Cutting #1

Cutting Analysis: Mechanics, Forces, and Power


GMT

What's so special about machining?

machining is fundamental to mechanical engineering because…

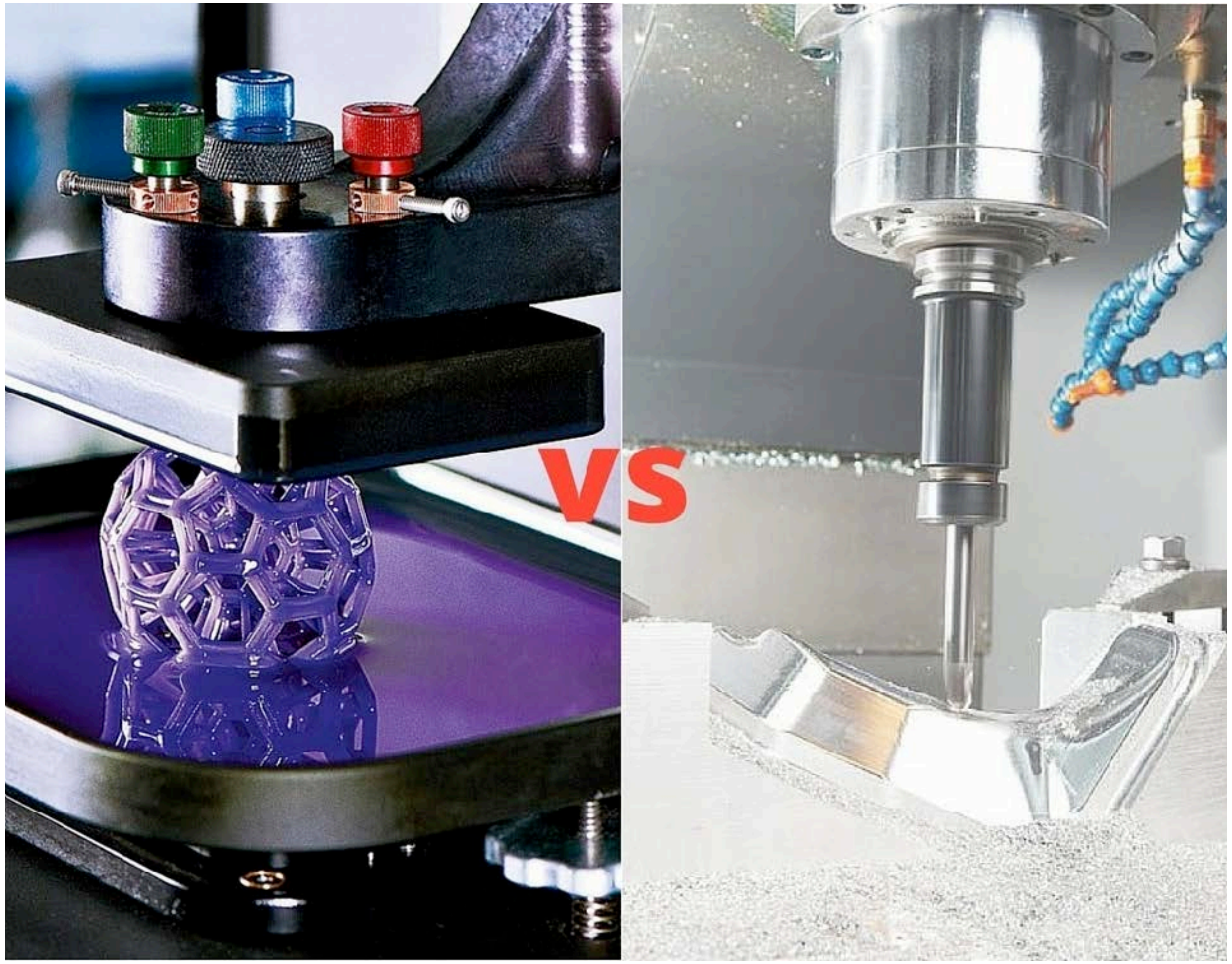
it is a major method of subtractive manufacturing

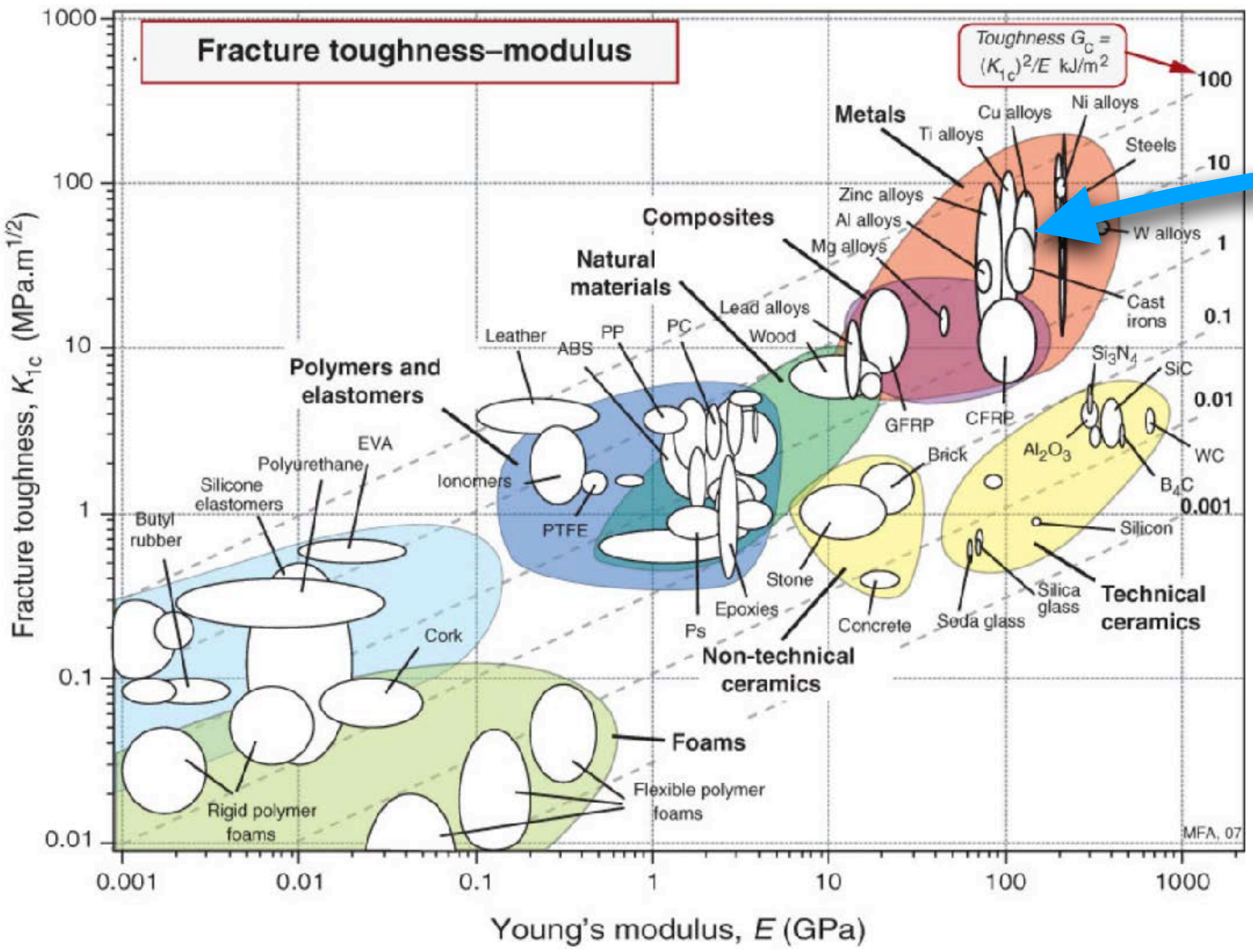
advantages of a subtractive process?

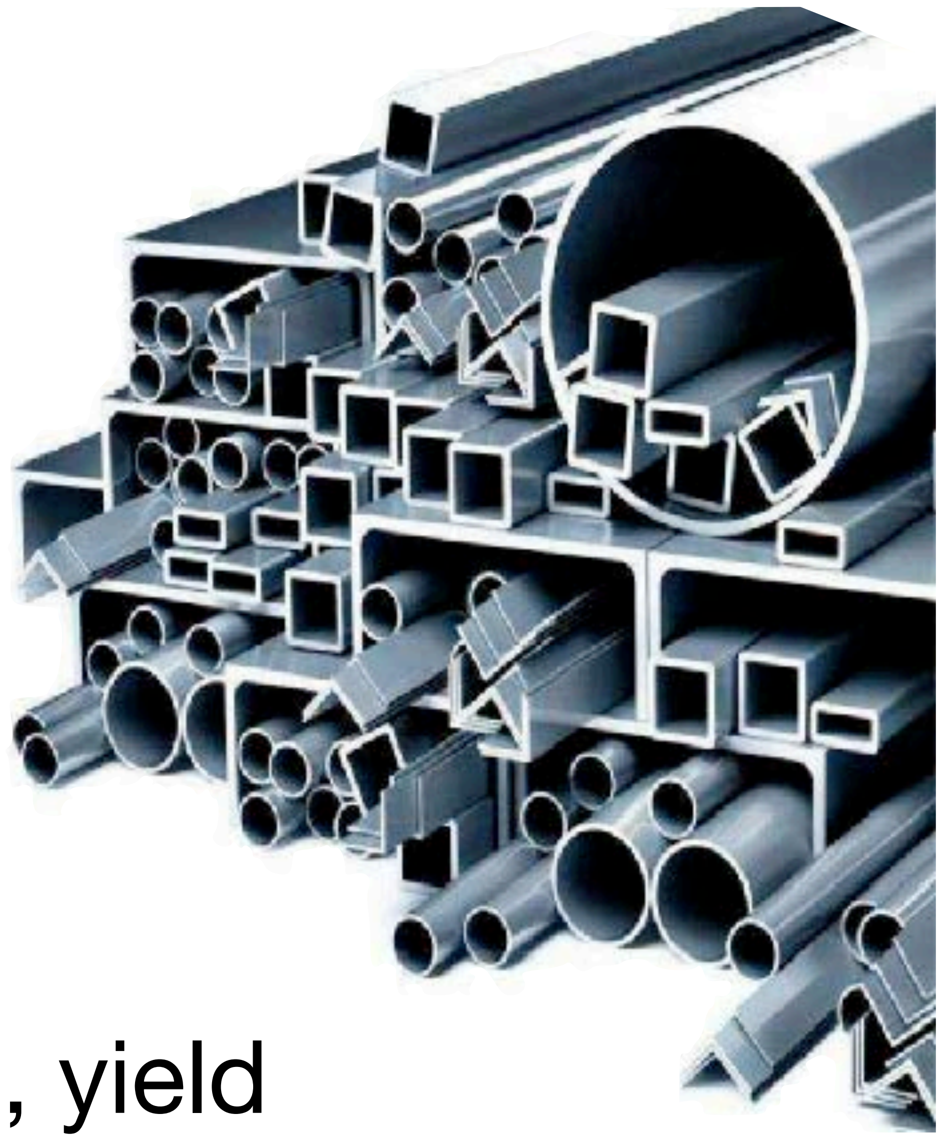
why not 3D print everything?

access to material properties

additive vs subtractive manufacturing







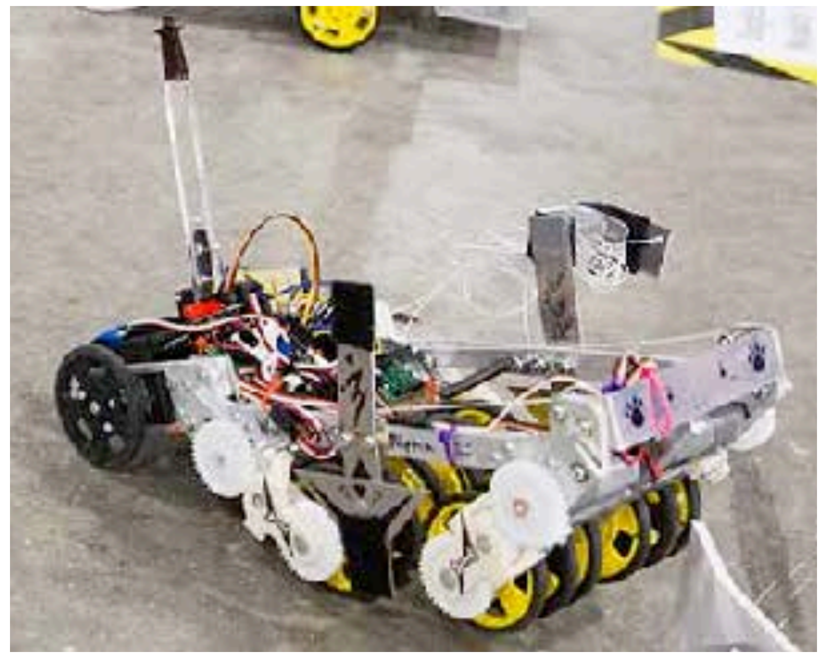
stiffness, toughness, yield strength, hardness, temperature resistance

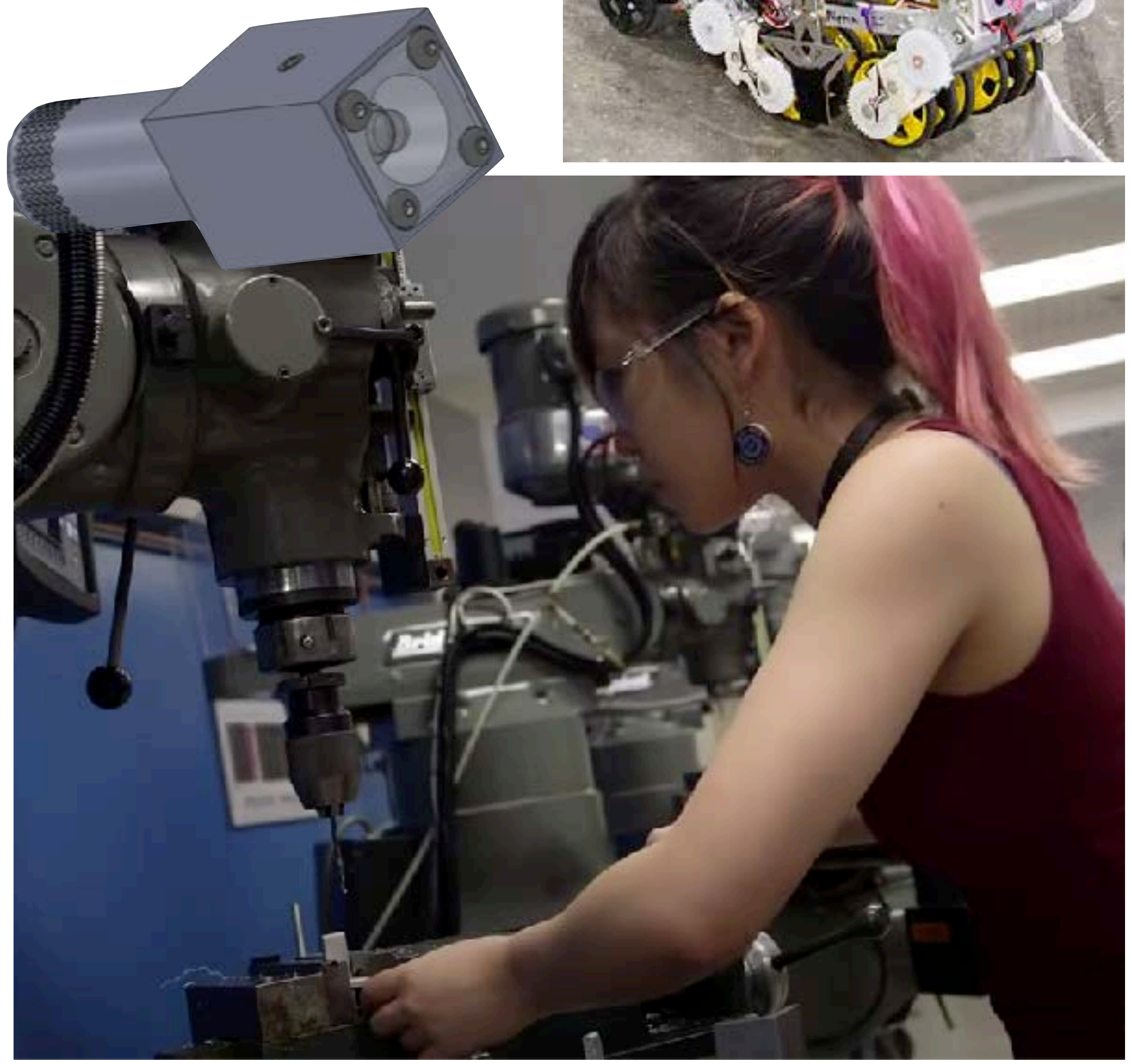
where do we see a big need for these material properties?

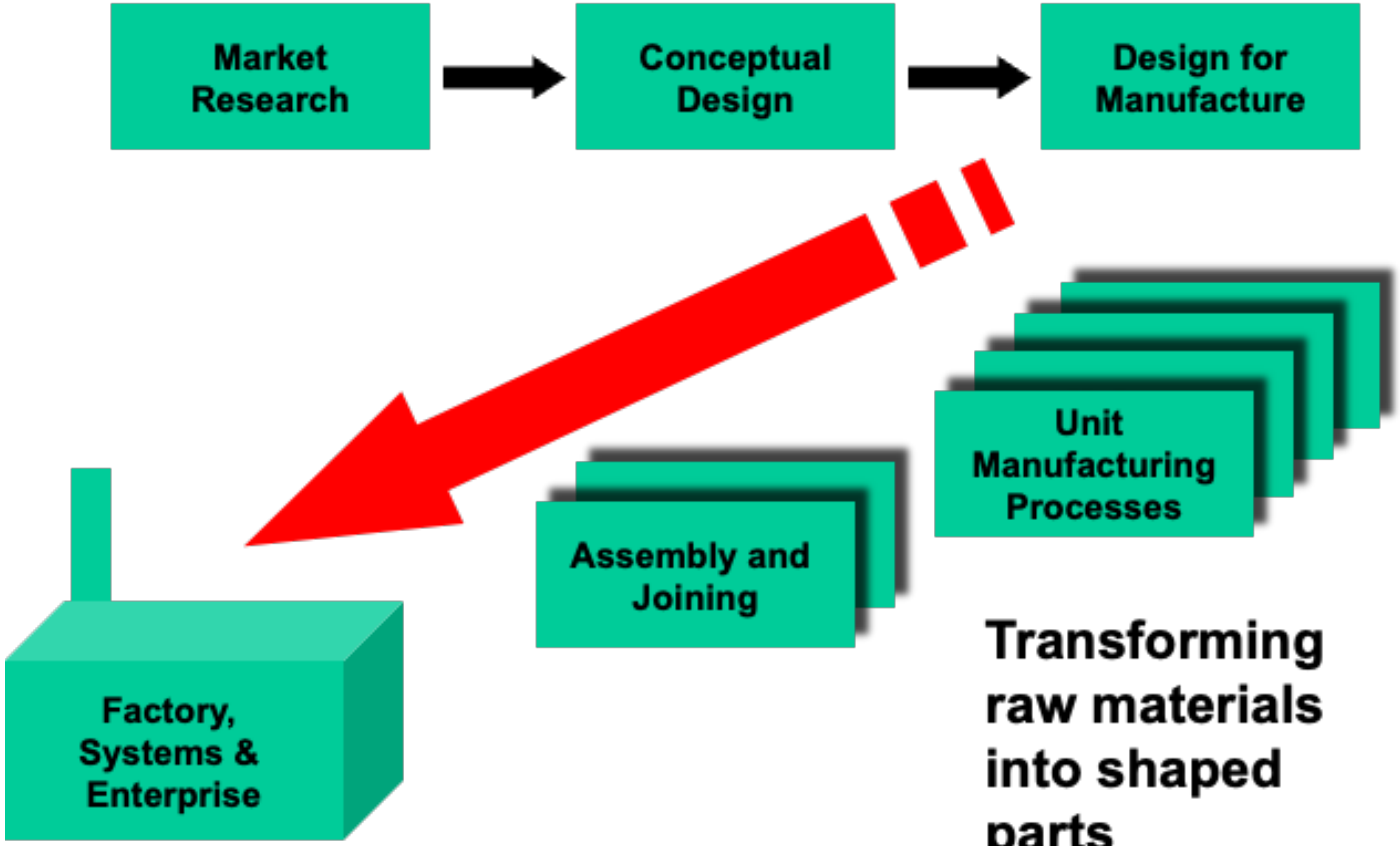
manufacturing

Design vs Manufacturing

your experience:
“prototype
machining”

