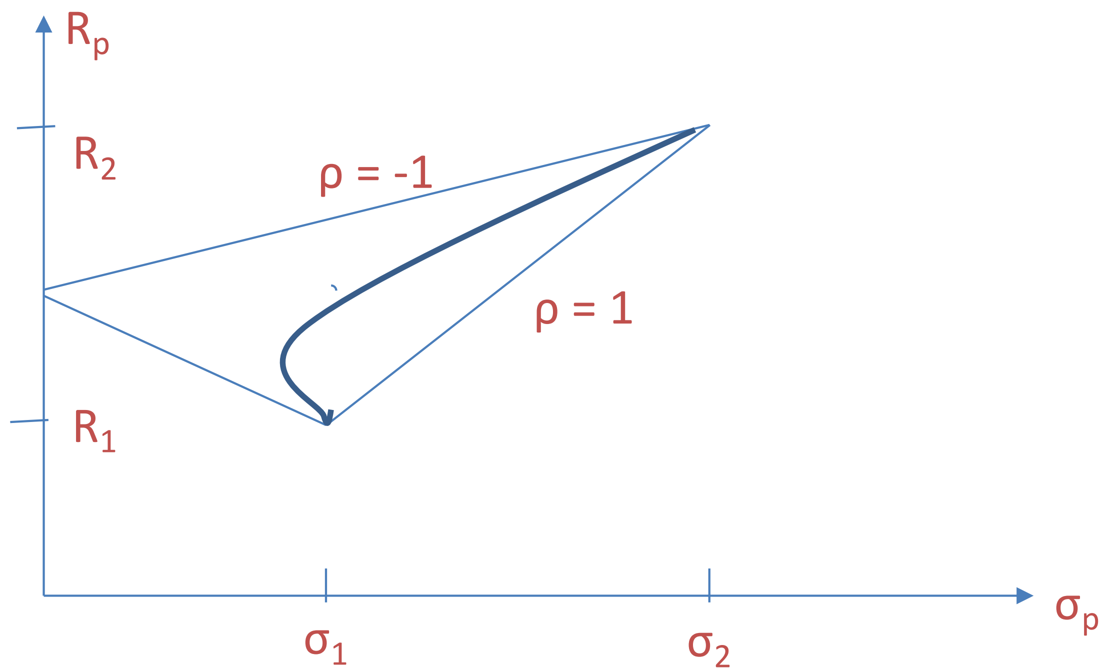
Two assets

$\rho = 1$, or $\rho = -1$, or $\rho = 0$

If $\rho = 1$, $\sigma_p = w_1 \sigma_1 + w_2 \sigma_2 = (1-w_2) \sigma_1 + w_2 \sigma_2$

