MIT 2.008 Design and Manufacturing II

Spring 2025

Homework 4 – Variation & Quality

Released: March 5, 2025 2:00 PM

Learning Objectives

- *Understand how well quality specifications from design will tolerate variation in manufacturing.*
- *Learn how to determine whether a process is capable, and how to determine whether a process is in control.*
- *Consider how differences between design specifications and manufacturing variation may lead to quality defects.*
- *Learn how principles of quality measurement for individual components can impact the fit between multiple components in an assembly.*

General Notes

- *For qualitative answers, we're not looking for long essays. Please answer using short (1-2 sentence per answer) bullet points.*
- *For quantitative answers, show your work as clearly as possible. When possible, keep answers in algebraic form until plugging in numbers at the very end; this way, it is much easier for graders to understand where you make mistakes and provide meaningful feedback.*

HOMEWORK TOTAL POINTS: 100 pts

Problem 1 - Variation - 30 pts

You're working on your yo-yo design and specifying bearings. You've determined that for a good press fit of the bearing into the housing, you require the outer diameter of the bearing to be 10 mm +/- 0.2 mm.

a) What is your Upper Specification Limit (**USL**) and Lower Specification Limit (**LSL**)?

b) You find a supplier who says they can provide a 10 mm bearing. If the bearings provided have a mean of 10 mm and a standard deviation of 0.15 mm, what **percentage of bearings will meet your specifications**? A Z-score table is provided in **Appendix I**.

c) Sketch this normal distribution, and indicate on the plot the **Mean, LSL, USL, Z_{USL}, Z_{LSL},** and **Z_{min}**.

d) After feedback from engineers in assembly, you determine that the specification for the bearings must be adjusted to 10 mm +0.1mm/-0.3mm. Given the same batch of bearings from the supplier, determine what **percentage will fit your system under the new requirement**.

Problem 2 - Process Capability and Control - 50 pts

a) You find the housing for the bearing to have an average inner pocket diameter of 10.1 mm with a standard deviation of 0.1mm. If your specification for the pocket is still 10 mm +/- 0.2 mm, what is the capability $\mathbf{C_p}$ and capability index $\mathbf{C_{pk}}$ of this process?

b) Are the capability C_p and capability index C_{pk} the same?

If not, why are they different?

Is this process capable?

c) Inspect the components of the equations for C_p and C_{pk}. Assuming the specifications and process mean cannot be modified, **what needs to change** to increase the capability of the process? Inspect the components of the equations for C_p and C_{pk}

The pocket is machined by making radial passes using a 6mm end mill. List **2 practical (qualitative) actions/tweaks** to the machining process for the housing which might achieve this change.

-

d) During the manufacturing of the housing, quality engineers determine the process statistics by measuring samples of the pocket diameters. Parts are measured in **sample sizes (n) of 9** each in order to check whether the process is stable. What is the **Upper Control Limit (UCL) and Lower Control Limit (LCL)** used during this sampling?

e) Will the pocket fail to meet specifications while still being considered in control, given the sample size of 9? Why or why not?

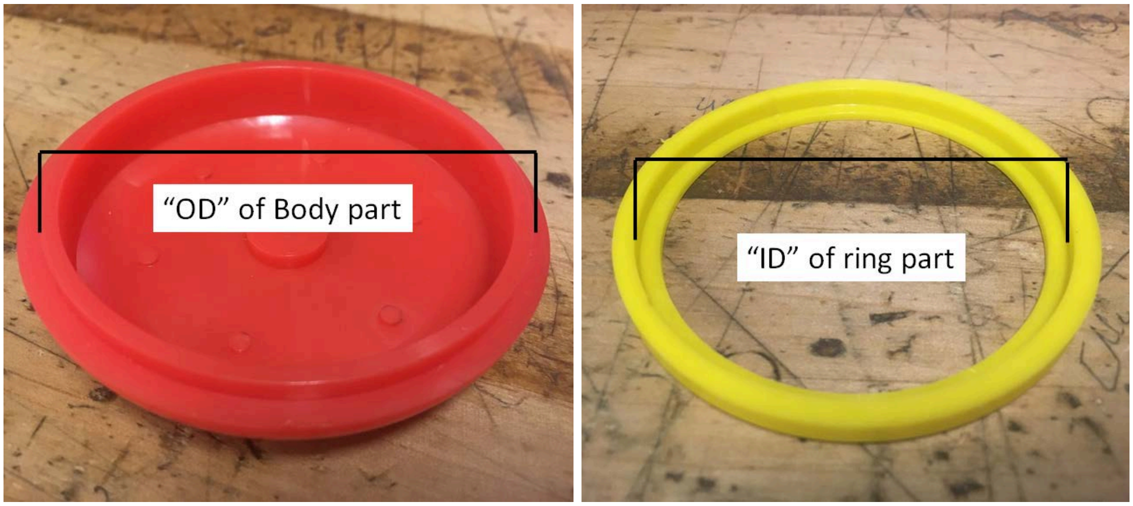
f) While operators are in the middle of making a large order of housings, the Shewhart Control chart shows that sample groups of the pocket diameters have **shifted to a mean of 10.15 mm**. Assume the process has the **same standard deviation**.