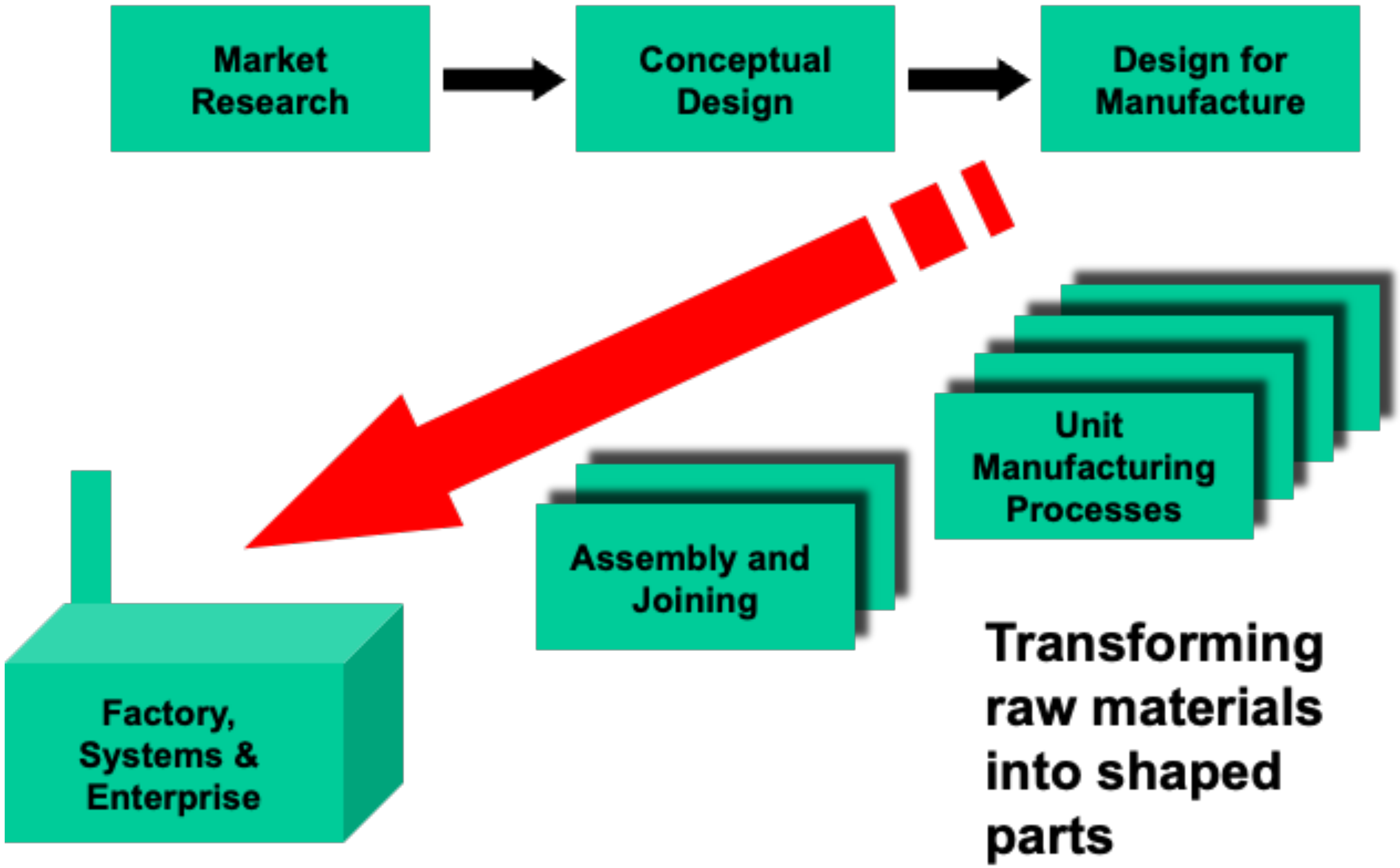






Transforming raw materials into shaped parts

but what does machining in manufacturing look like?



injection molding

"net shape"

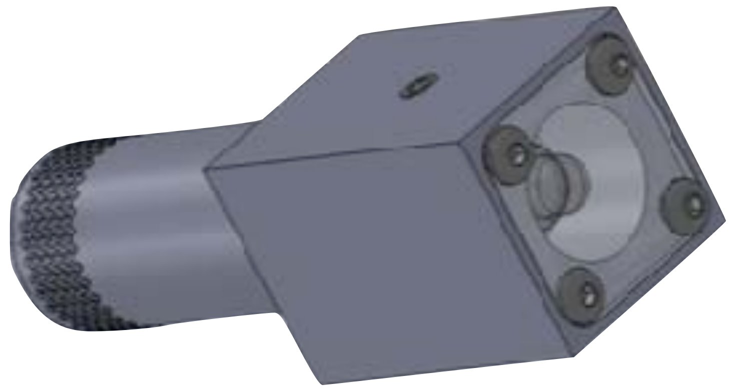
thermoforming

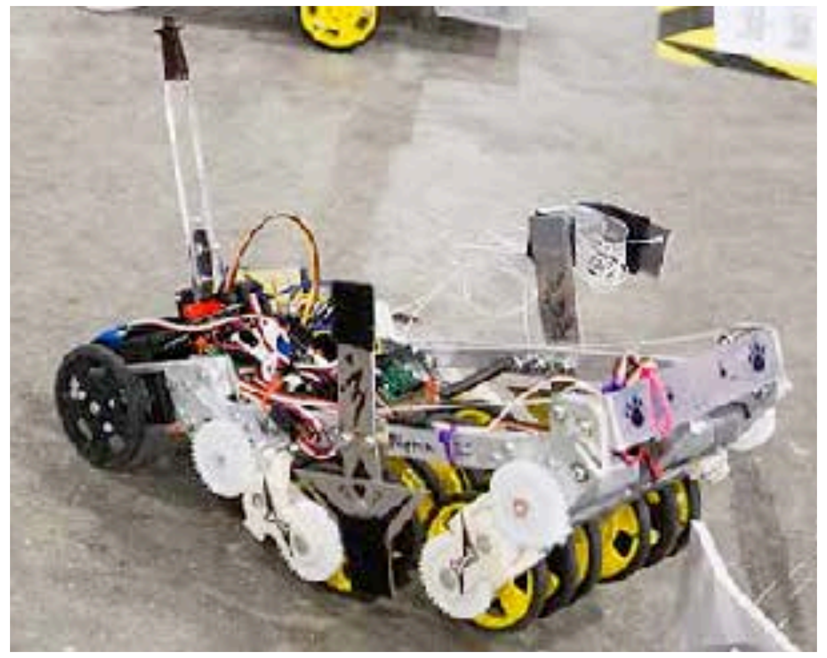
include

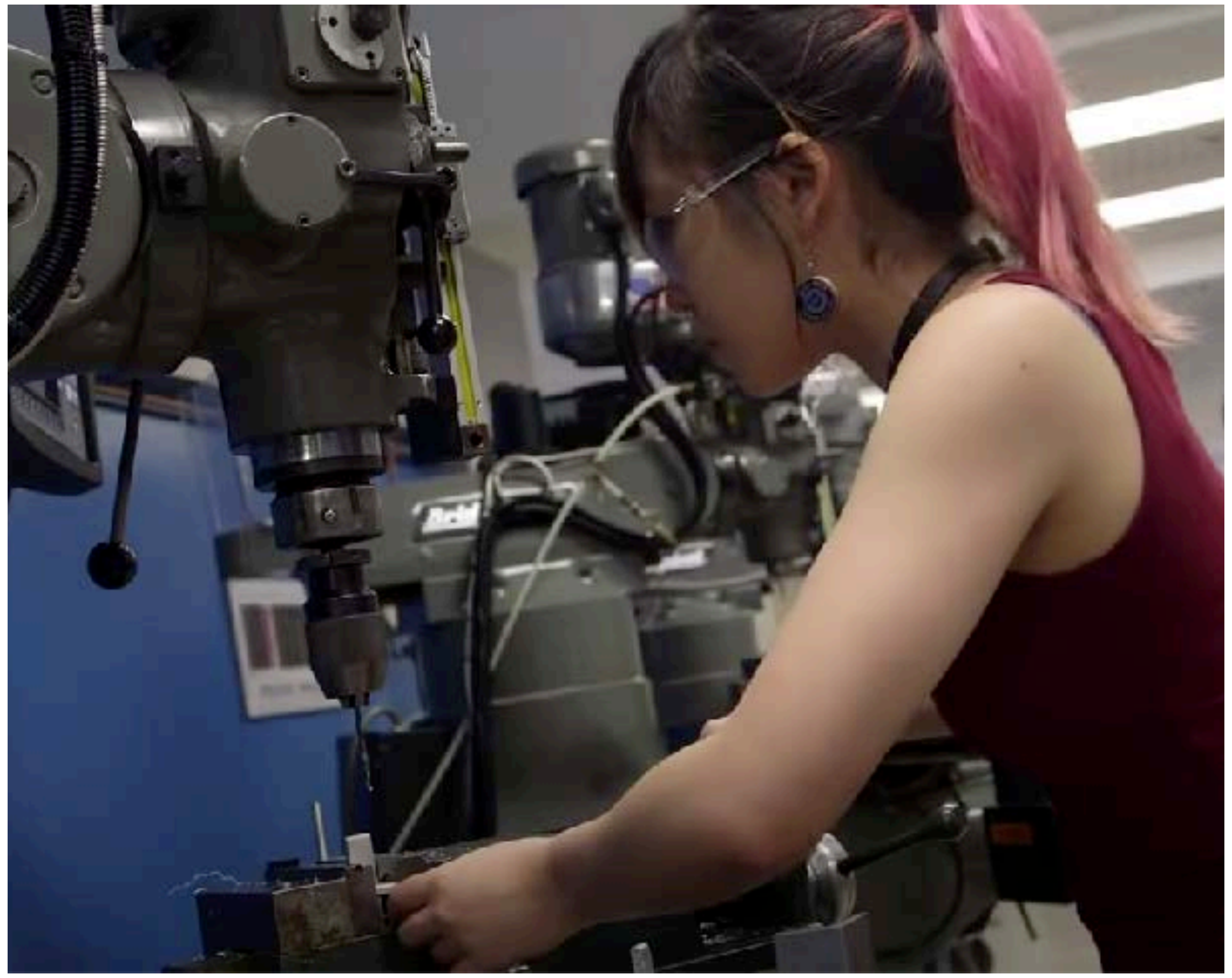
machining "subtractive"

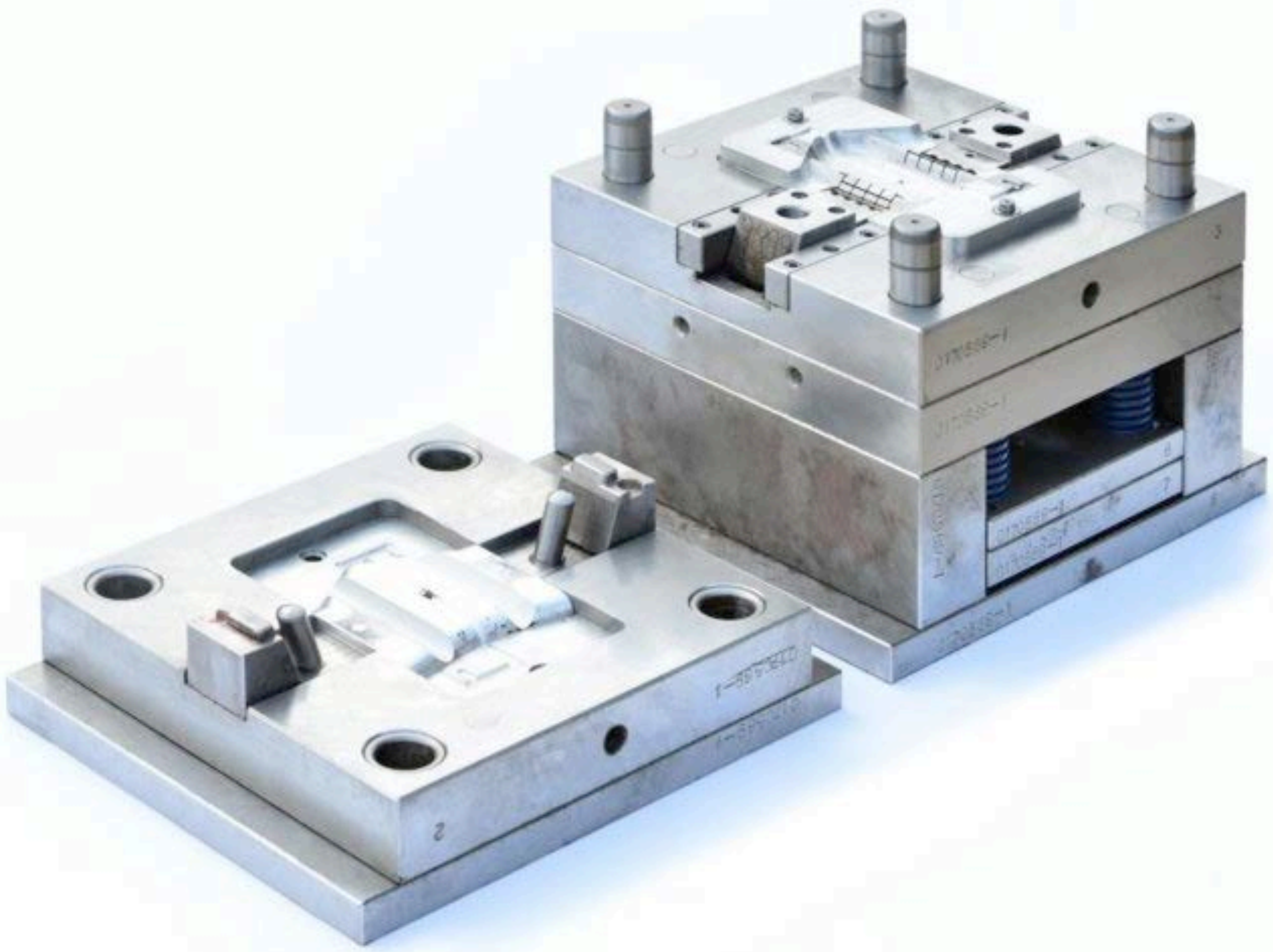
but what does machining in manufacturing look like?

prototype machining **vs.** manufacturing: tooling production









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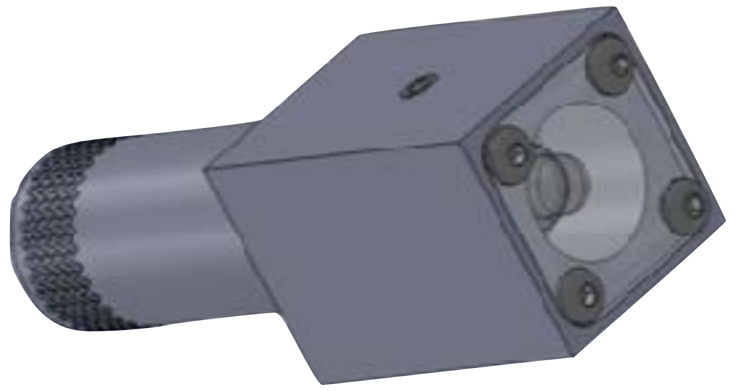
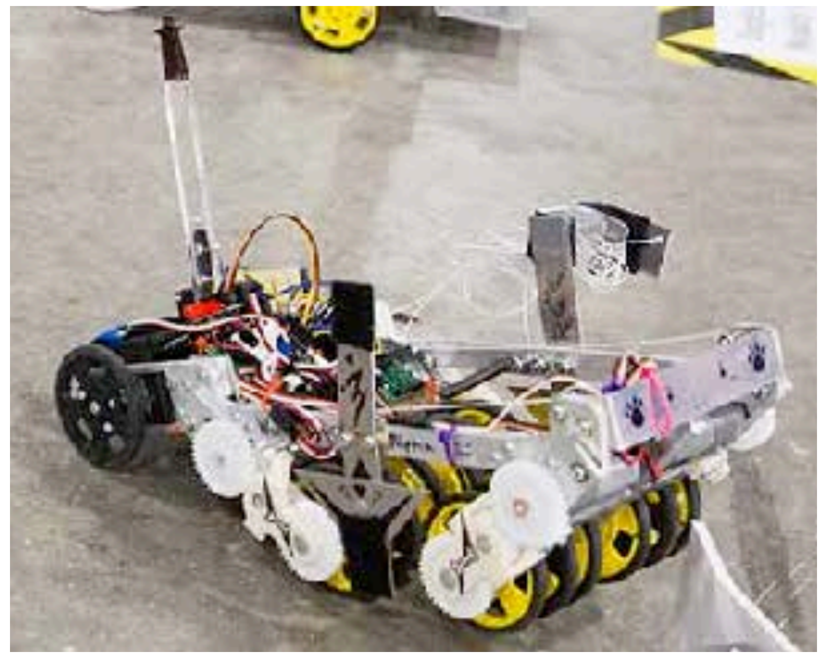
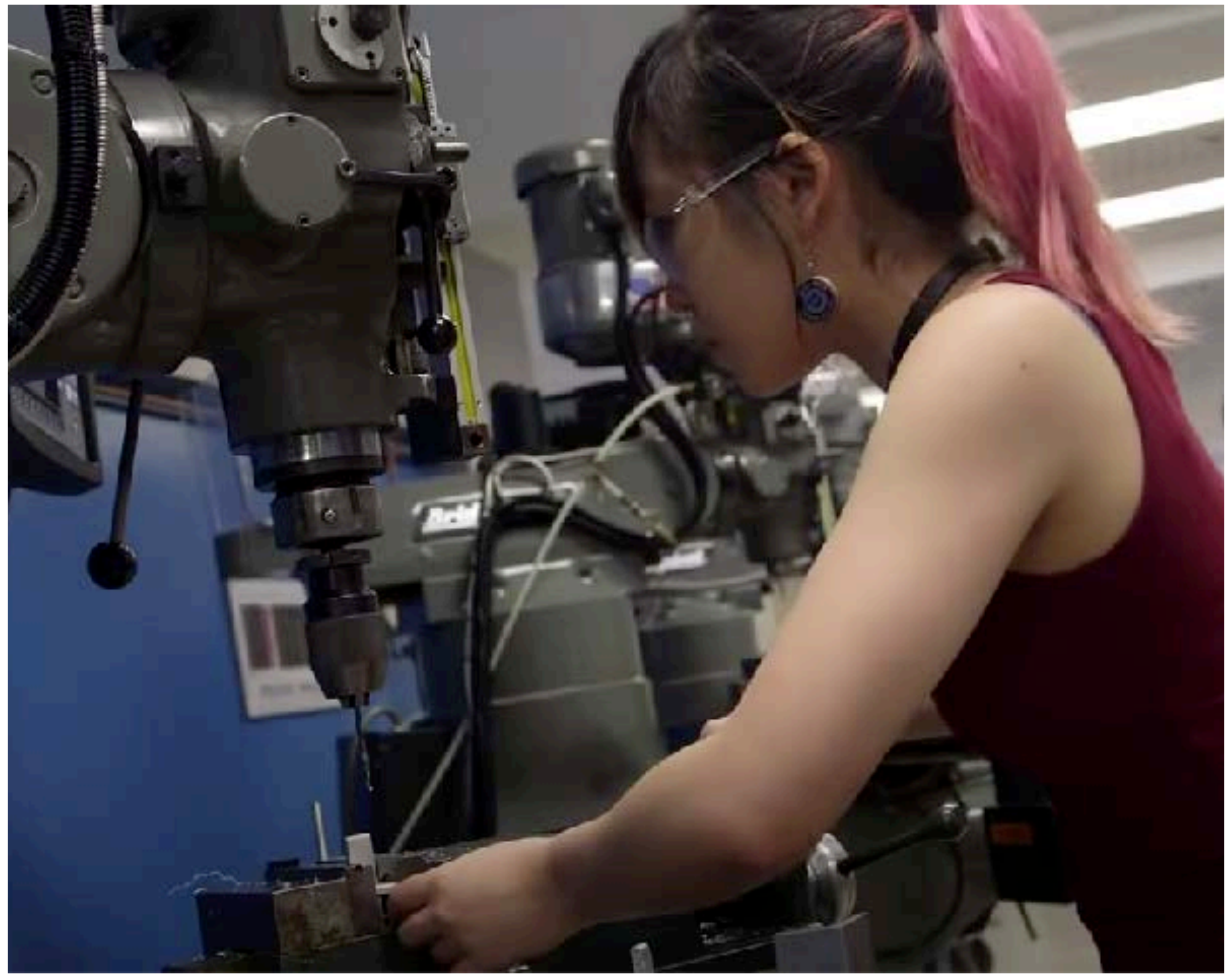
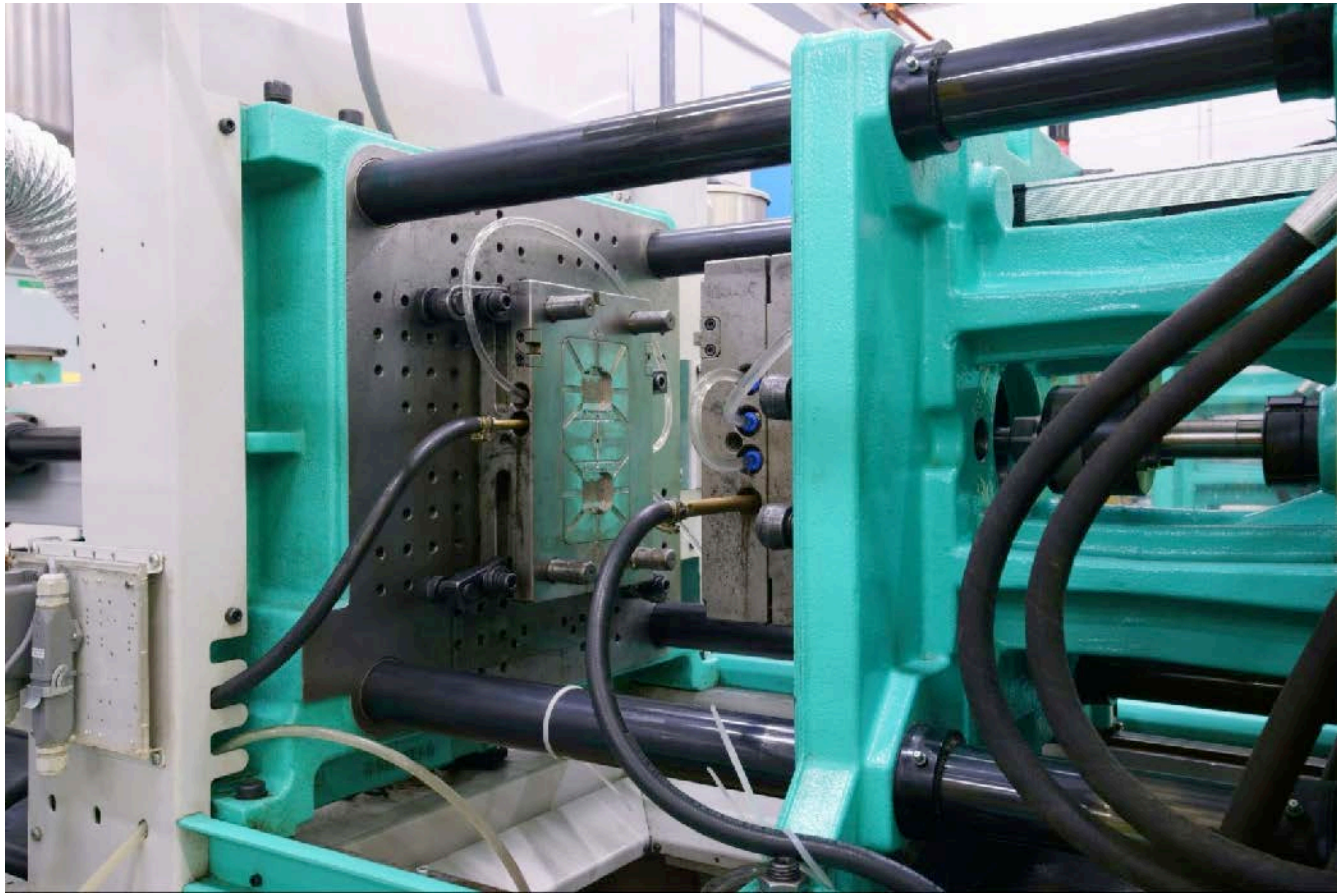
prototype machining **vs.** manufacturing: machine production