$w_2 = (\sigma_p - \sigma_1)/(\sigma_2 - \sigma_1)$

$R_p = (1-w_2) R_1 + w_2 R_2$

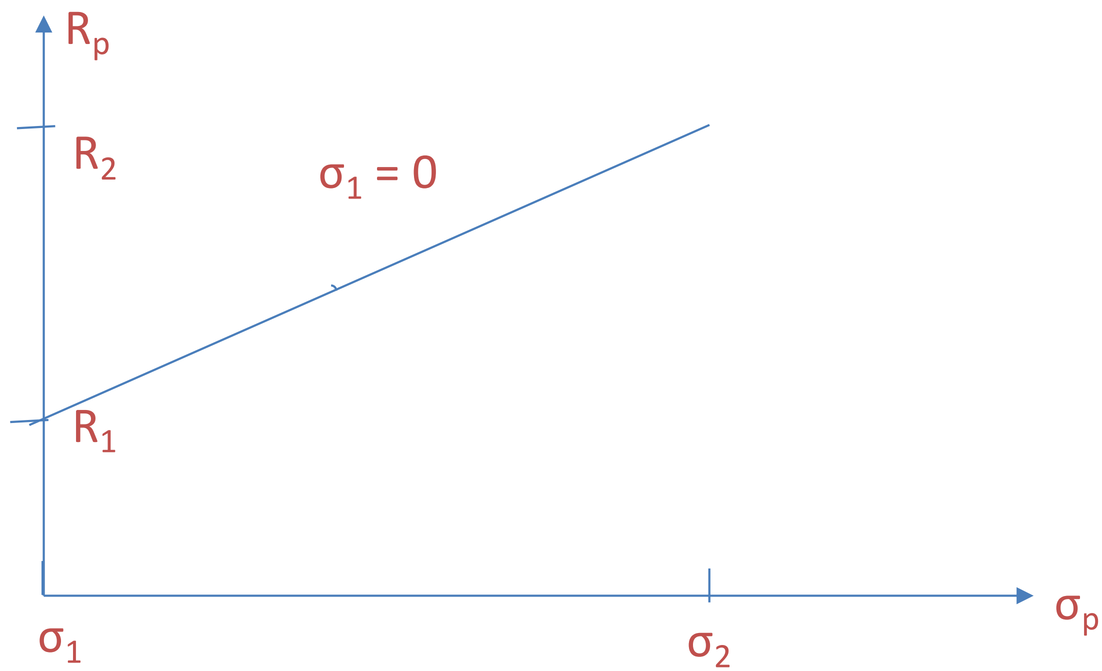


Portfolio Theory – Illustrated in special cases

Two assets

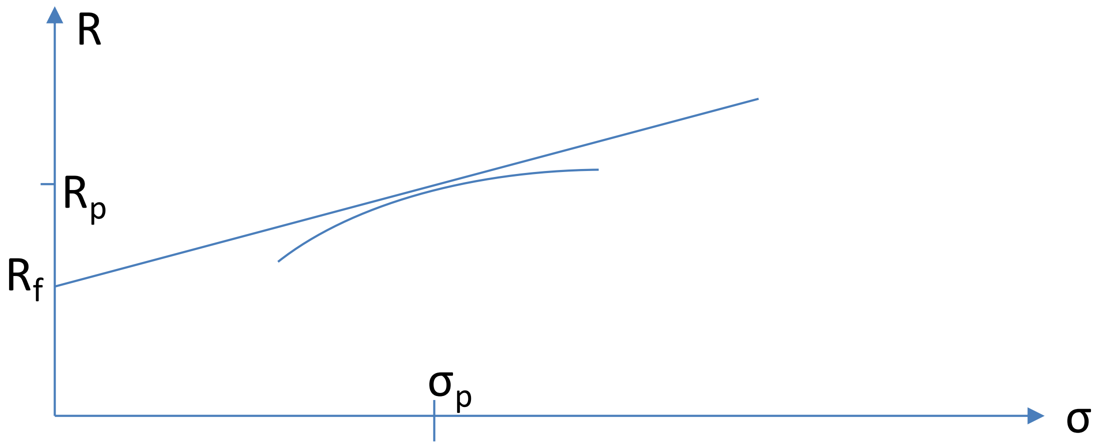
If $\sigma_1 = 0$, then $\sigma_p = w_2 \sigma_2$, $w_2 = \sigma_p / \sigma_2$

$R_p = (1-w_2)R_1 + w_2 R_2 = R_1 + (R_2 - R_1)\sigma_p / \sigma_2$



Capital Allocation Line

What if we add a risk-free asset on top of a portfolio?



$R = R_f + \sigma (R_p - R_f)/ \sigma_p$,

Sharpe Ratio $SR = (R_p - R_f)/ \sigma_p$

$R_p - R_f = \alpha + \beta (R_{BM} - R_f)$,

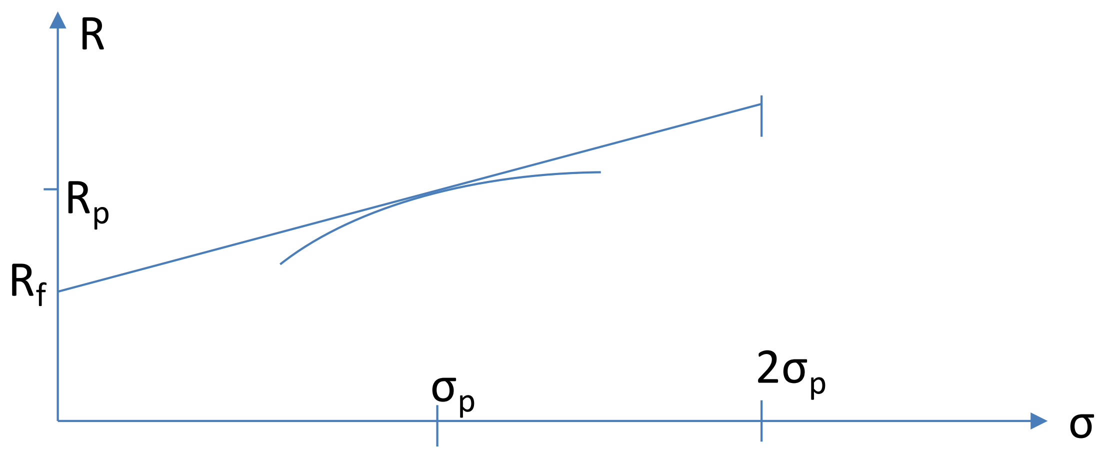
$\beta = \rho \, \sigma_p / \sigma_{BM}$

Portfolio Leverage

What if we borrow money to increase risk of a portfolio?