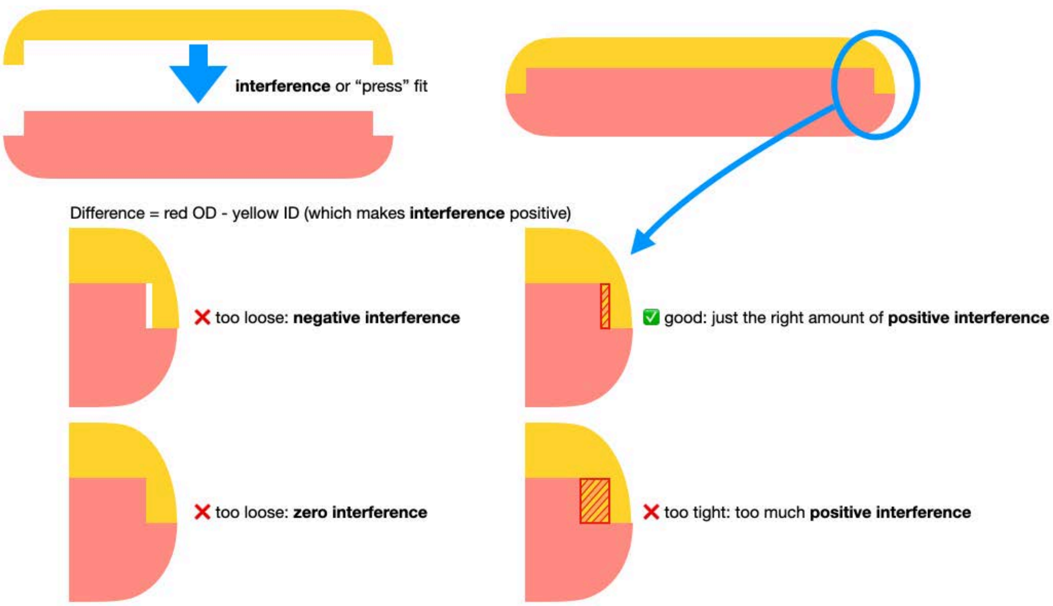
What are the **new control limits**?

What percentage of parts are expected to be out of spec without engineers detecting a loss of control (i.e. above USL or below LSL but within UCL and LCL)?

<u>Problem 3 - Variation for an Assembly - 20 pts</u>

During your 2.008 projects, you are making yo-yos which contain injection-molded red bodies and yellow outer rings. You need to ensure that the rings will snap-fit onto the bodies.





Assume that the yellow ring ID is specified to be 70.00 +/- 0.10 mm (USL 70.10, LSL 69.90), and the red body OD is specified to be 70.20 +/- 0.10 mm (USL 70.30, LSL 70.10. This means a specified interference fit of 0.20 +/- 0.20 mm (USL 0.40, LSL 0.00) for when these two parts are press-fit together.

After carefully measuring a small sample batch of bodies and rings, you have determined that the measured dimensions of the parts vary according to the following:

Dimension	Mean [mm]	Standard Deviation [mm]
OD of red body	70.05	0.16
ID of ring (yellow)	70.45	0.34

a) Considering the failure modes of injection molding:

List **2 potential common (random) causes** for variation in dimensions

List **2 potential assignable (systematic) causes**

Answer generally about the injection molding process, not separately for each component.

b) Using the same logic you followed in Questions 1 and 2, you determine that only 9.89% of rings and 31.89% of red bodies meet their specifications, respectively.

If you randomly select one of the yellow rings and one of the red bodies, what is the chance that *both* will be within their specifications?

c) You realize that although you may have very low odds of finding 2 parts both within spec, you might have better odds of randomly selecting a ring and a body and having them fit together, since out-of-spec parts can fit out-of-spec parts as long as the interference is within spec. So let's consider the **distribution of the interference** in the assembly, rather than the distributions of each individual part's dimension.

Combining Normally Distributed Data Sets

added data sets:

$$\mu_{new} = \mu_1 + \mu_2$$

$$\sigma_{new} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

subtracted data sets:

$$\mu_{new} = \mu_1 - \mu_2$$

$$\sigma_{new} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

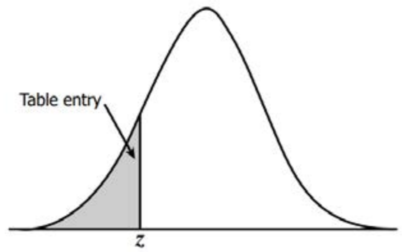
Using the formulas for combining normally distributed data sets above, calculate the **mean and standard deviation of the interference (red OD - yellow ID).**

d) If you randomly select one of the yellow rings and one of the red bodies, what is the chance that they will fit together within the specified range of interference (USL 0.40, LSL 0.00)?

What accounts for the difference between this answer and your answer to part (b)?

Appendix I: Z-score Table

$$z = \frac{x - \mu}{\sigma}$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998