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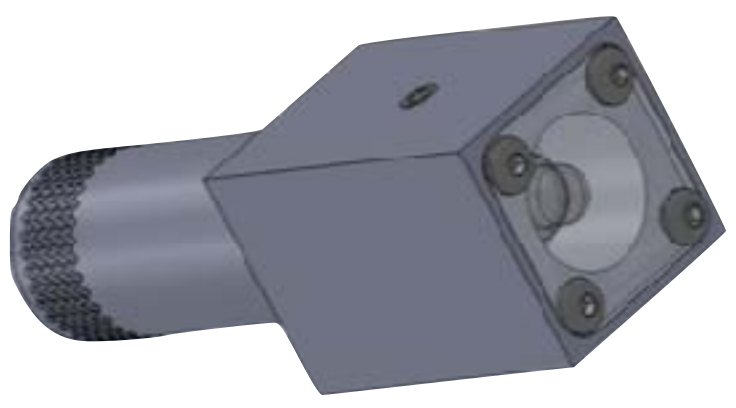
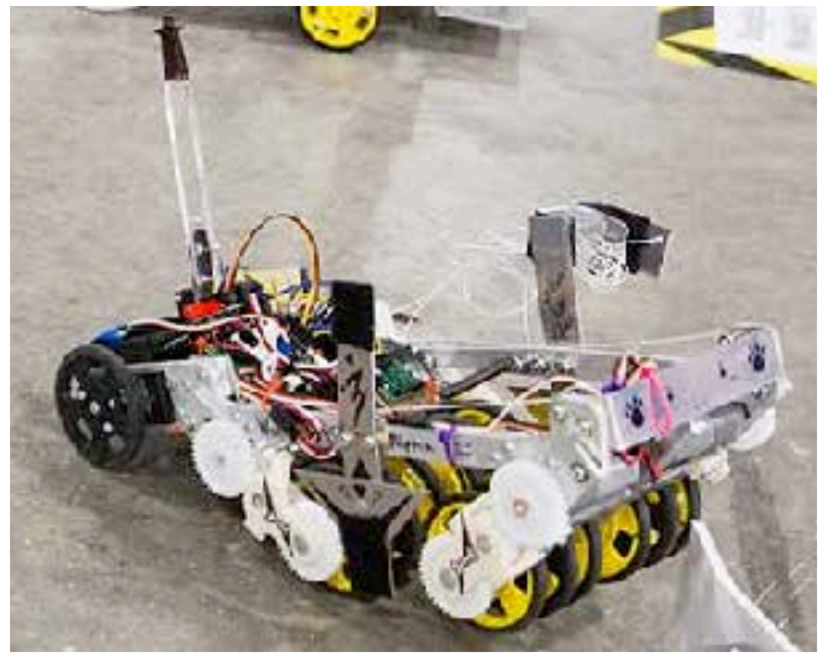
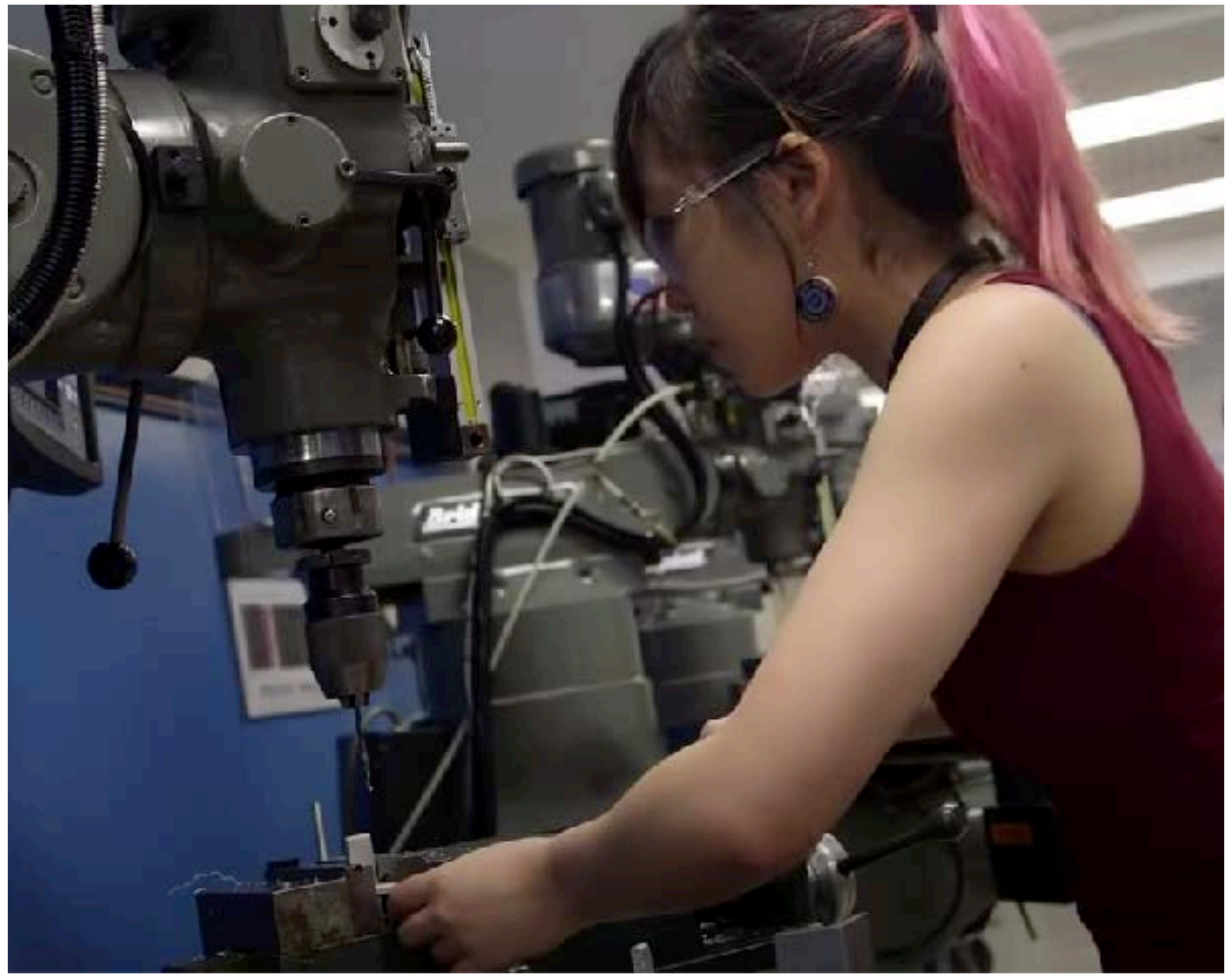
Cutting Analysis: Mechanics, Forces, and Power

prototype machining **vs.** manufacturing: primary manufacturing process

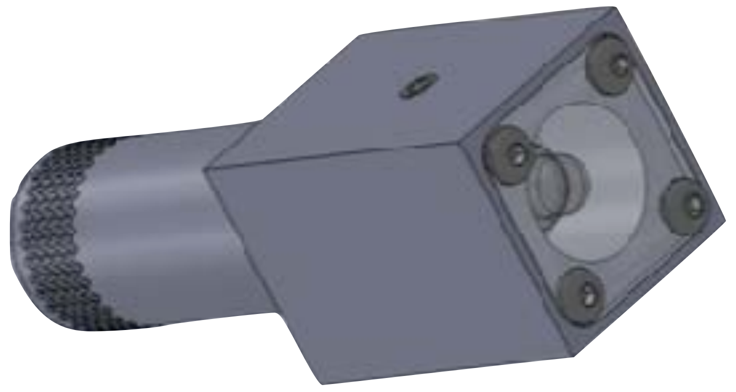


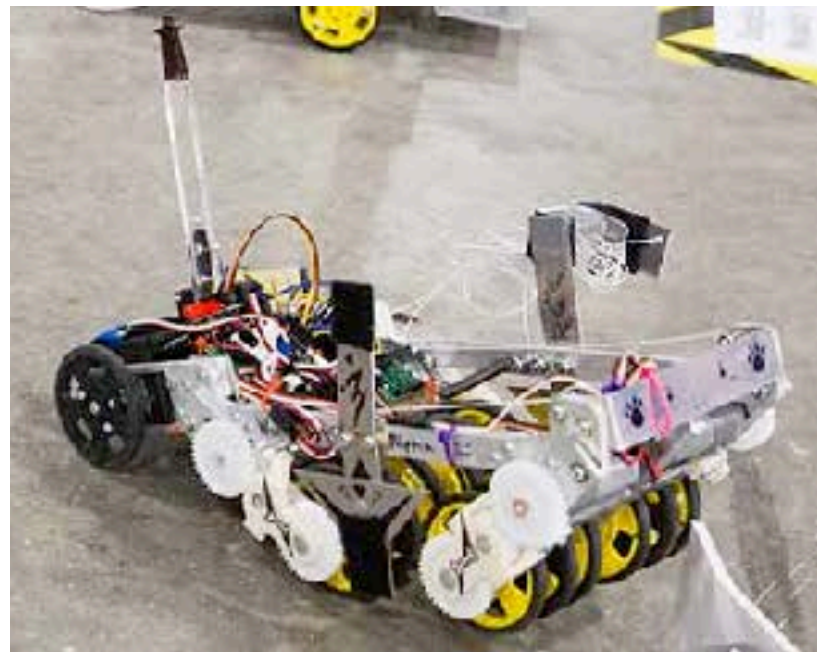


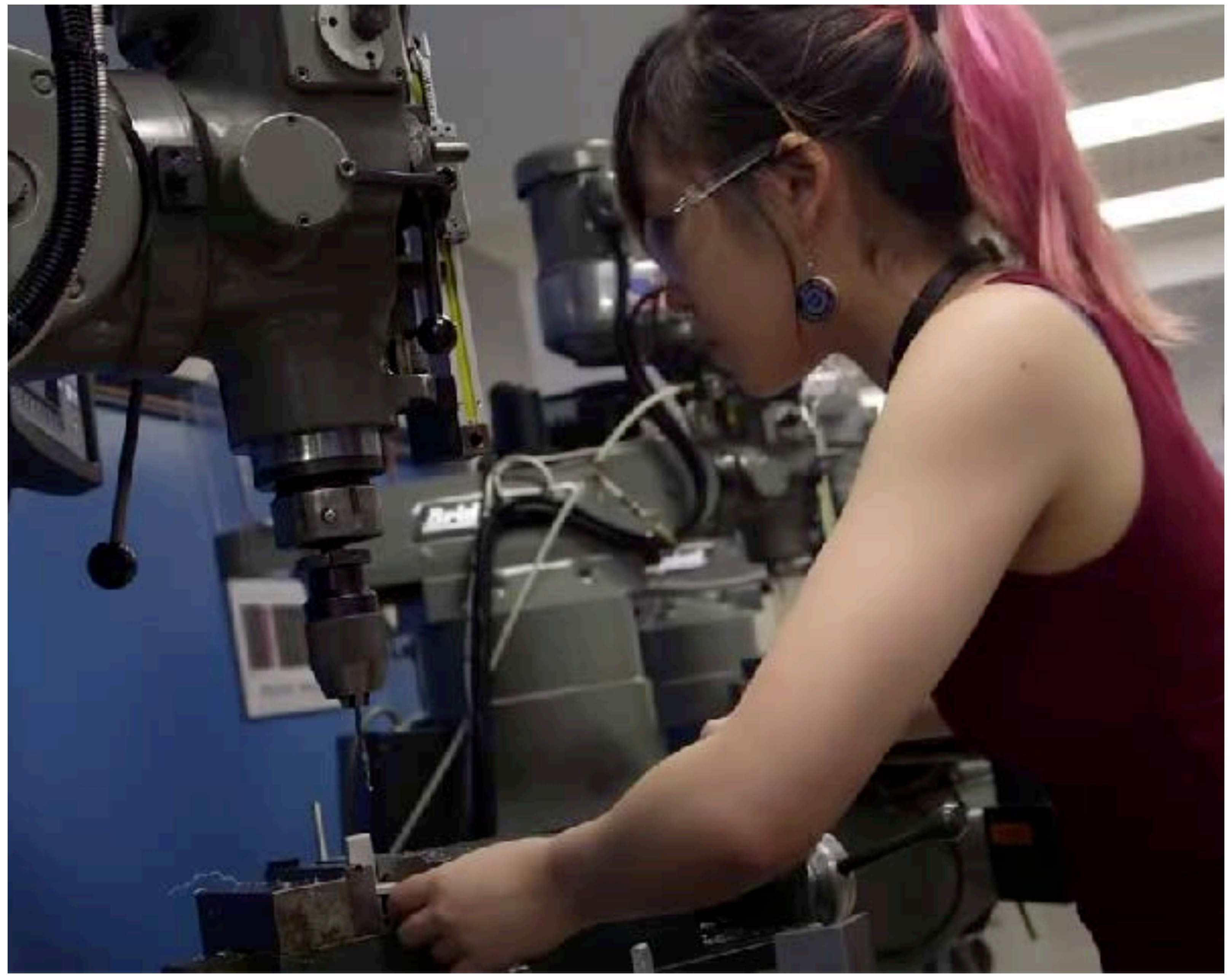



SLIDING HEADSTOCK
2160137

prototype machining **vs.** manufacturing: secondary manufacturing process







2.008 Coverage of Machining

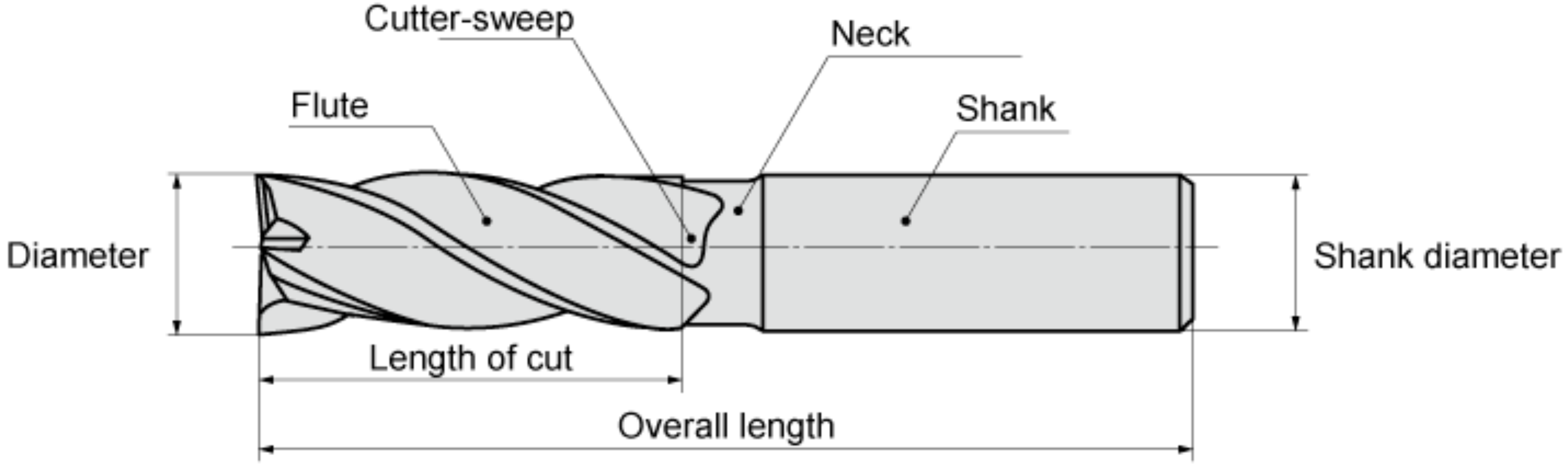
Cutting #1: Cutting Analysis geometry & motion, cutting forces, energy and power