$\sigma = 2\,\sigma_p$, $R = 2R_p - R_f$

$w_1 = -100\%$, $w_2 = 200\%$



Examples of portfolio with 2 assets:

60/40 Equity/Bond reference portfolio

Levered risk parity portfolio:

levered bond allocation to achieve equal risk contribution from equity and bonds

Importance of Rebalancing – To Enjoy the Only "Free Lunch" from Diversification

Portfolio with 2 assets

Example of 2 year performance.

W_1 =50%, W_2 =50%

Year 1, R_1 = +100%, R_2 = -50%

Year 2, R_1 = -50%, R_2 = +100%

Year 1: R_p = 0.5*1+0.5*(-0.5) = +25%

Year 2: R_p = 100/125*(-0.5)+25/125*1 = -20%

if no rebalancing, 2-year return = 0%

If rebalanced, w_1 = w_2 = 62.5/125

Year 2: R_p = 62.5/125*(-0.5)+62.5/125*1=+25%

Limitation of the “Model Portfolio Theory” (MPT)

Confidence in Capital Market Assumptions?

- Return
- Volatility
- Correlation matrix

Instability of Mean Variance Optimization

- Unlimited solutions
- Artificial constraints

Sample of my research work

"How Much Should You Bet? -- A Sizing Method Based on Expected Gain and Loss", J. Xia, working paper, 2010

"A Model of Synchronization for Self-organized Crowding Behavior", J. Xia, 2016

https://arxiv.org/abs/1612.01132

"Emergence of Power Law Distribution from Crowd with Heterogeneous Agents", J. Xia, working paper, 2014

MPT Issue Number 1:

1. Volatility is a poor measure of risk. It does not differentiate upside and downside deviations (or skew) in the distribution
2. Sharpe Ratio uses volatility and suffers from the same issue
3. Sizing is not directly linked with quality of the investment. (*Sortino Ratio distinguishes skew but does not provide sizing)

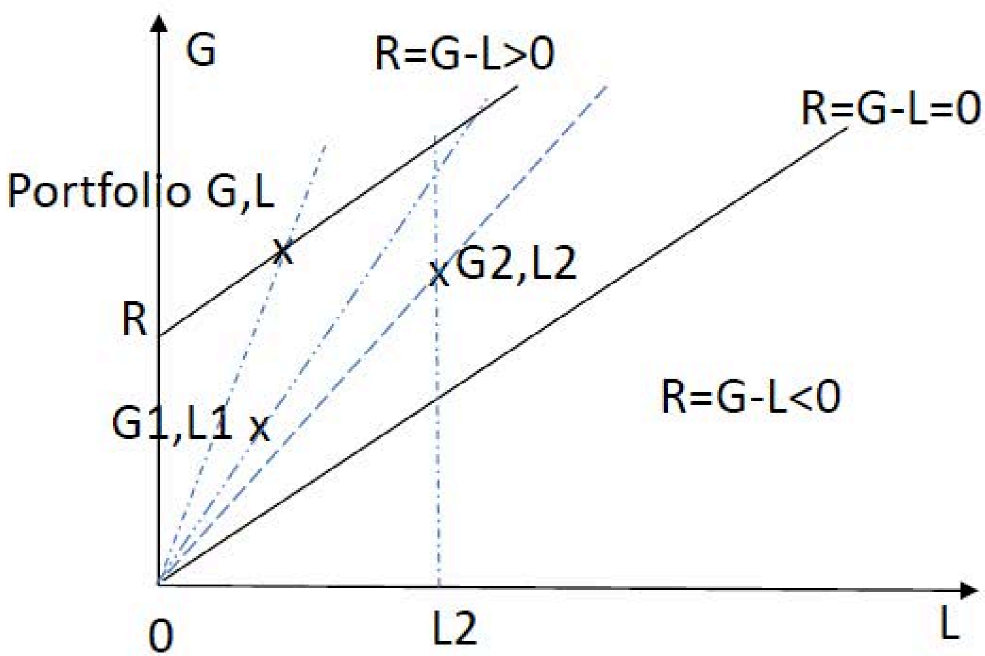
For example, if you are long an out-of-the-money call option, higher vol is not risk, but source of profit.