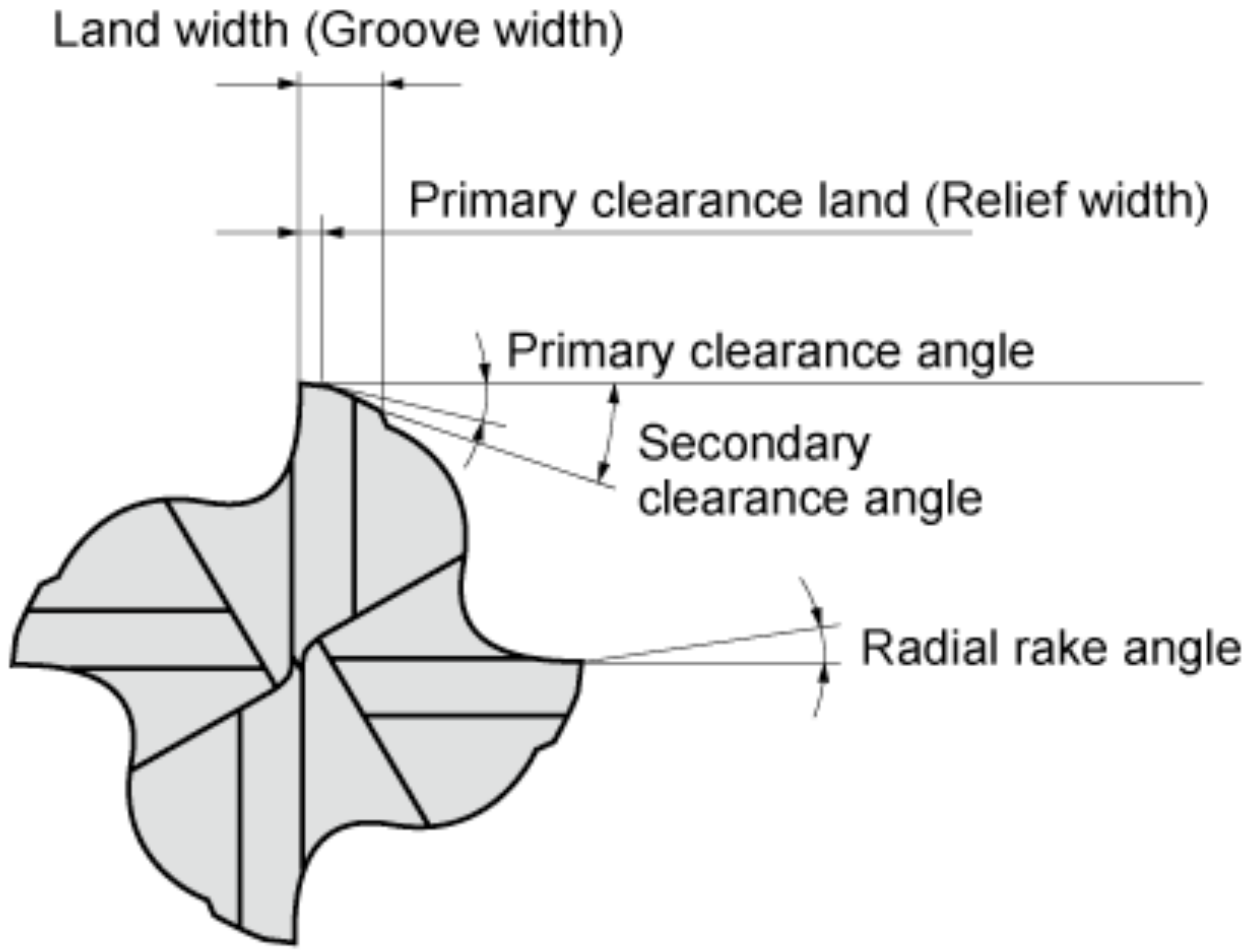
Cutting #2: Forces and Power Demos

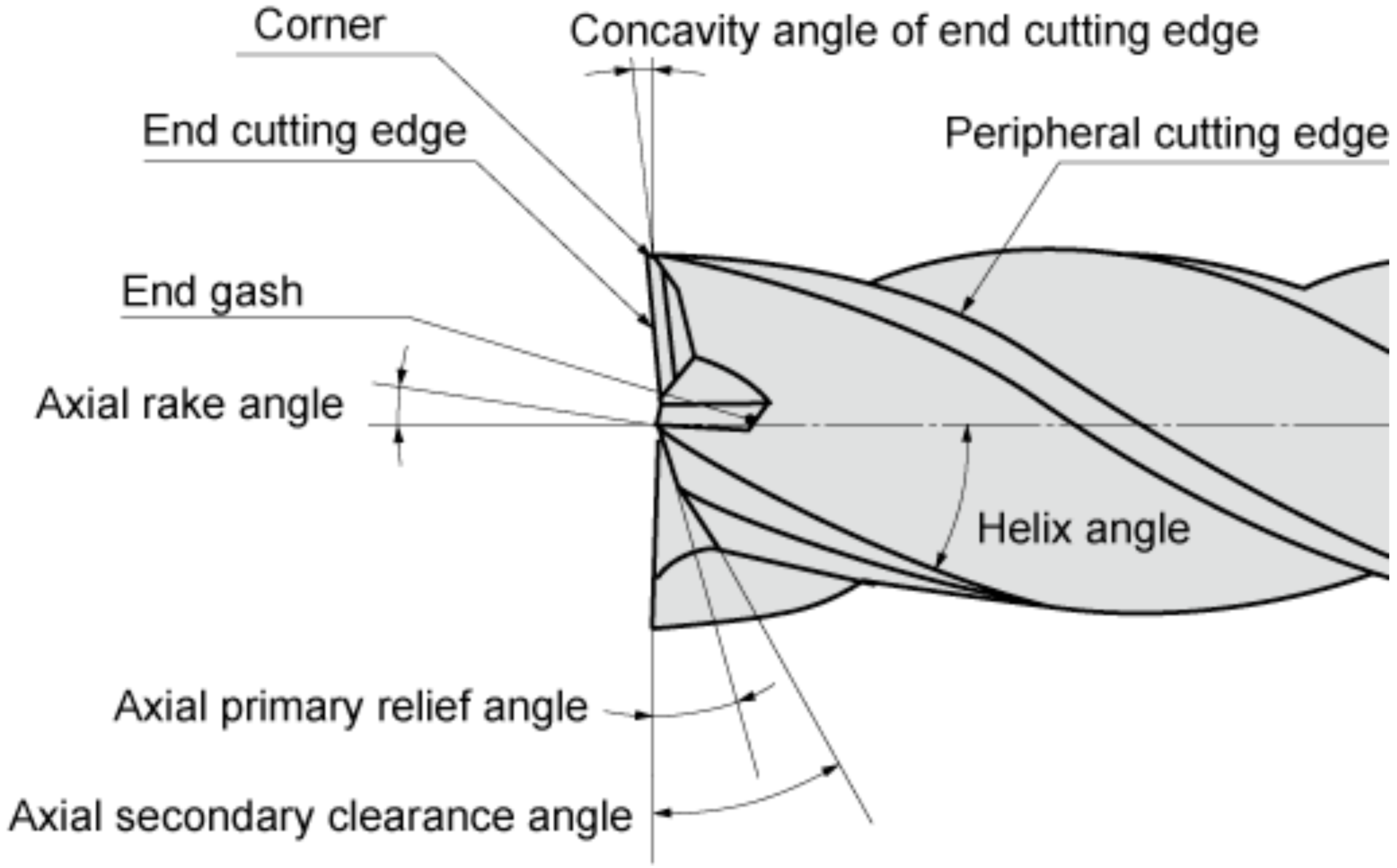
Cutting #3: Practical Considerations

1. Geometry and Motion

Cutting Geometry



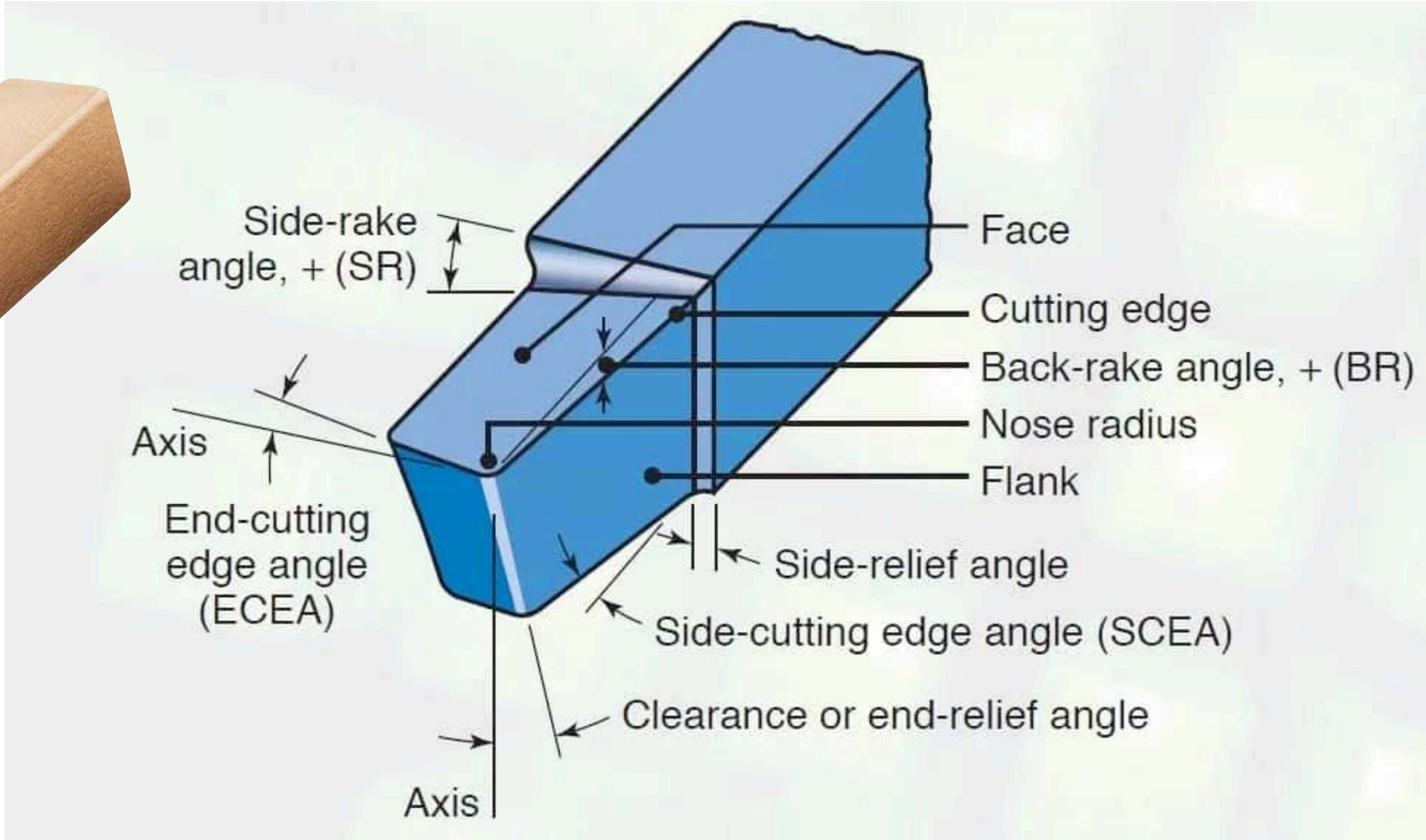




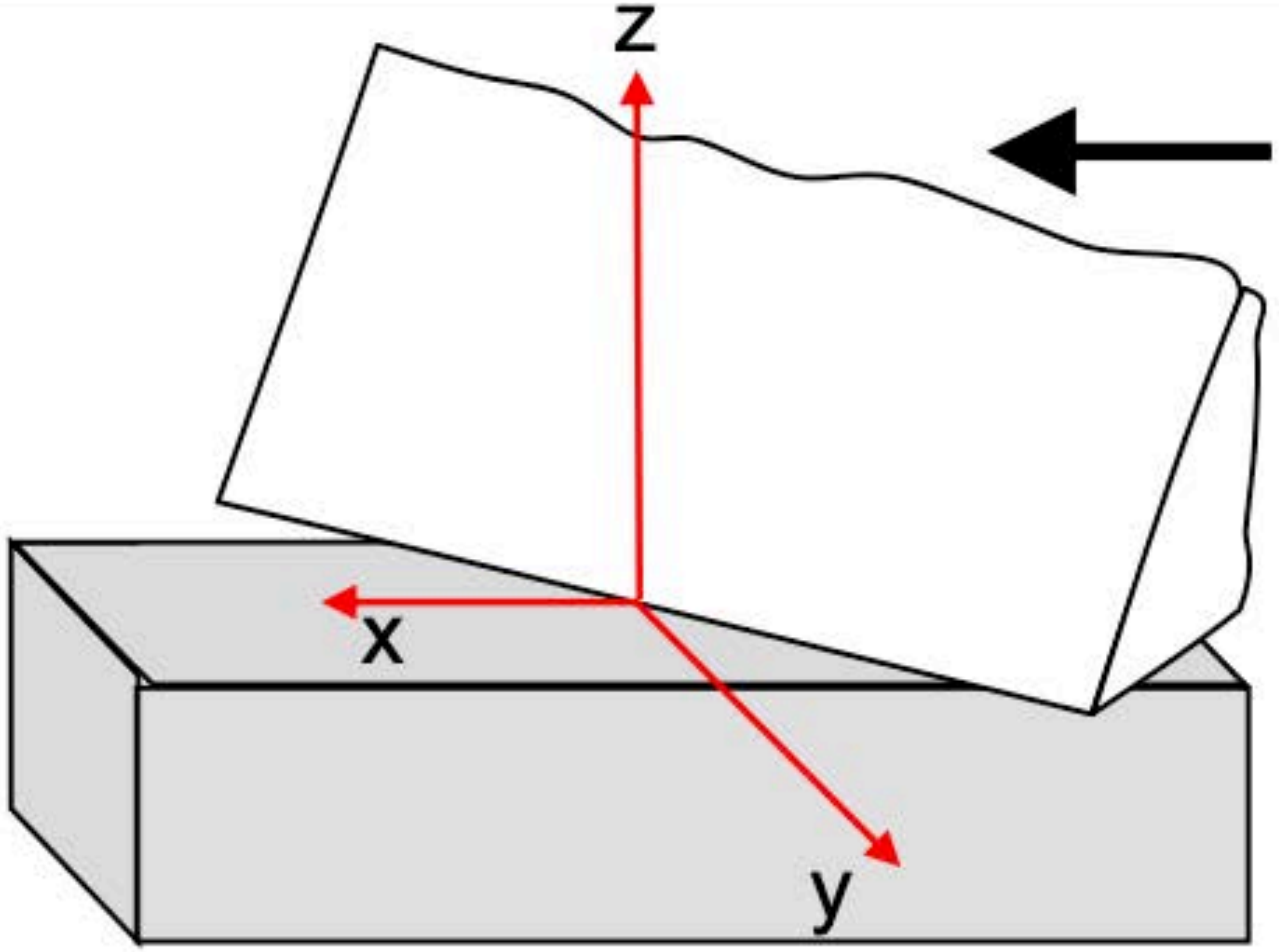
Cutting #1

Cutting Analysis: Mechanics, Forces, and Power

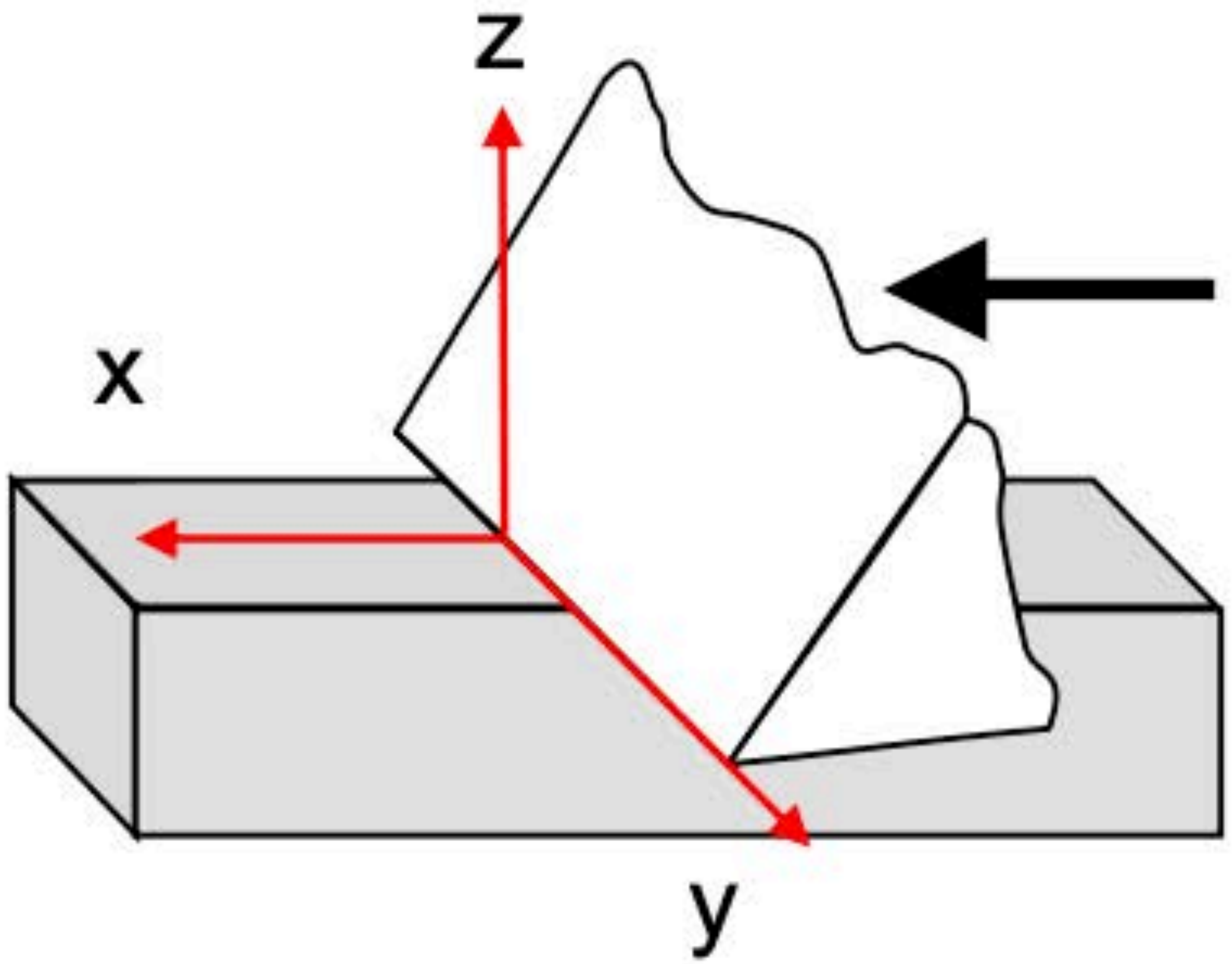
Cutting Geometry



Cutting Model