New way to compare different investments

Expected Gain G and Expected Loss L, both are positive numbers

Looking for the best skew G >> L or highest **new ratio**, (G-L)/(G+L)=1-2L/(G+L)=Size(%), bounded in [-1,+1]

same as Kelly Criterion for binary bets



Optimization Problem:
Find the set of weights W1,W2 such that (G-L)/(G+L) is maximized for the portfolio
$R_p(t) = w_1 R_1(t) + w_2 R_2(t)$
$w_1 + w_2 = 1$ or levered >1
G and L are calculated from the portfolio return $R_p(t)$

MPT Issue Number 2:

How do we predict future based on the past?

Observations →

Measurable Data →

Useful Information or Pattern Recognition →

Model (inputs, conditions, outputs) →

Prediction of outputs →

Repeat observations →

Control conditions and inputs

Stationary?

Are historical patterns repeatable?

Human factor, crowding behavior

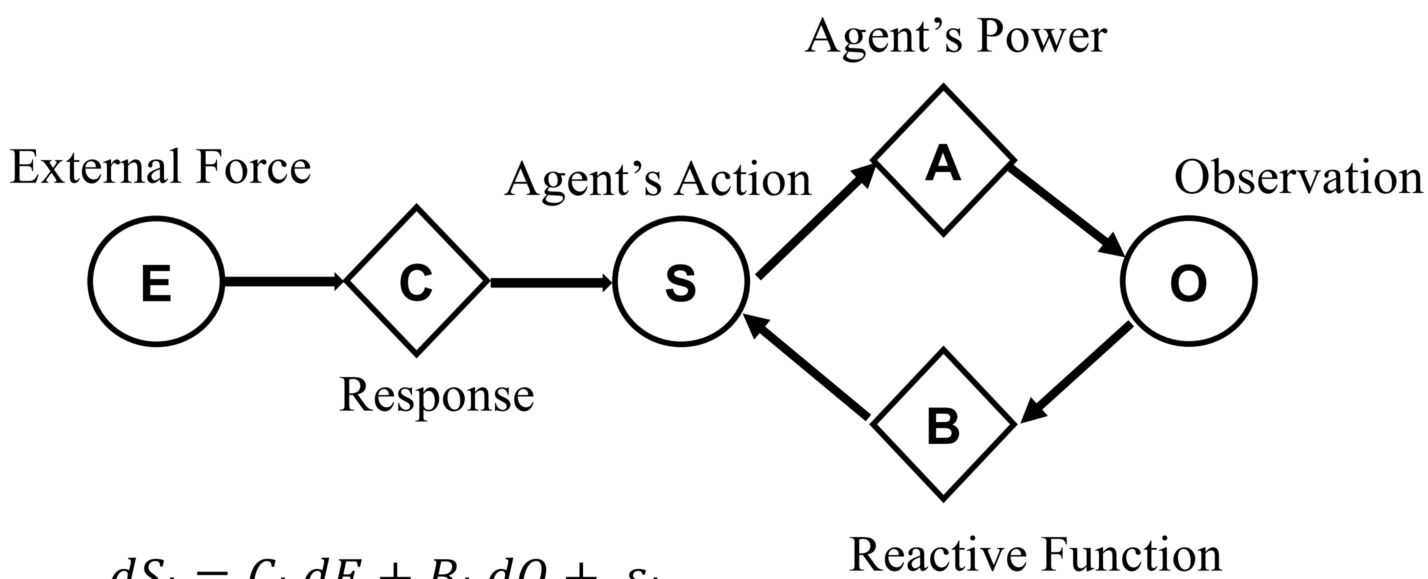
Adaptive approach

Let’s watch a video:

https://www.youtube.com/watch?v=KnndQgIUraQ&ab_channel=GerritStassyns

Crowd Behavior Modeling

Interaction Loop of Agents, Observation and External Force



$$dS_i = C_i\, dE + B_i\, dO + \varepsilon_i$$

$$dO = \sum_{i=1}^{N} A_i dS_i$$

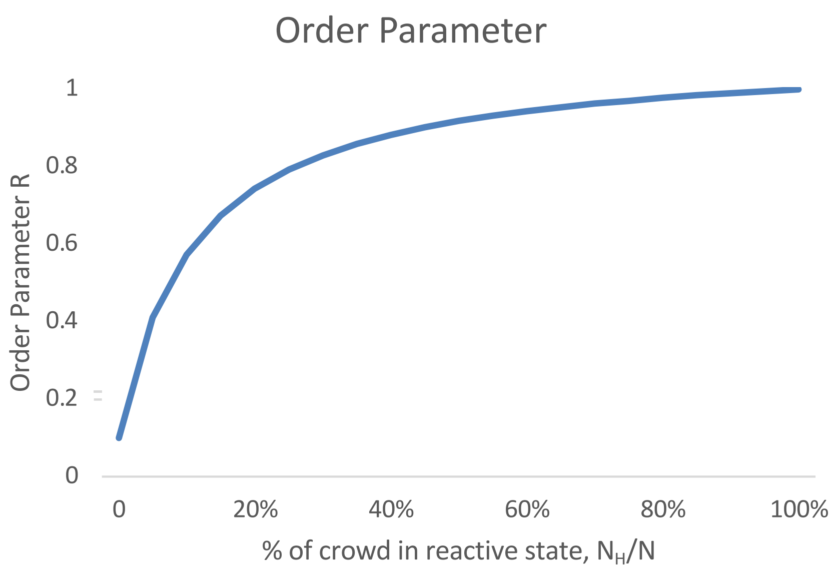
When in rational state, B_i takes a set of lower values of $\{B_L\}$. When in reactive state, B_i takes a set of higher values of $\{B_H\}$.

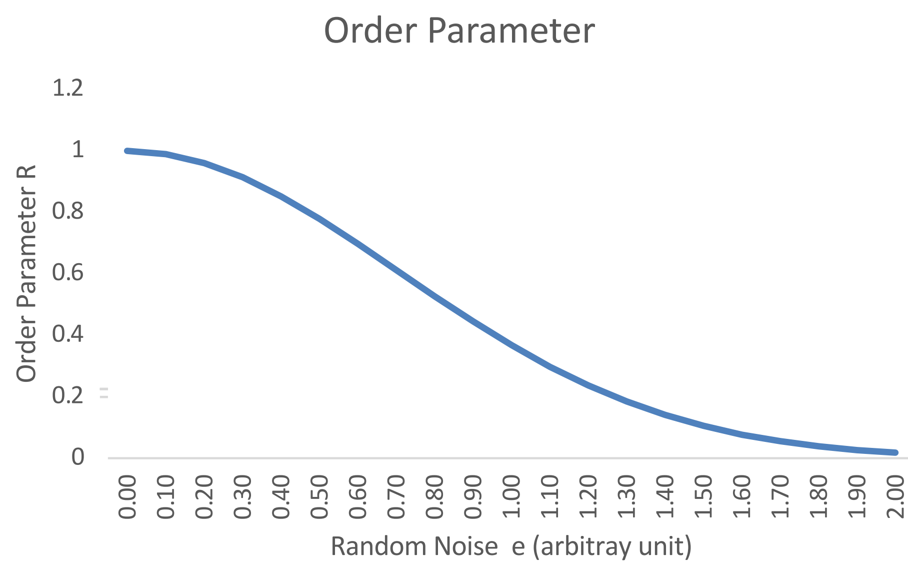
N_H is number of agents in reactive state, induced by large dO moves