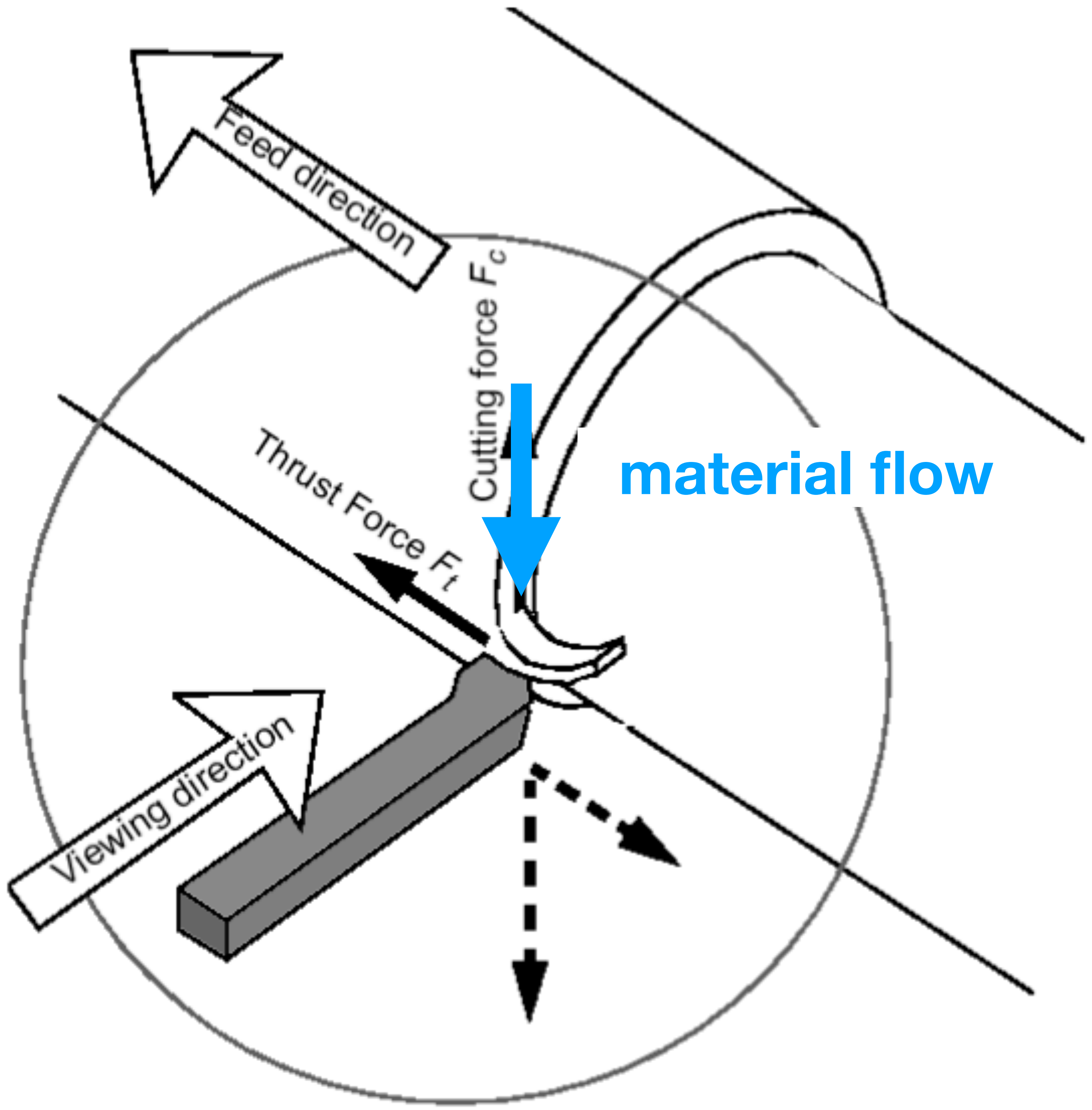


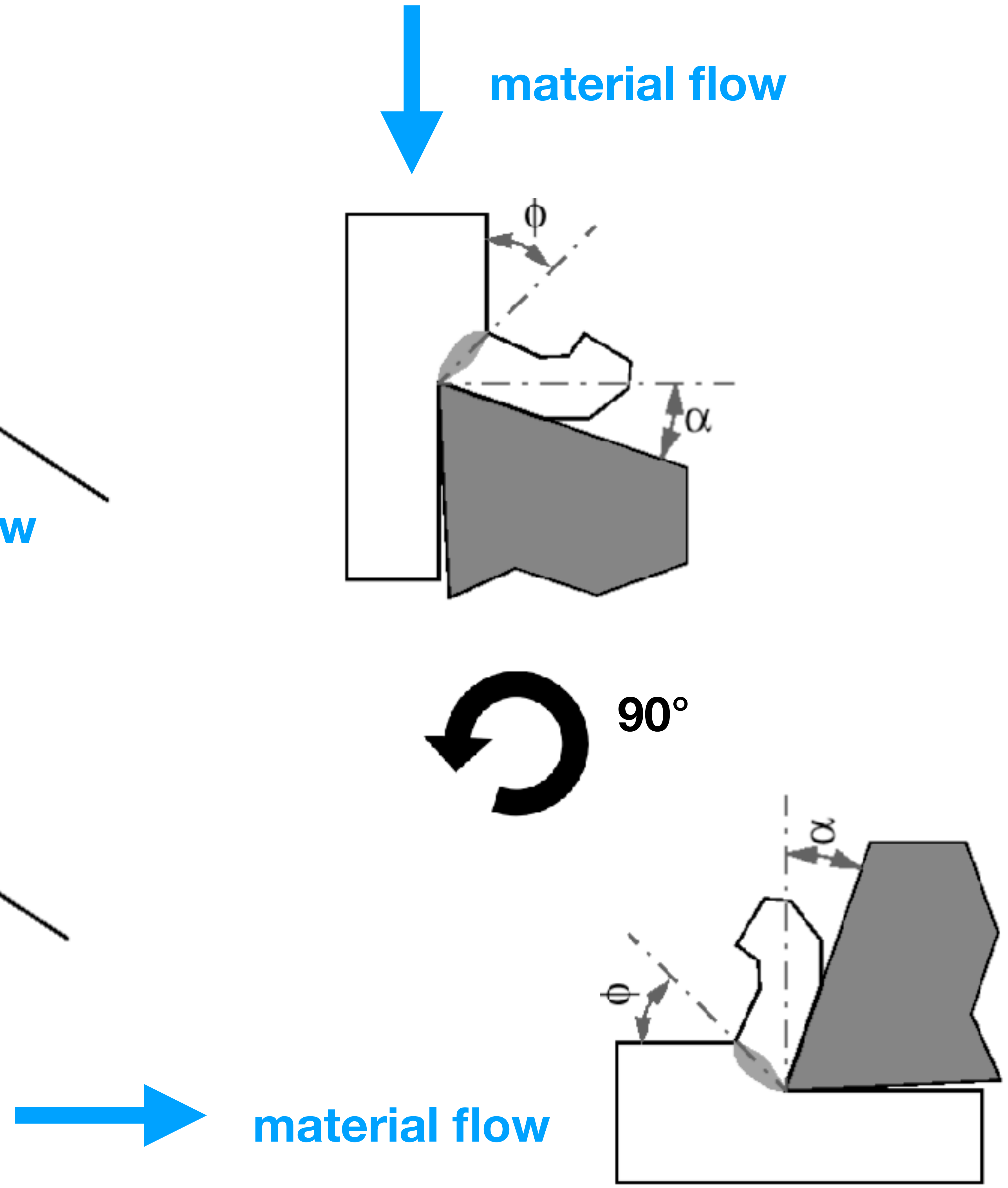
Oblique (3D)



Orthogonal (2D)

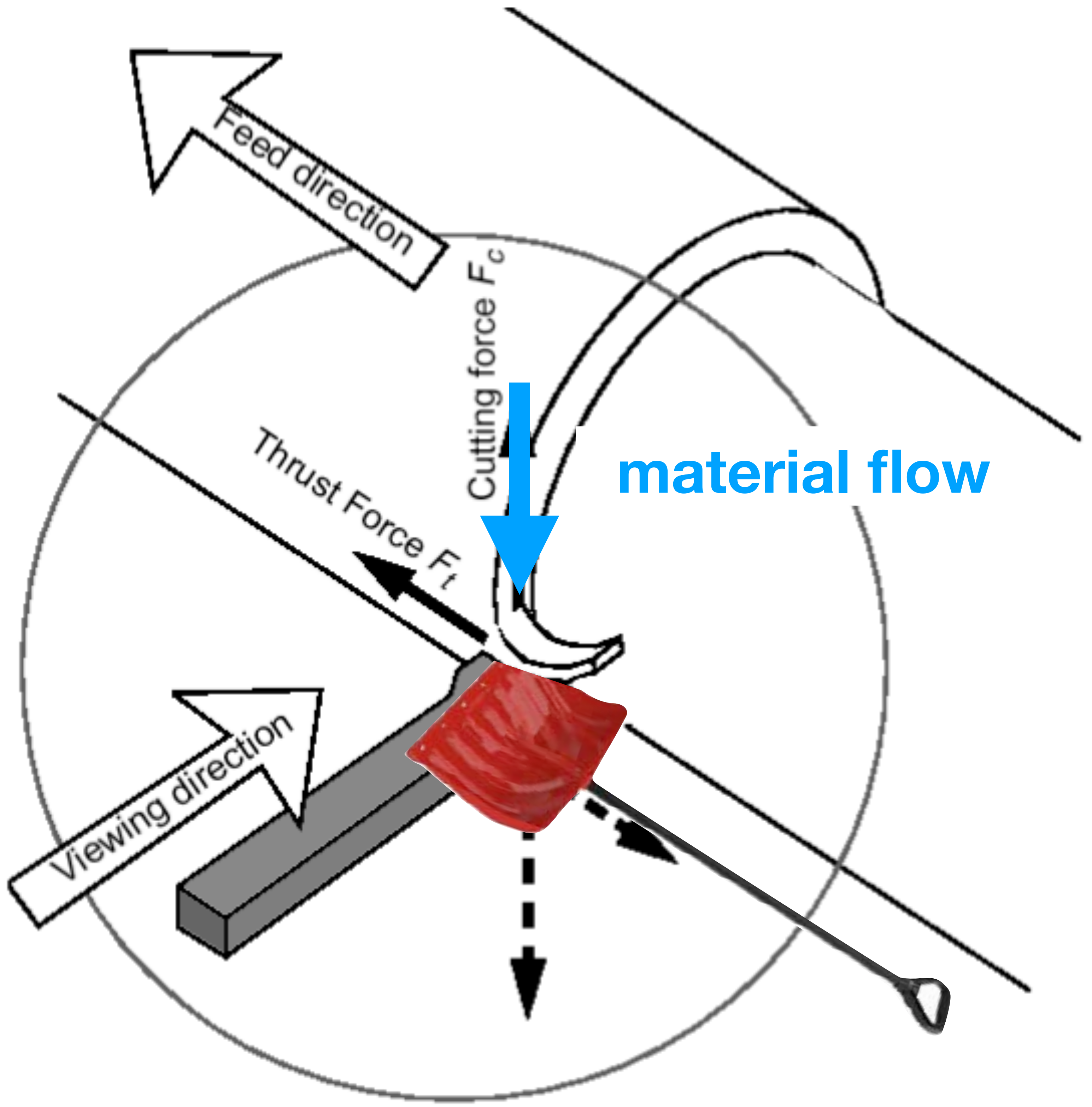
Cutting Theory on a Lathe

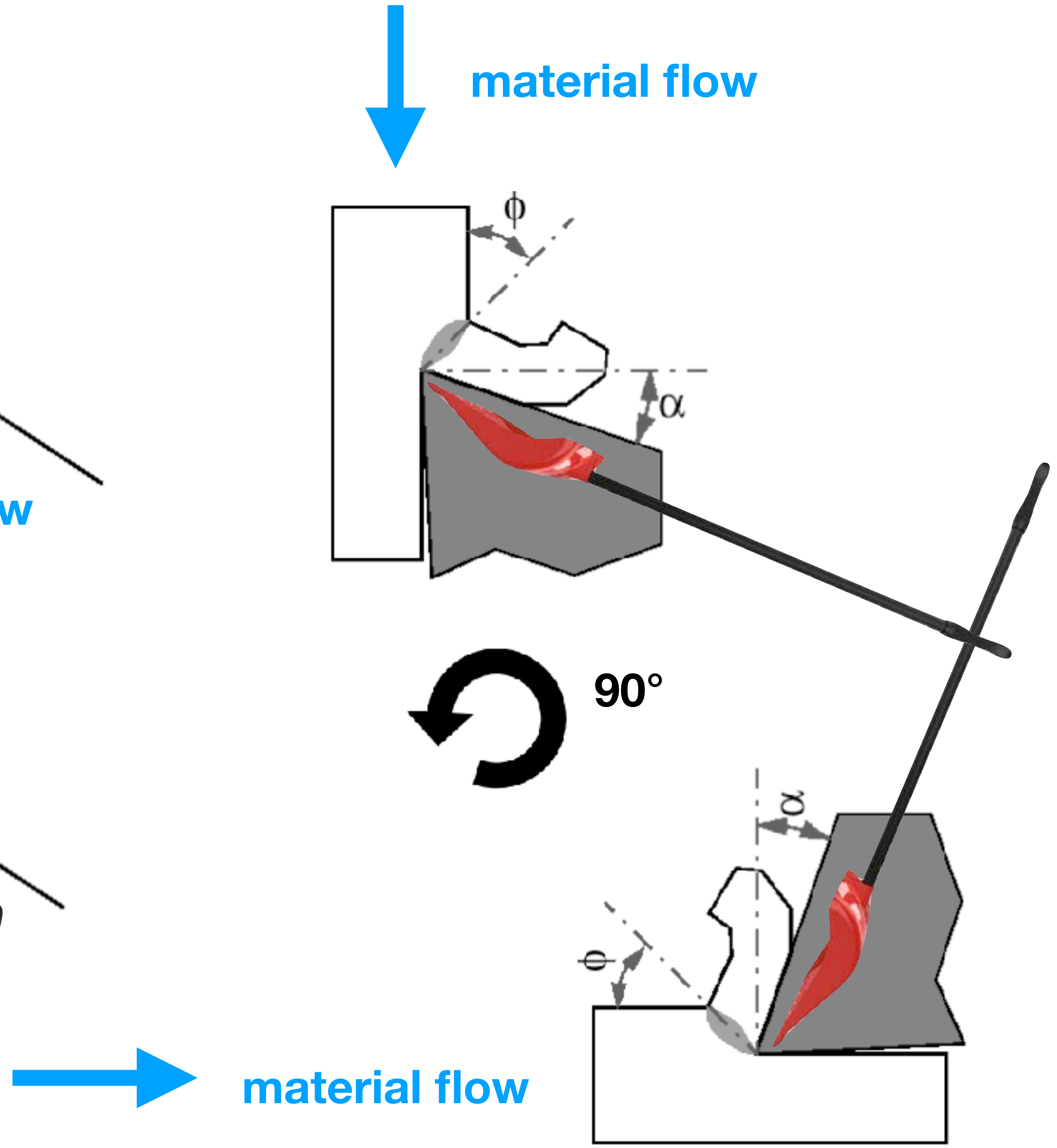




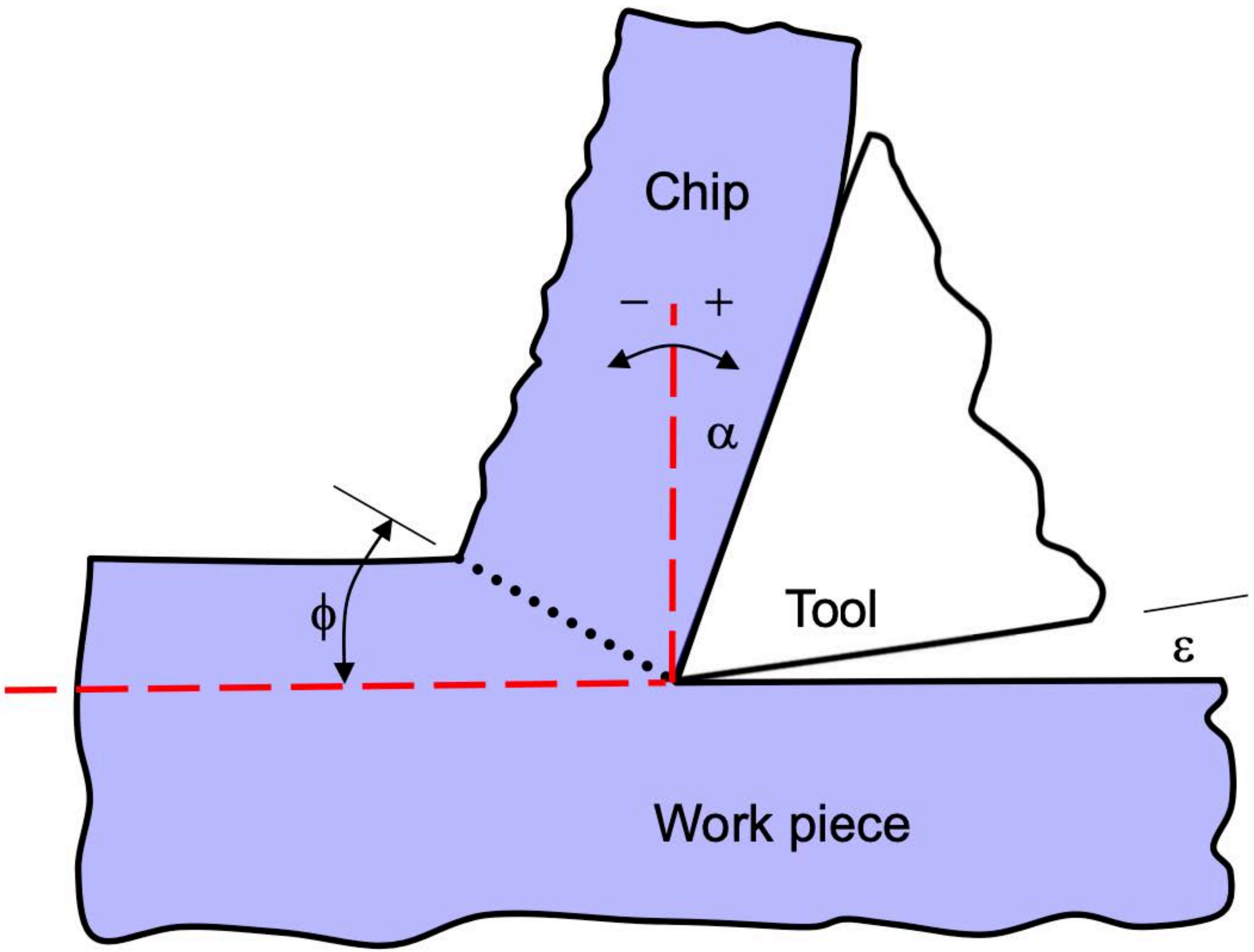
~~Cutting Theory~~ on a Lathe

Shovel Theory





Cutting Model



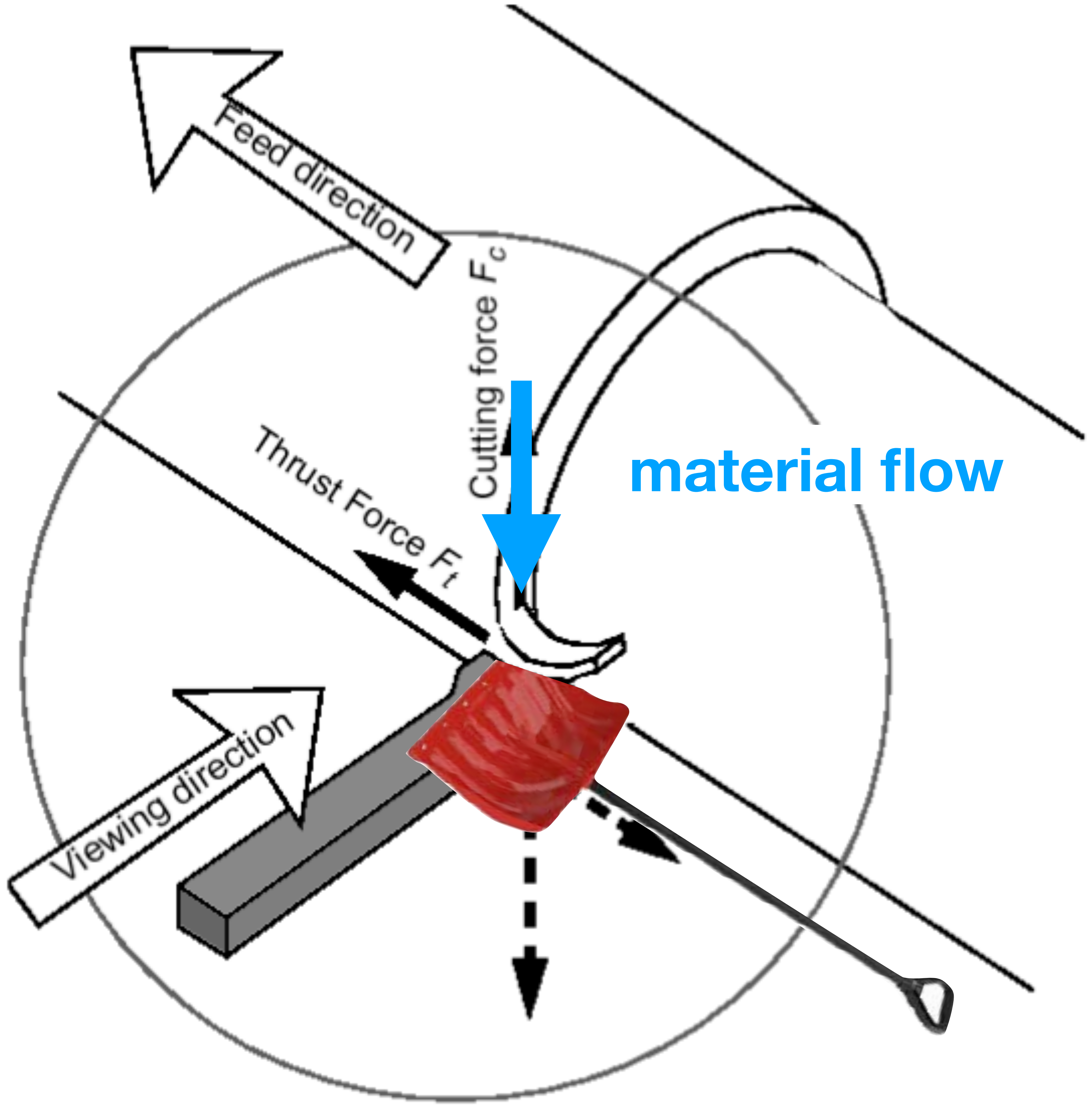
Φ: shear angle
α: rake angle
ε: relief angle