Crowd Behavior Modeling

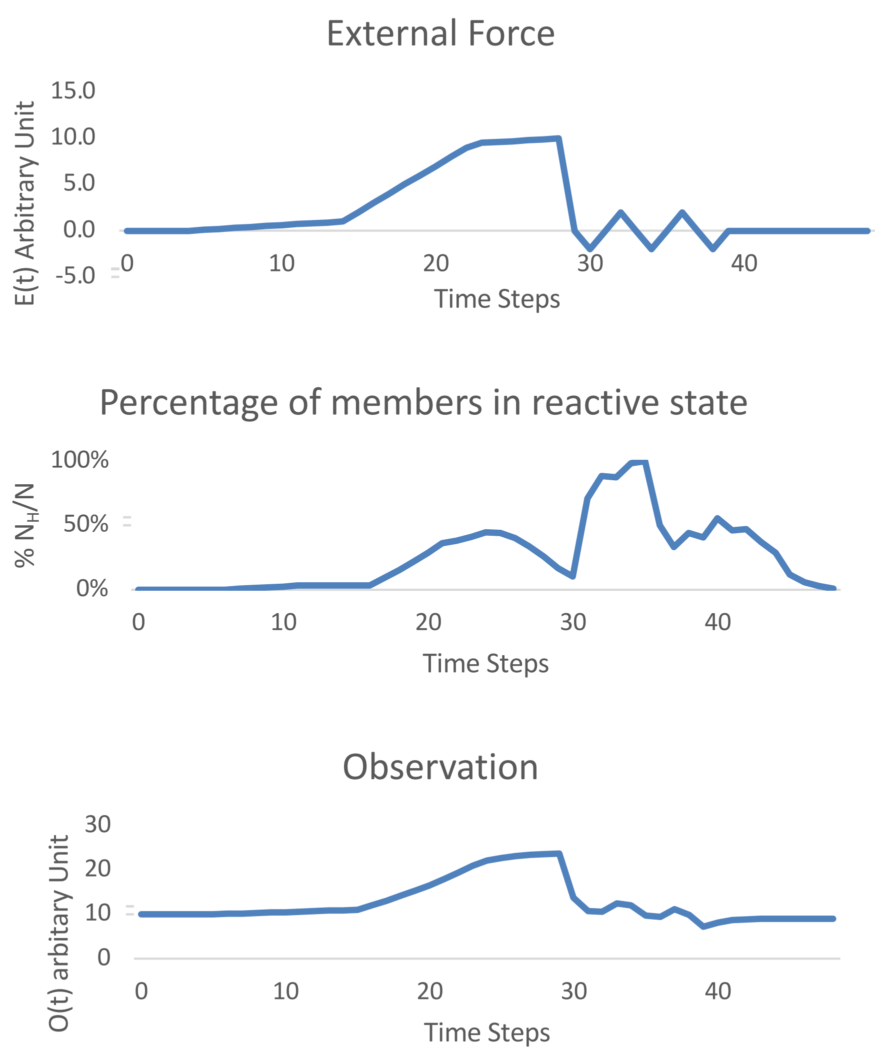
Order parameter measures synchronization

$$R(t) = \frac{|\sum_{i=1}^{N} dS_i(\mathrm{t})|}{\sum_{i=1}^{N} |dS_i(\mathrm{t})|}$$

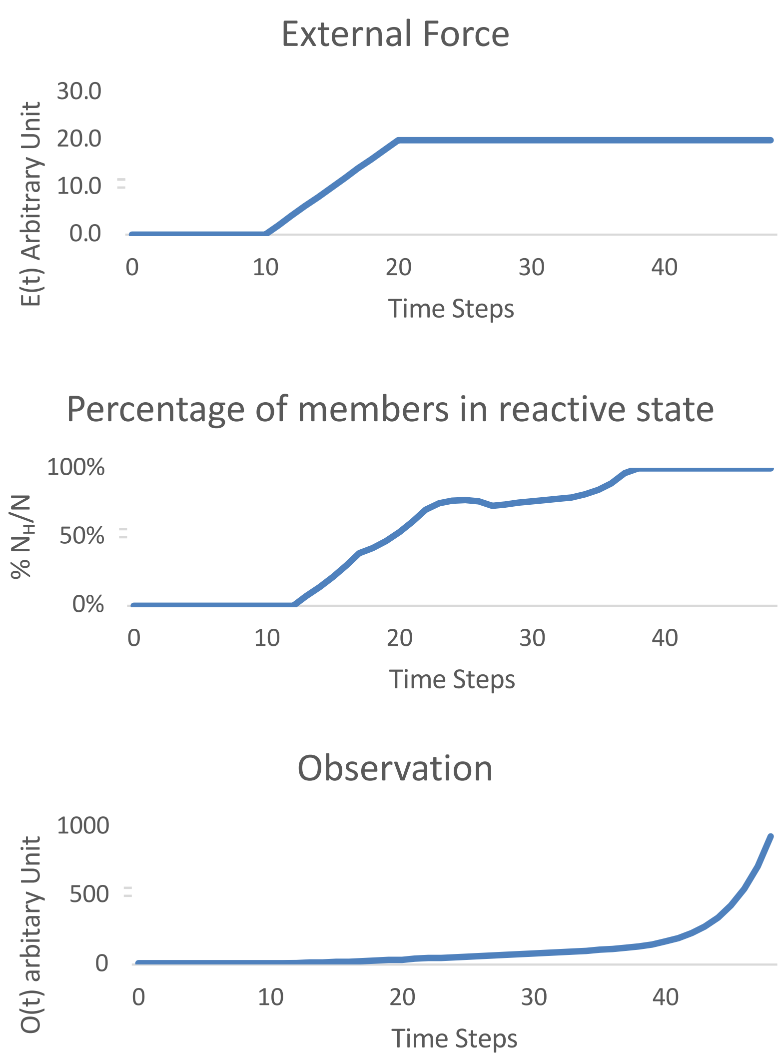




Bubble Simulation



Tipping Point when A * B(max) >1



Power Law Distribution

What is Power Law Distribution?