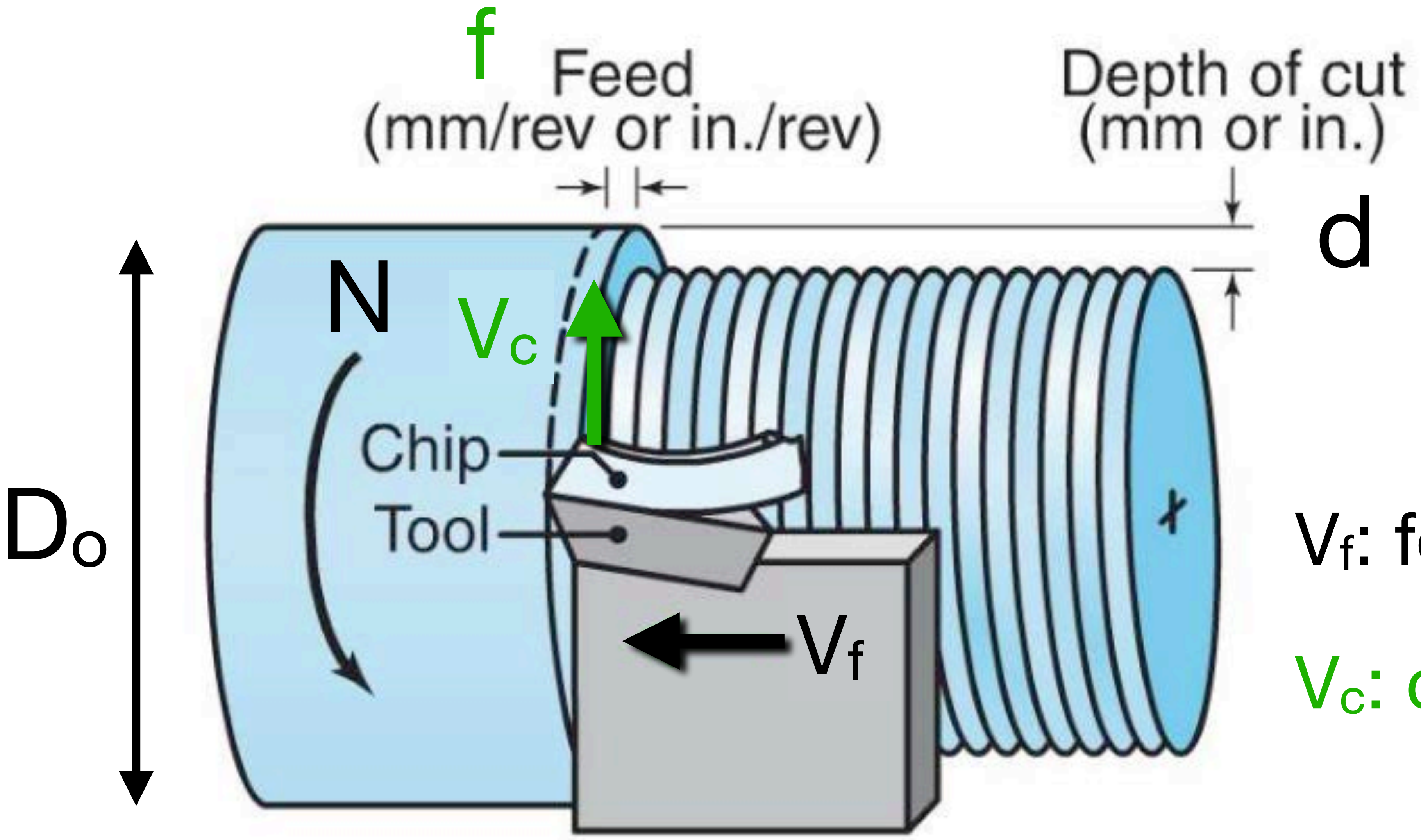

LIVE
EDS
0.322
100µm
Mag.
250 ×
FW
2076 µm
HV
5kV
Int.
Image
Det.
BSD Full
WD
11.317 mm
Pres.
0.32 Pa
2023/5/26 18:45
in-situ-m2

Cutting #1

Cutting Analysis: Mechanics, Forces, and Power

Lathe Parameters



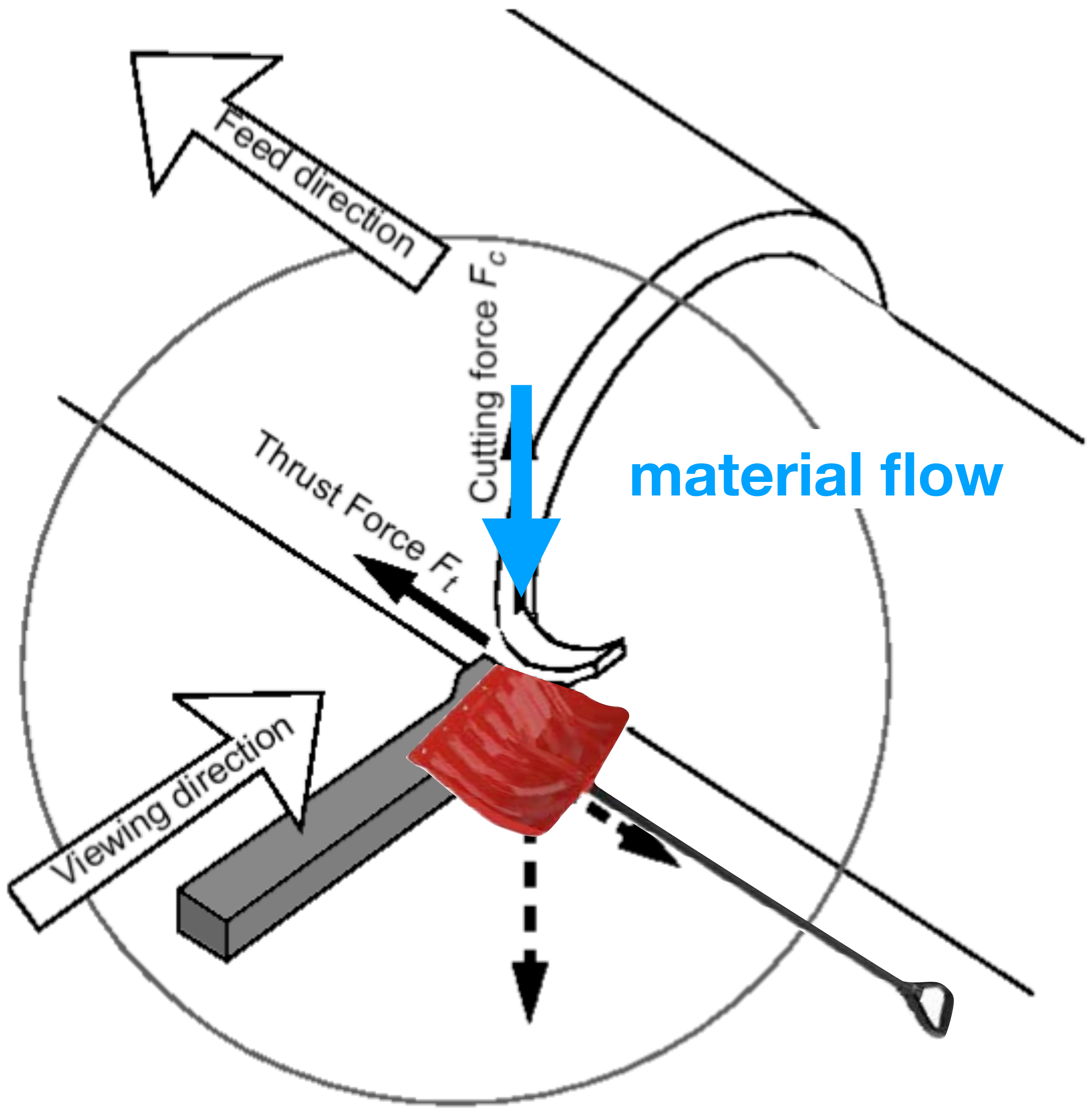


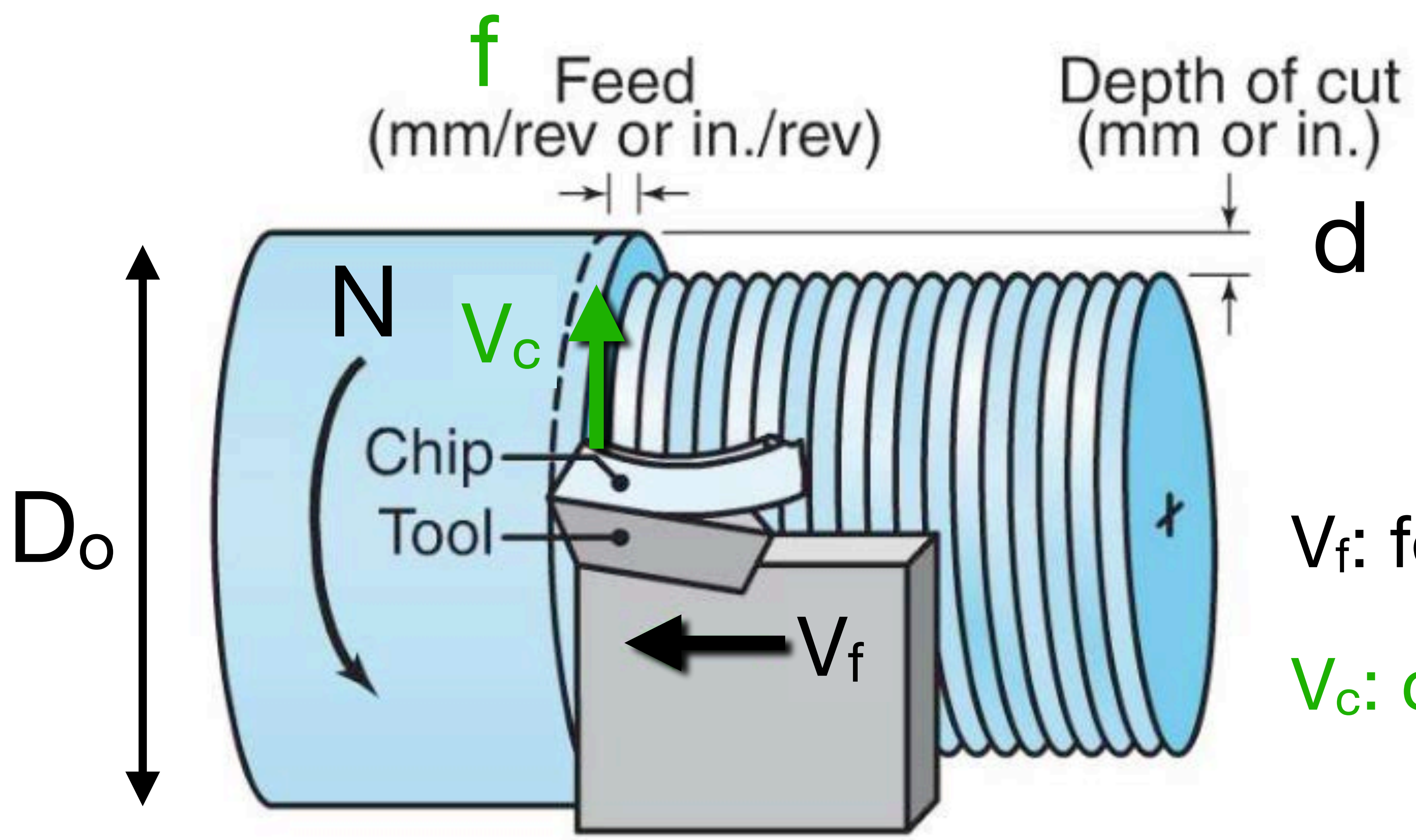
d: depth of cut [in]
f or t_0: feed [in/rev]
N: spindle speed [rev/min]
Do: original diameter [in]

V_f: feed rate = f*N [in/min]

V_c: cutting velocity = π*D*N [in/min]

Lathe Parameters





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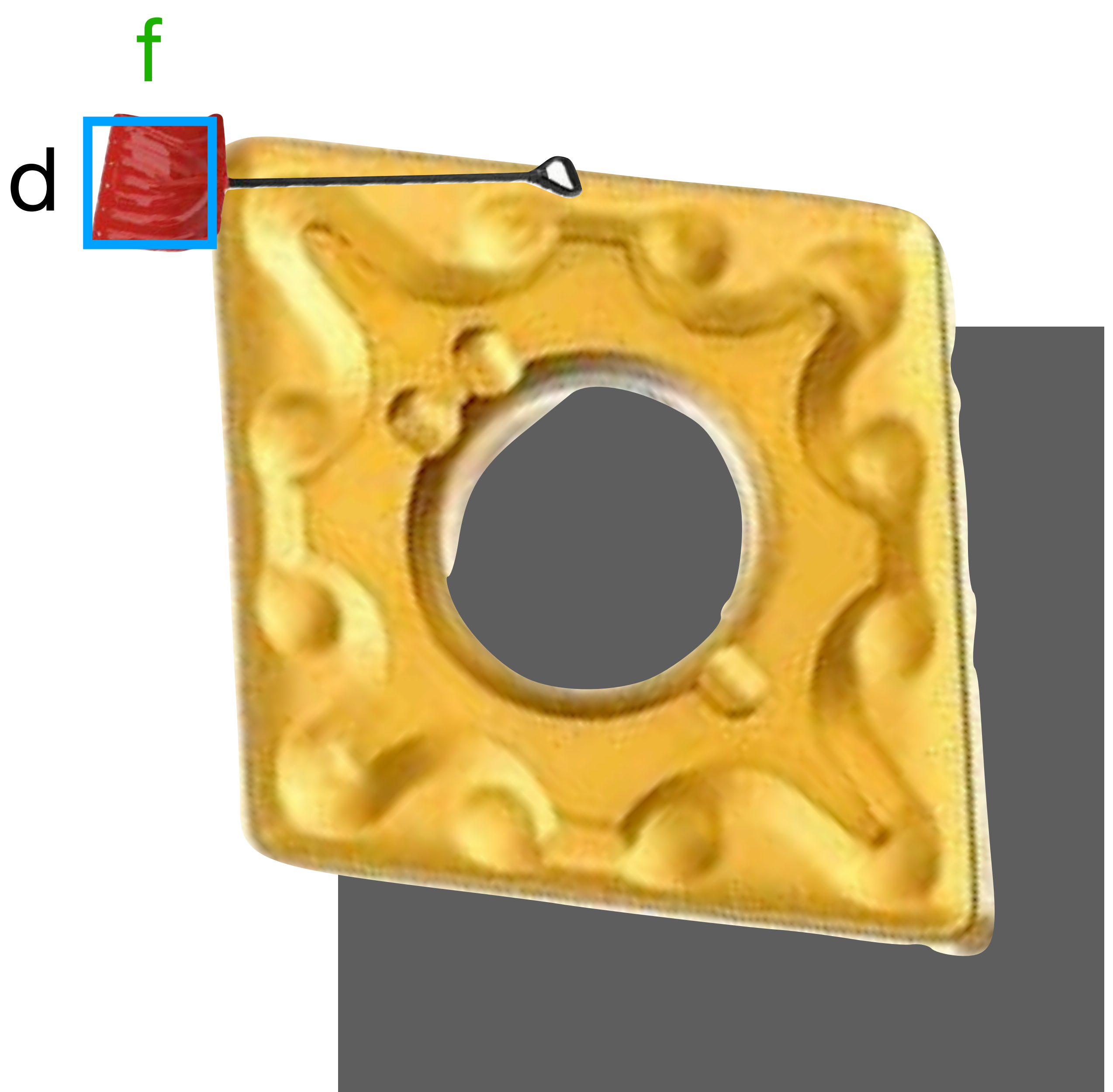



Blondihacks.com

Cutting Velocities

v_{chip} vs v_{cut}?

mass conservation: $\dot{m}_{in} = \dot{m}_{out}$

$$\rho \frac{Volume_{in}}{time} = \rho \frac{Volume_{out}}{time}$$

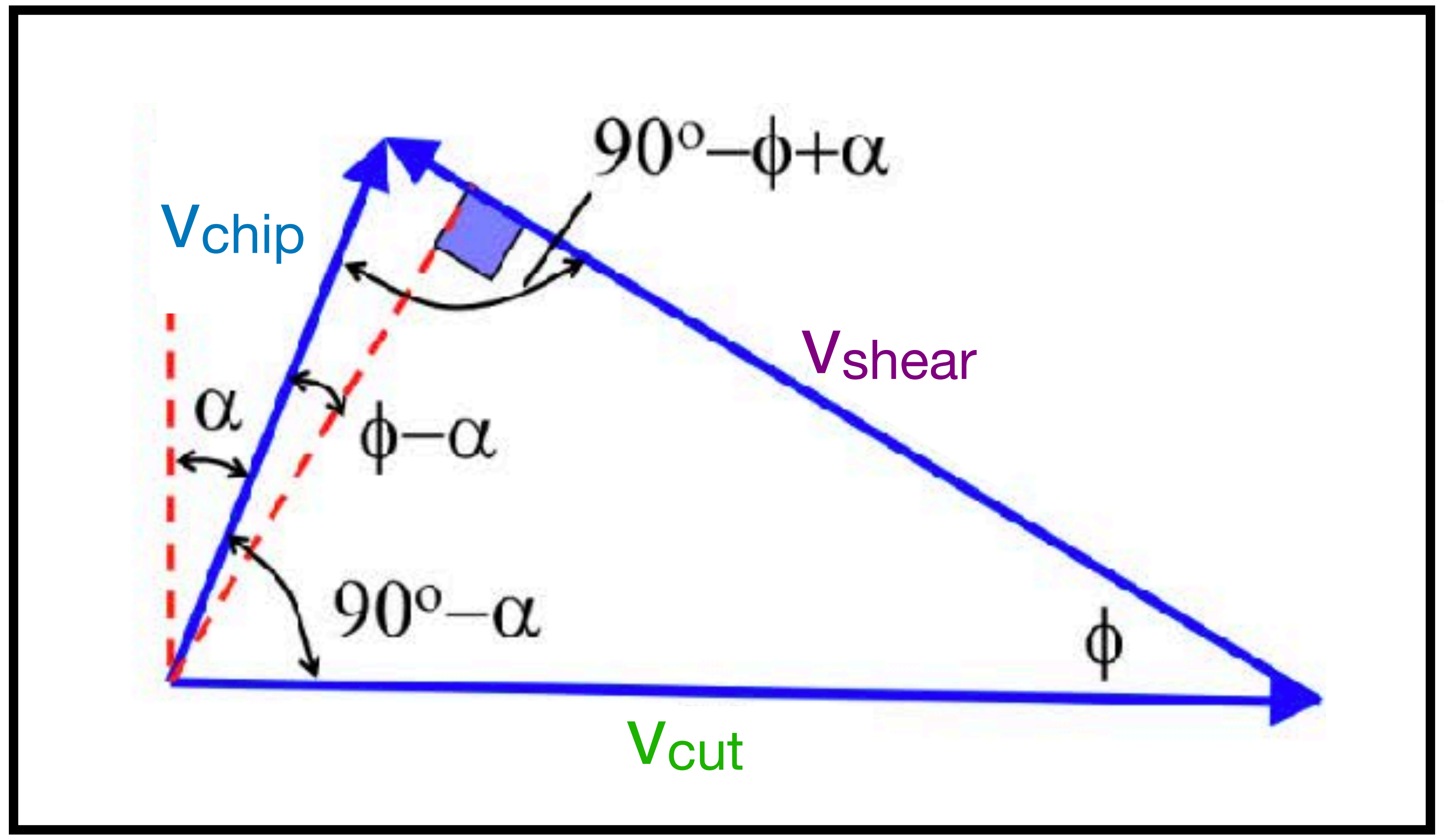
$$\rho t_0 d v_{cut} = \rho t_c d v_{chip}$$

⚠ notation: t_0 instead of f

$$\frac{v_{chip}}{v_{cut}} = \frac{t_0}{t_c}$$

$$\frac{v_{chip}}{v_{cut}} = \frac{t_0}{t_c} = r = \frac{sin(\phi)}{cos(\phi - \alpha)}$$

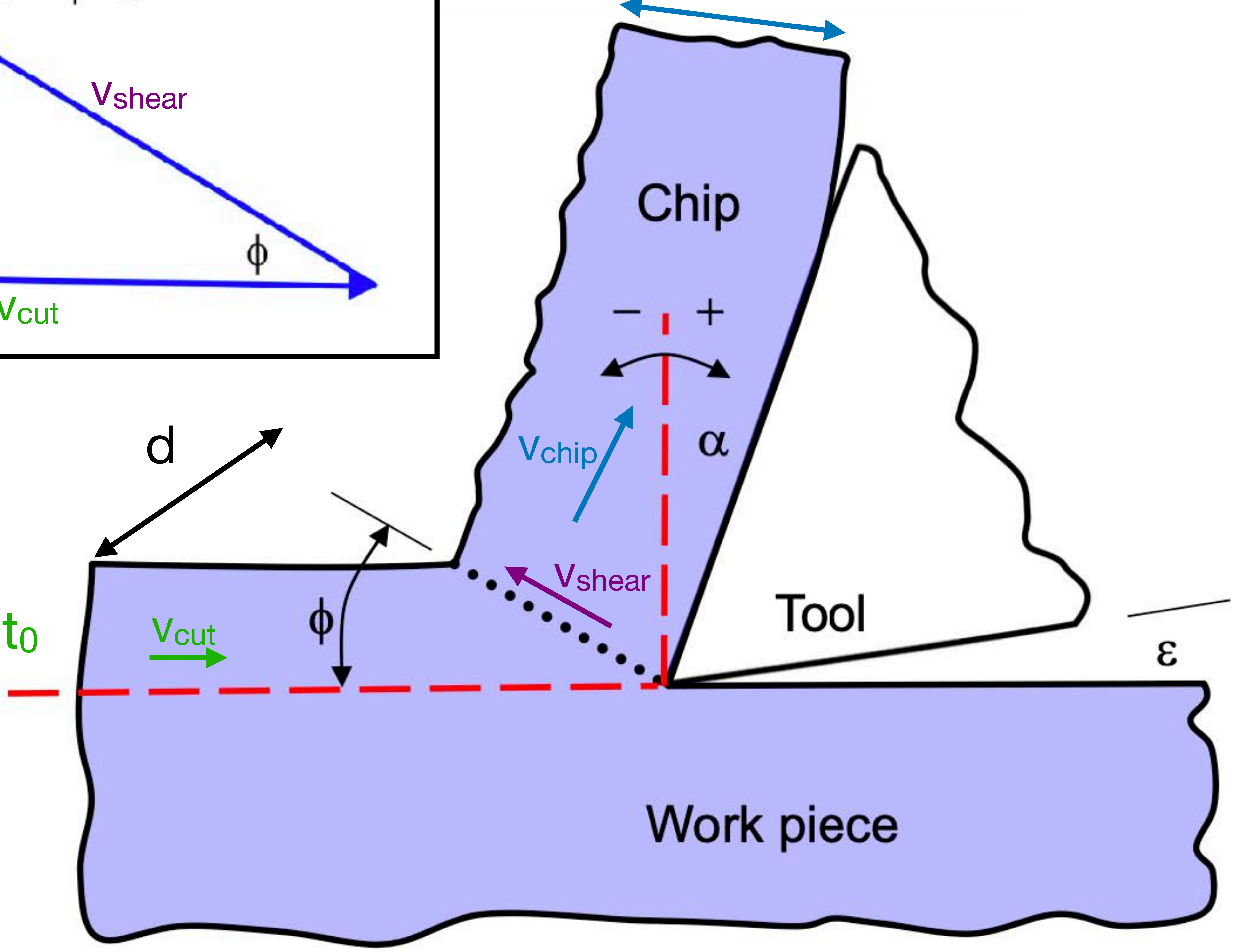
measure chip to get shear angle!



law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

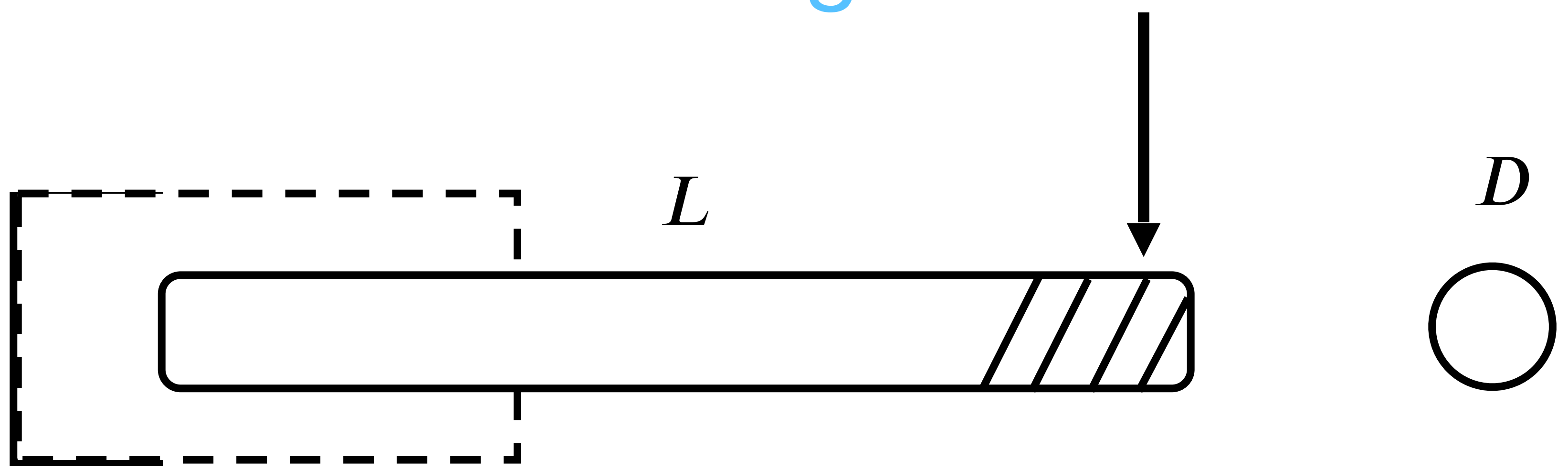
Φ: shear angle
α: rake angle
ε: relief angle
t_c or t_{chip}: thickness of the chip
f or t_0: feed, or material that becomes the chip
d: depth of cut (into the page)



2. Cutting Forces

Cutting boils down to two things:

shear and… beam bending!

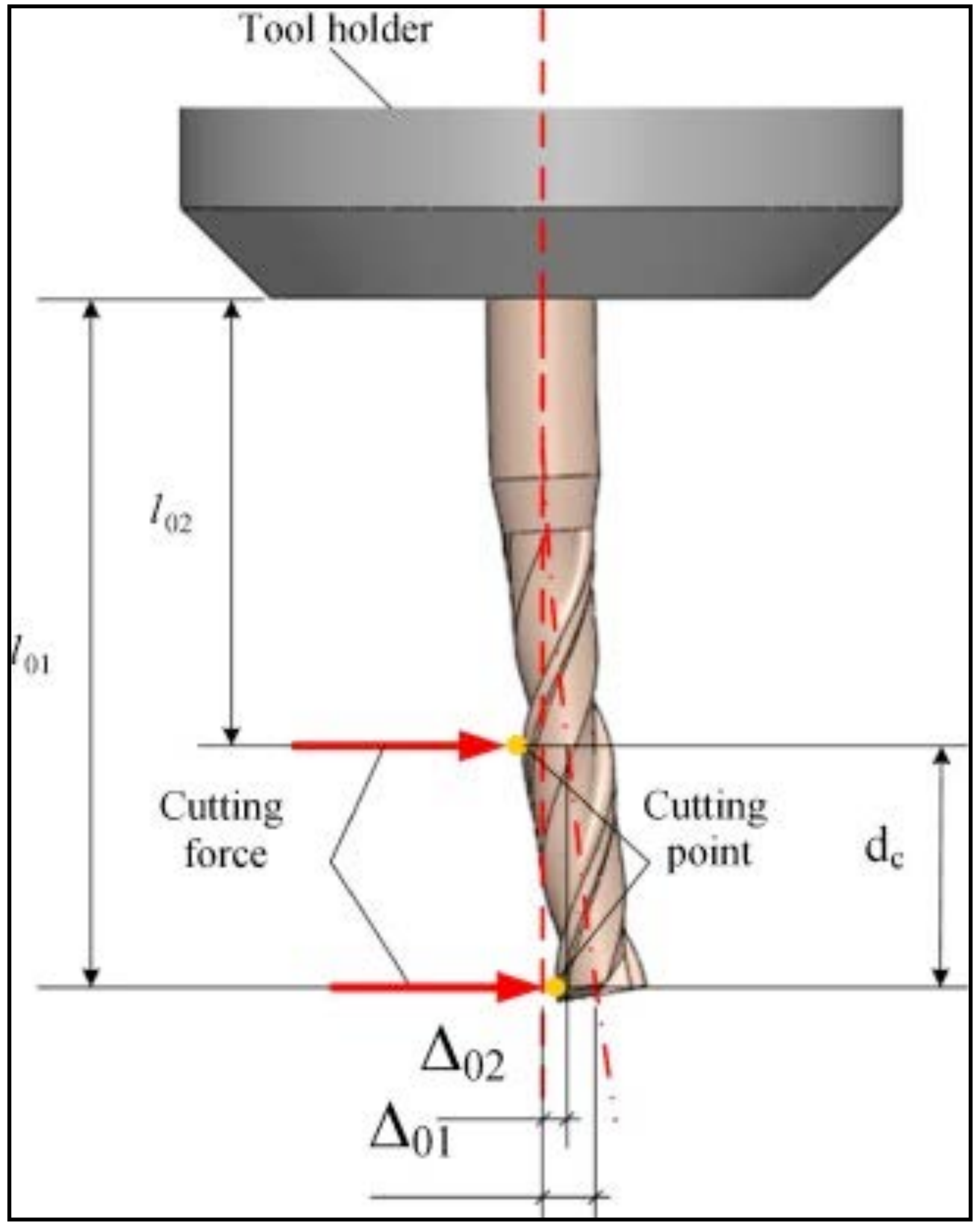


beam bending:

$$\delta = \frac{FL^3}{3EI} \qquad F = \frac{3\delta EI}{L^3}$$

$$k = \frac{\partial F}{\partial \delta} = \frac{3\pi}{64}\frac{D^4}{L^3}E$$

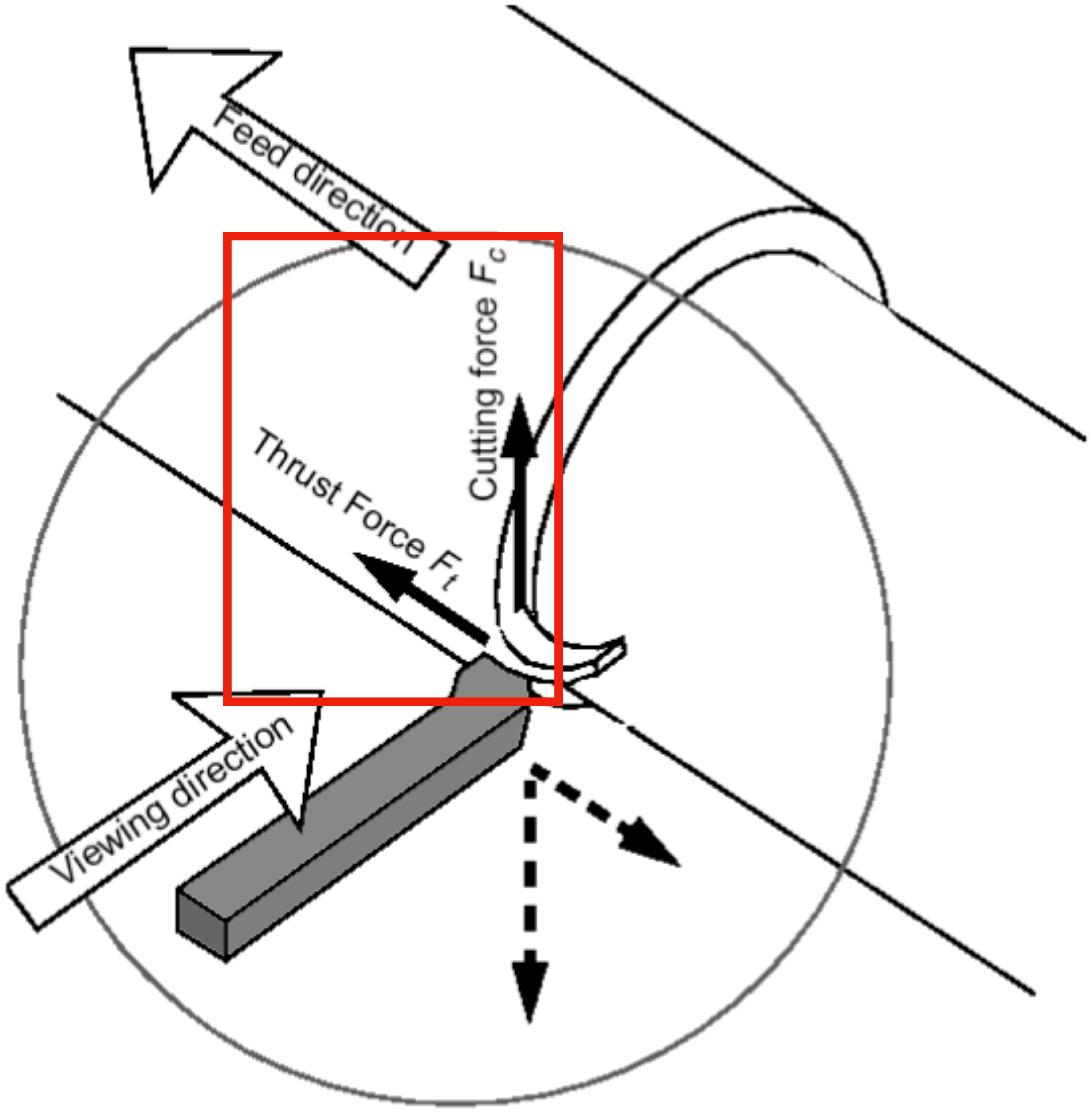
δ: amount of deflection
F: force
L: length
E: elastic modulus of the tool material
I: area moment of inertia
k: stiffness



so, we need to know about forces

Cutting Force and Thrust Force

let’s start by examining these two forces



Cutting #1

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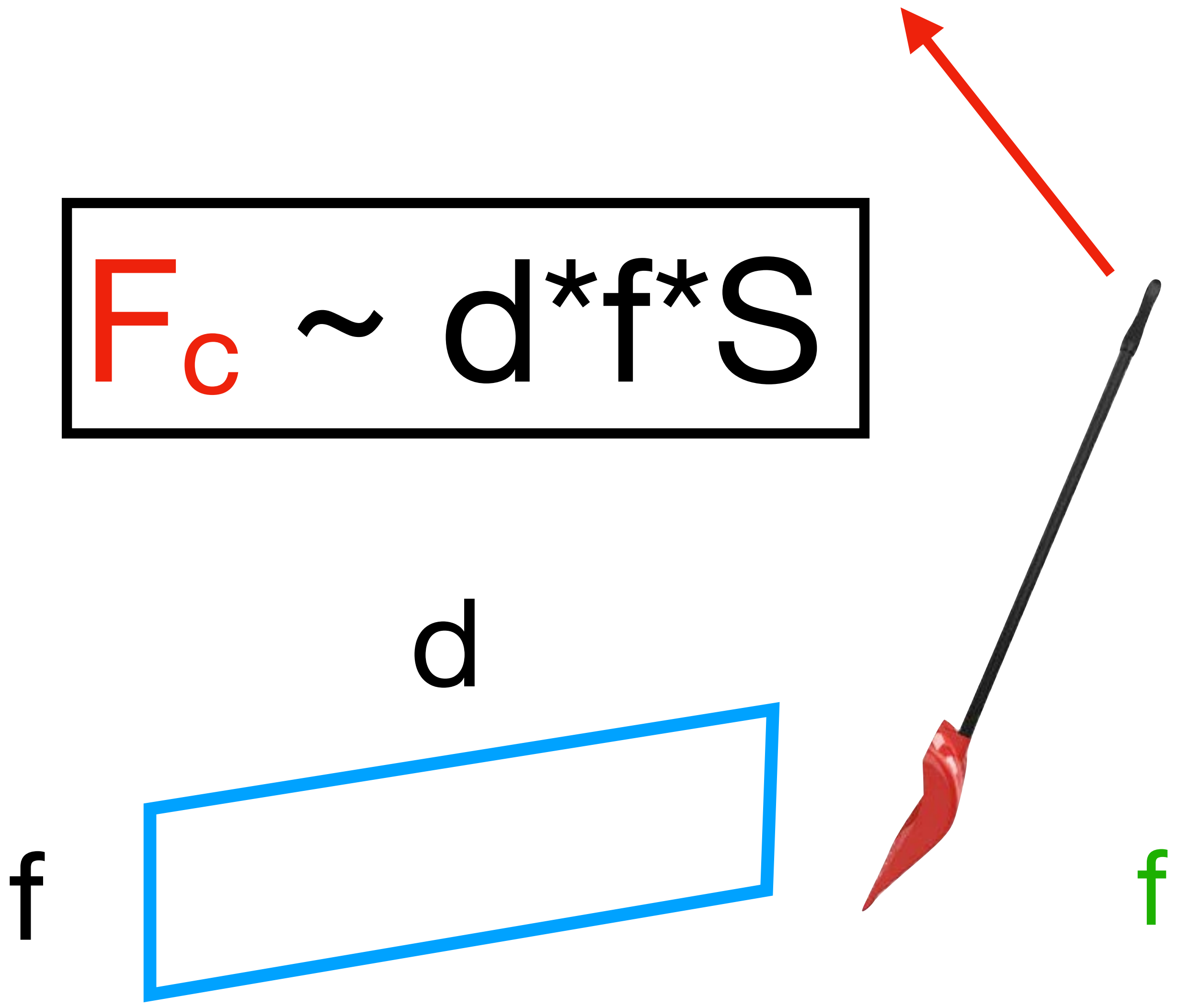
Cutting Force

more snow in contact with shovel: ⬆ force

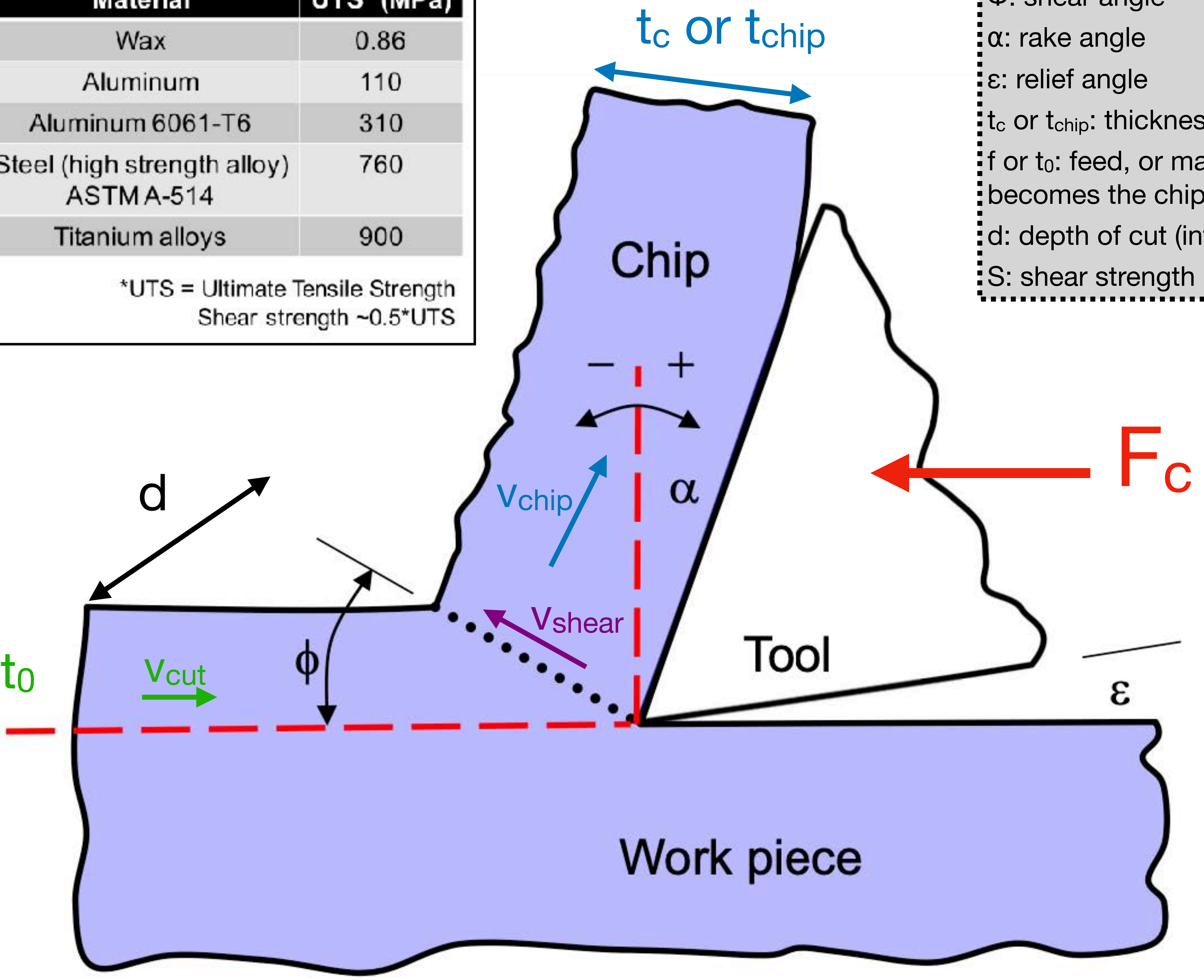
moving faster: ⬆ power (minimal ⬆ force)

$$F_c \sim d*f*S$$



Material	UTS* (MPa)
Wax	0.86
Aluminum	110
Aluminum 6061-T6	310
Steel (high strength alloy) ASTM A-514	760
Titanium alloys	900