Winner-take-all, 80-20 rule, Matthew Effect

Scale-free or scale invariance:

$p(bx)=g(b)p(x)$

if and only if $p(x)$ is power law distribution.

Examples of Power Law Distribution: social phenomenon, wealth distribution, VC, city sizes, viral posts

Compared with Gaussian Distribution: natural phenomenon, height distribution

Many Social Phenomena Obey Power Law Distribution

But why?

What's the cause of power law distribution? For example, why does wealth distribution follow power law?

It has to do with crowd interaction!

Agents have different powers in influencing the market, rich get richer.

We need to model each agent differently, i.e. heterogeneity.

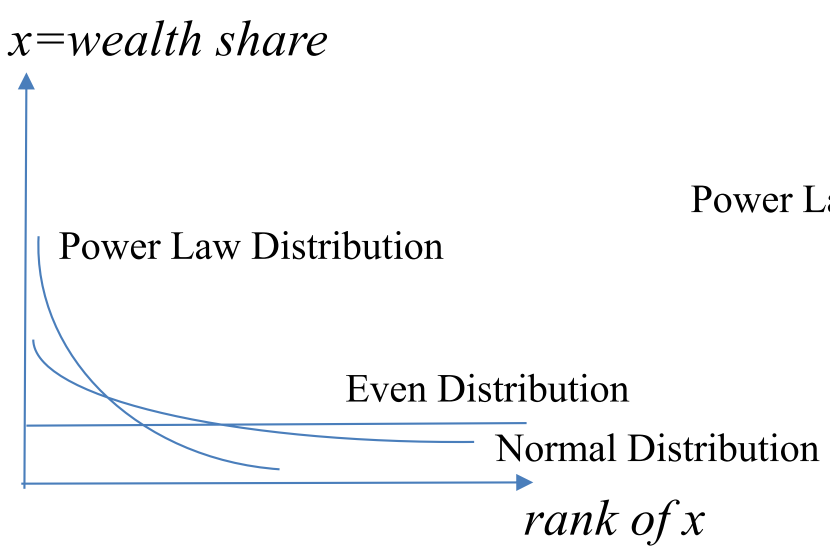
For random variable x, Normal distribution is described by PDF (probability density function) as the Gaussian function

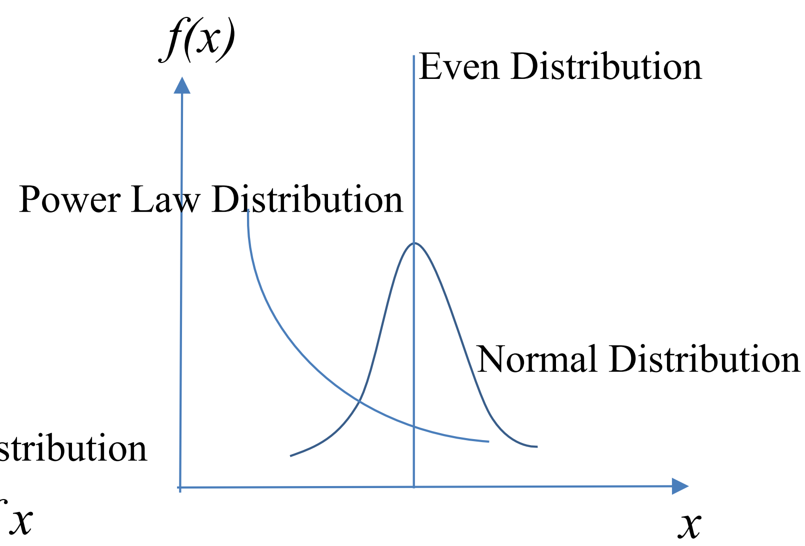
$$f(x) \sim e\text{^}[-(x-m)^2/(2\sigma^2)]$$

CDF for Power Law Distribution, $P[X>x] \sim 1/x^k$, Pareto's Law

Expected Value $E[x]$ vs rank r: $E[x] \sim 1/r^b$, Zipf's Law

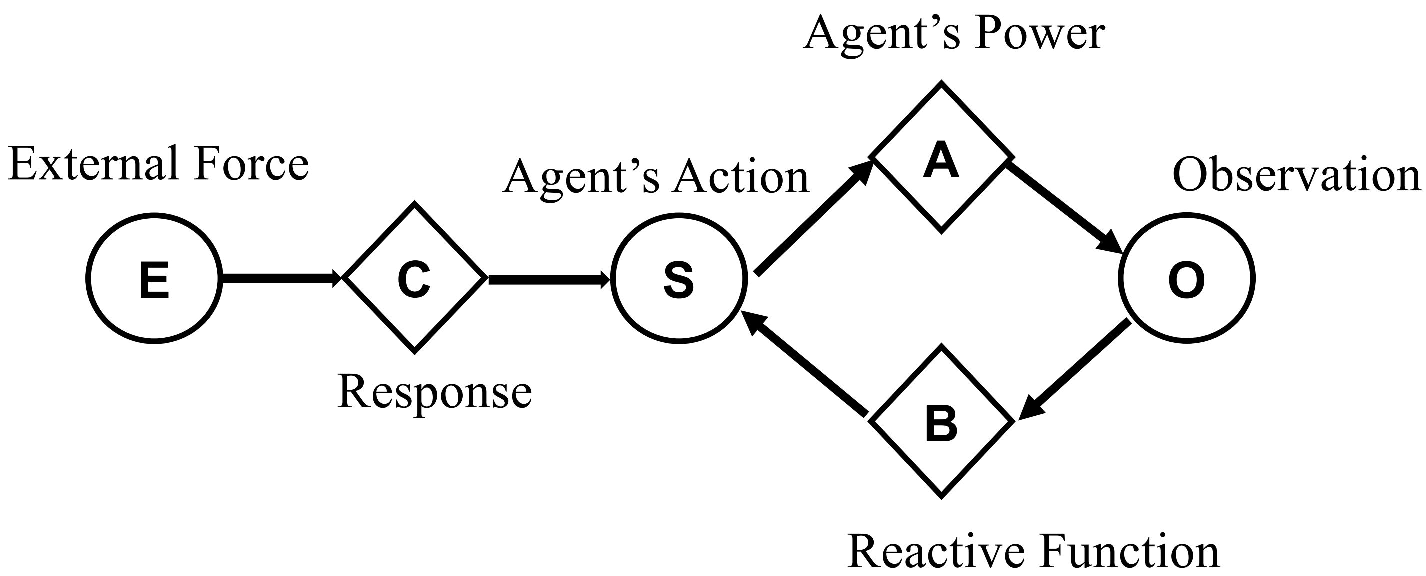
PDF for Power Law distribution: $P[X=x]=f(x) = C/x^a$, $a=1+k=1+1/b$





Crowd Modeling with Heterogeneous Agents

Introduce agents with different powers A(i,t)



Agents gain power by betting in the right direction of observation

Concentration of Power

Super Agents

System Stability

Entropy

What we learned today?

- Portfolio management is about how to size each investment
- Clarify your objectives
- Know your loss tolerance
- Diversification helps but you need to rebalance
- Volatility is not risk. Sharpe Ratio doesn't capture skew nor provide sizing – Use New GL Ratio
- Capital market assumptions are not reliable
- Crowd behavior can push markets to extremes
- Pay attention to powerful super-agents
- Understand key drivers of your portfolio

Investment Game

You are given $10,000 hypothetically

1. You need to pick to buy a publicly traded stock, ETF, currency, bond. Email us your selection before Sept 14
2. The price you buy is at the close of Friday Sept 13
3. The price you sell is at the close of Friday Nov 15

You need to track on a spread sheet:
1. How many shares you have bought
2. Daily profit and loss in $ and %
3. After closing out on Nov 15, total profit or loss in $, sum up all daily gains (G) and losses (L), calculate (G-L)/(G+L)

You are given one time chance to switch your entire position on Friday Oct 11’s close. Carrying the $ balance from previous position and roll it into a new stock or ETF. Email us your decision before Oct 12

Games for future portfolio managers (PMs) or traders:

1. Paper trading games

2. Lectures on research, data collection, signal generation, prediction models, back testing, sizing of single trades, portfolio optimization

3. Road to become a fund manager:
team building, fund raising, term sheet negotiation, systems and technology, back office and accounting, legal and compliance, tax, etc