*UTS = Ultimate Tensile Strength
Shear strength ~0.5*UTS



Φ: shear angle
α: rake angle
ε: relief angle
t_c or t_{chip}: thickness of the chip
f or t_0: feed, or material that becomes the chip
d: depth of cut (into the page)
S: shear strength

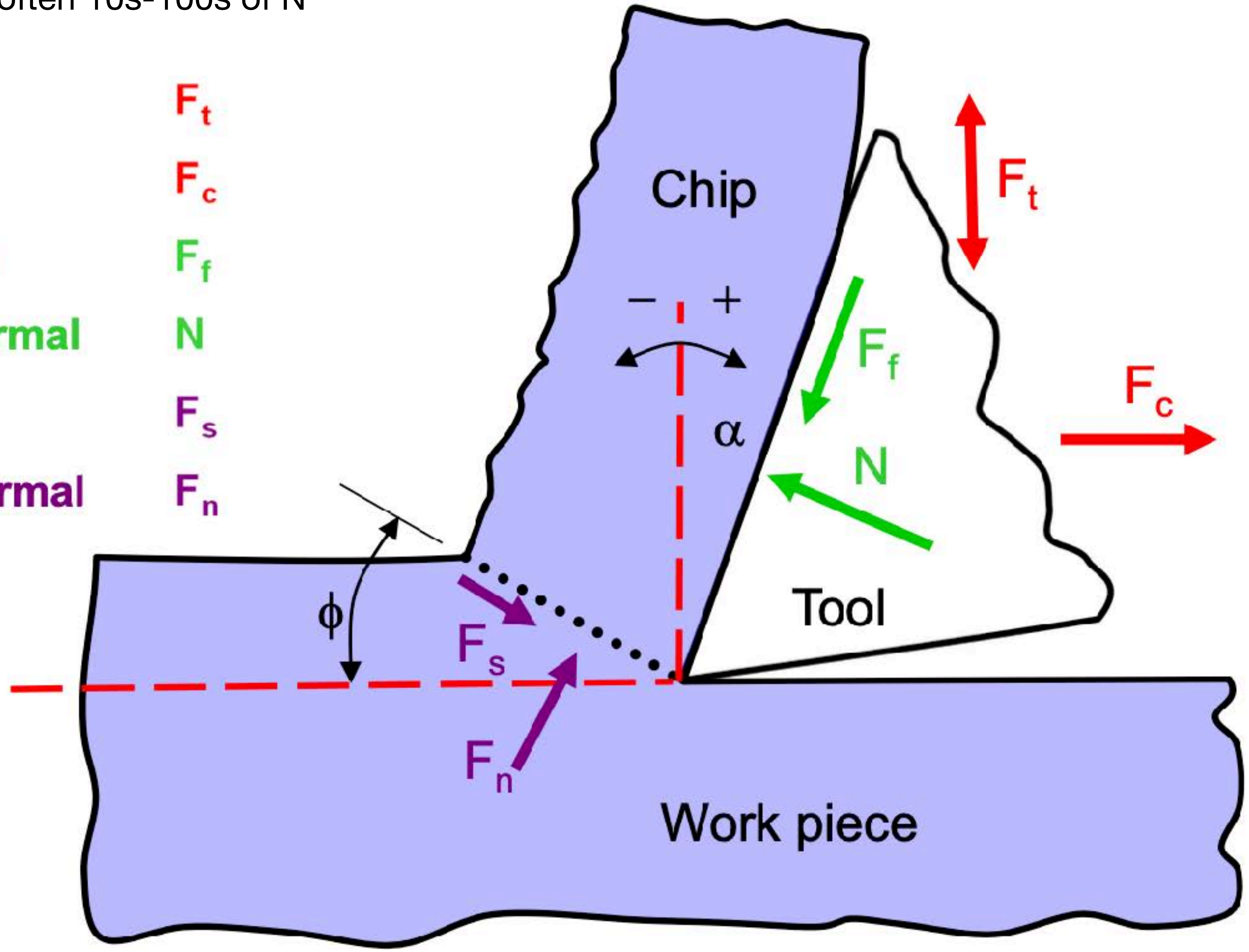
assumes all the cutting force goes directly into shearing the plane

underestimate: lower bound

Forces in Cutting

cutting forces: often 10s-100s of N

Thrust	F_t
Cutting	F_c
Friction	F_f
Tool normal	N
Shear	F_s
Chip normal	F_n



Forces in Cutting

machine ⟷ tool

cutting forces: often 10s-100s of N

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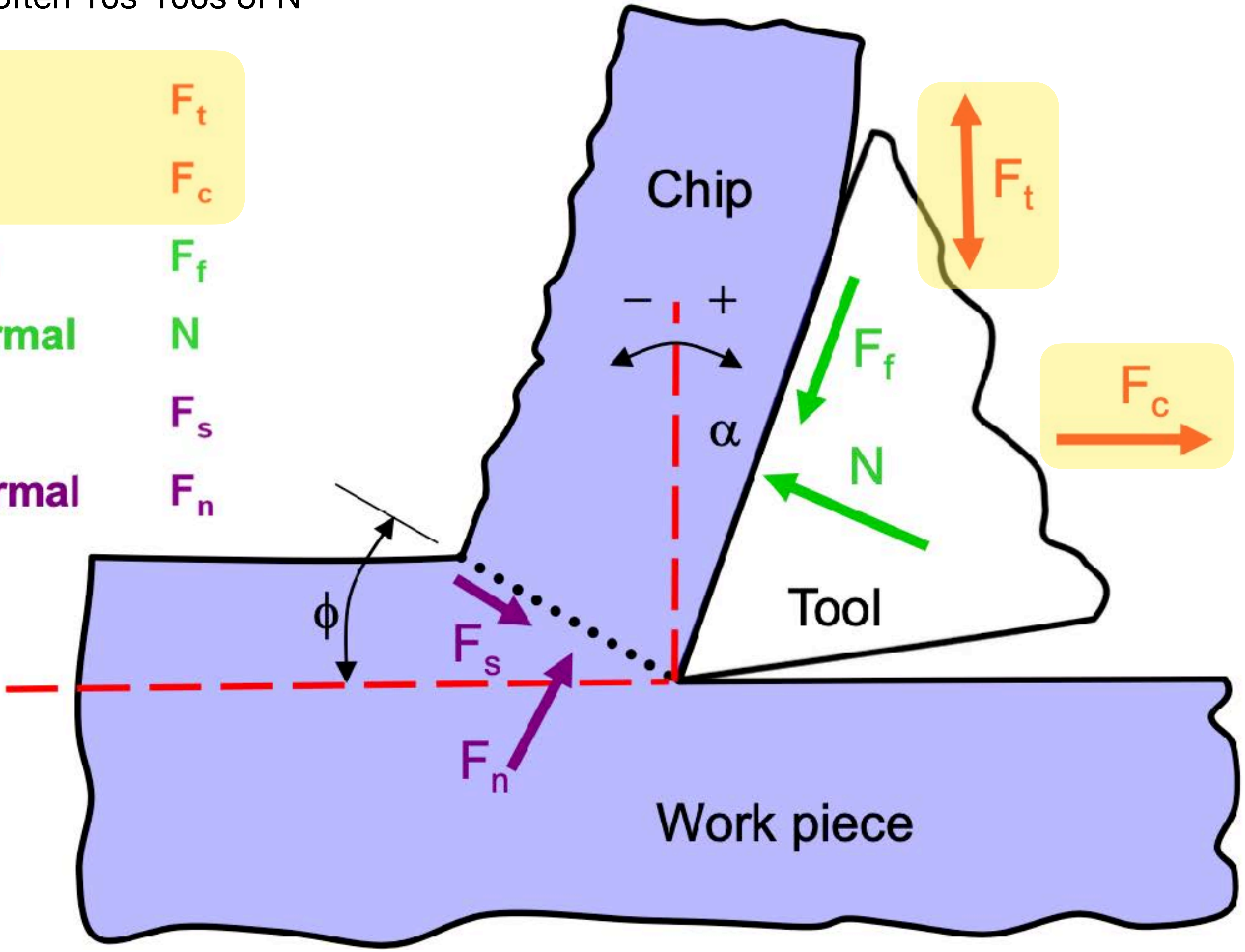

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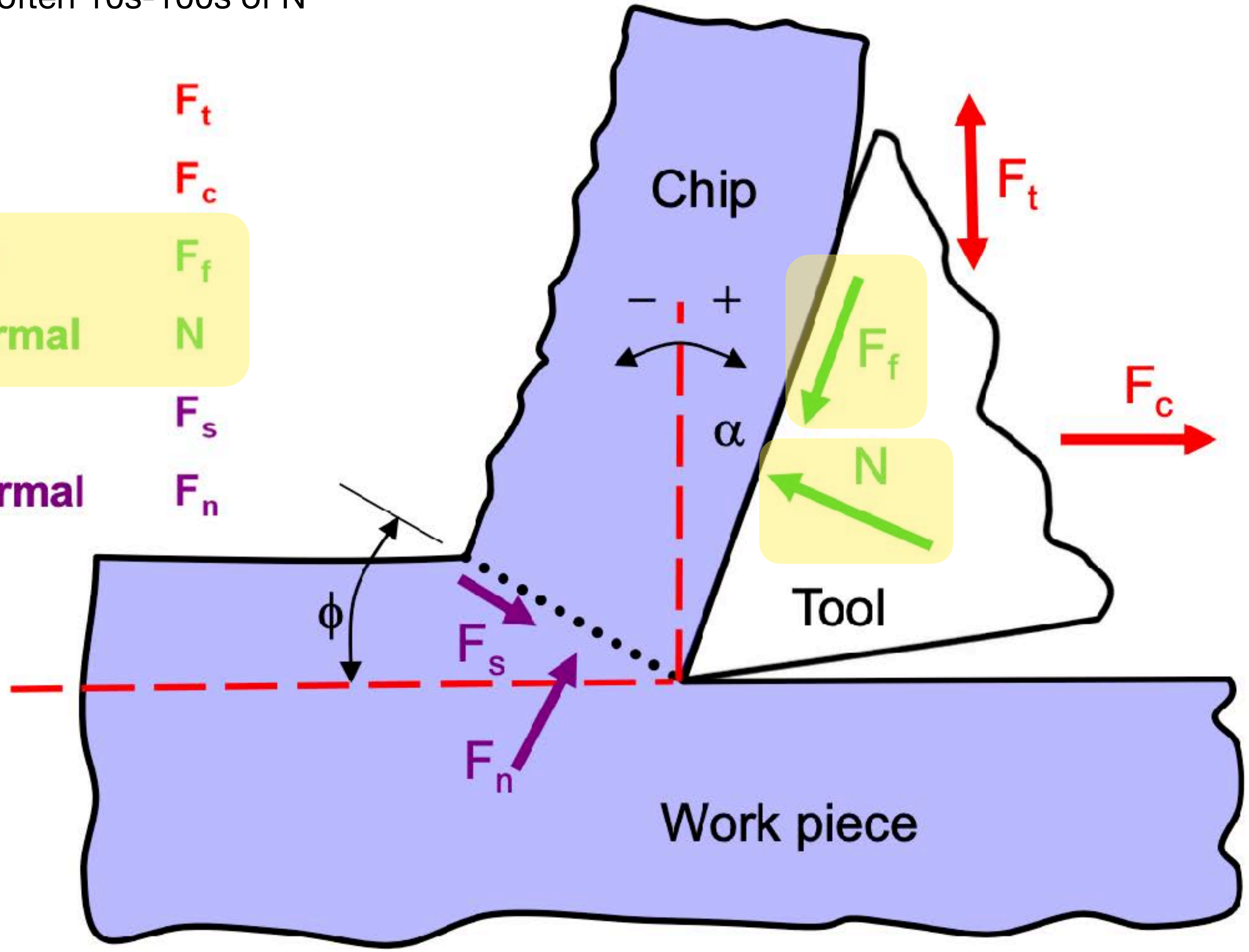
Chip normal	F_n

tool ⟷ chip



Forces in Cutting

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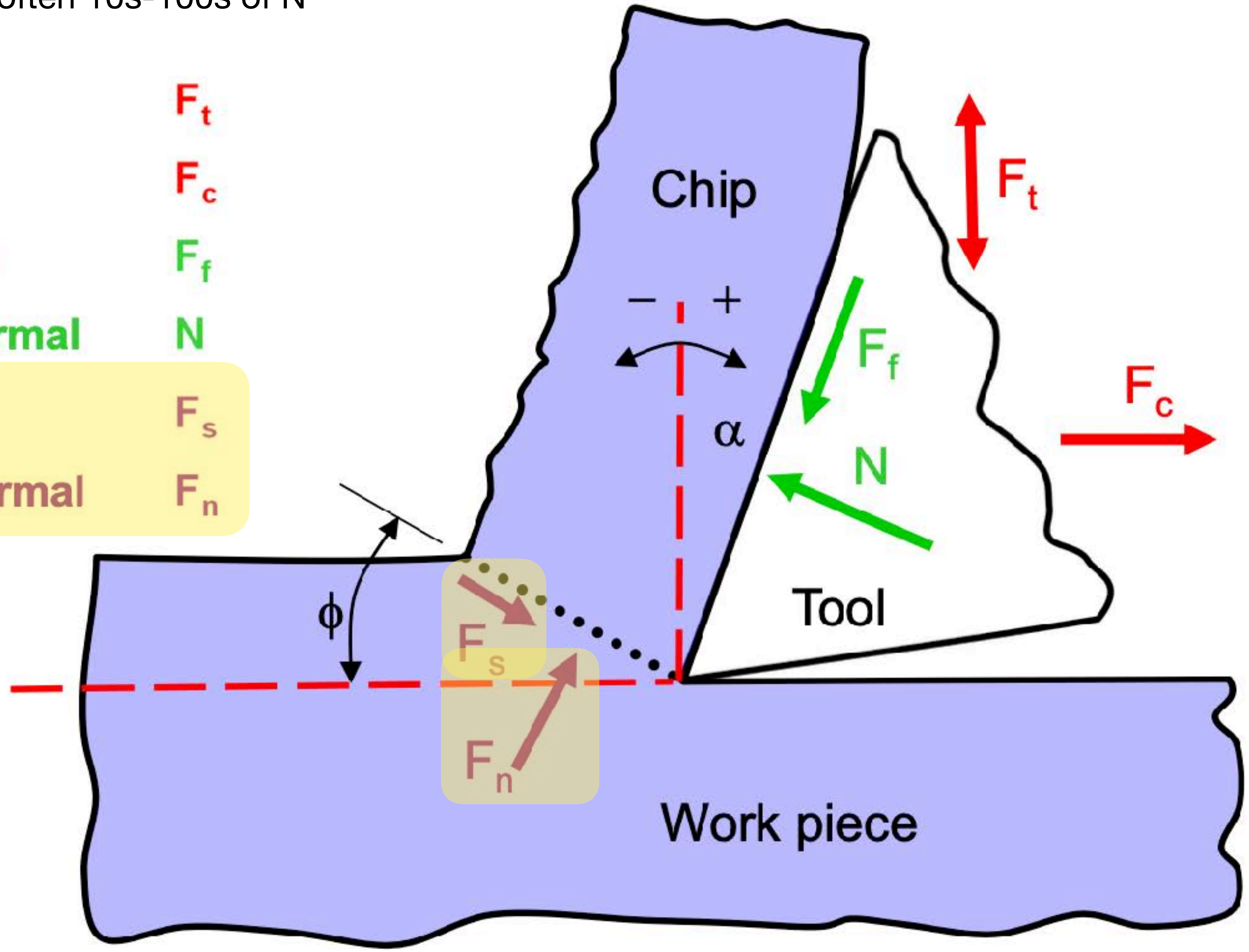
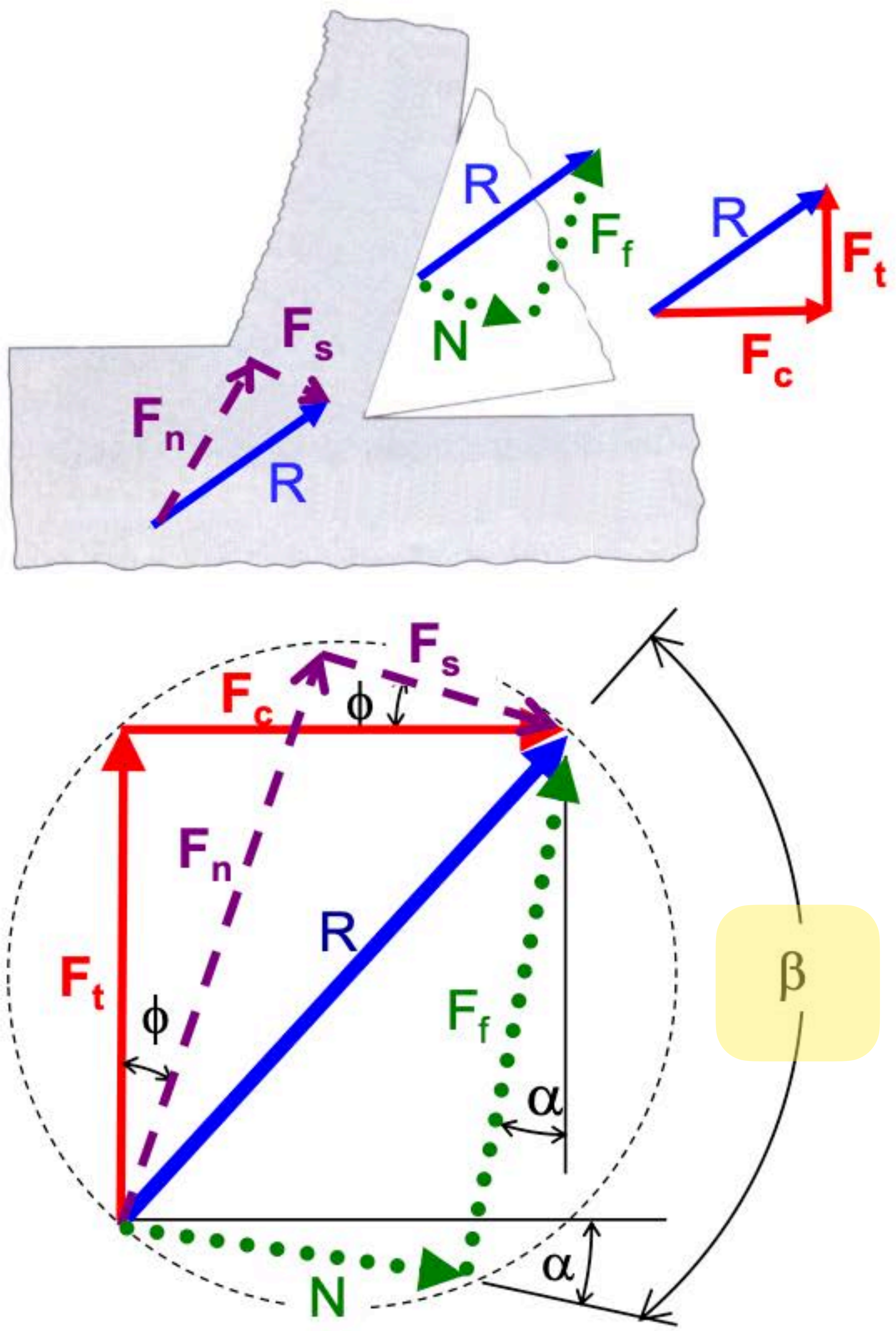
Cutting	F_c
Friction	F_f
Tool normal	N
Shear	F_s
Chip normal	F_n

chip ⟷ workpiece



Merchant's Diagram



Shear plane forces:

$$F_s = F_c \cdot \cos(\phi) - F_t \cdot \sin(\phi)$$

$$F_n = F_c \cdot \sin(\phi) + F_t \cdot \cos(\phi)$$

Tool-chip forces:

$$F_f = F_c \cdot \sin(\alpha) + F_t \cdot \cos(\alpha)$$

$$N = F_c \cdot \cos(\alpha) - F_t \cdot \sin(\alpha)$$

$$\mu = \frac{F_f}{N} = \tan(\beta)$$

$$Typcially : 0.5 < \mu < 2$$

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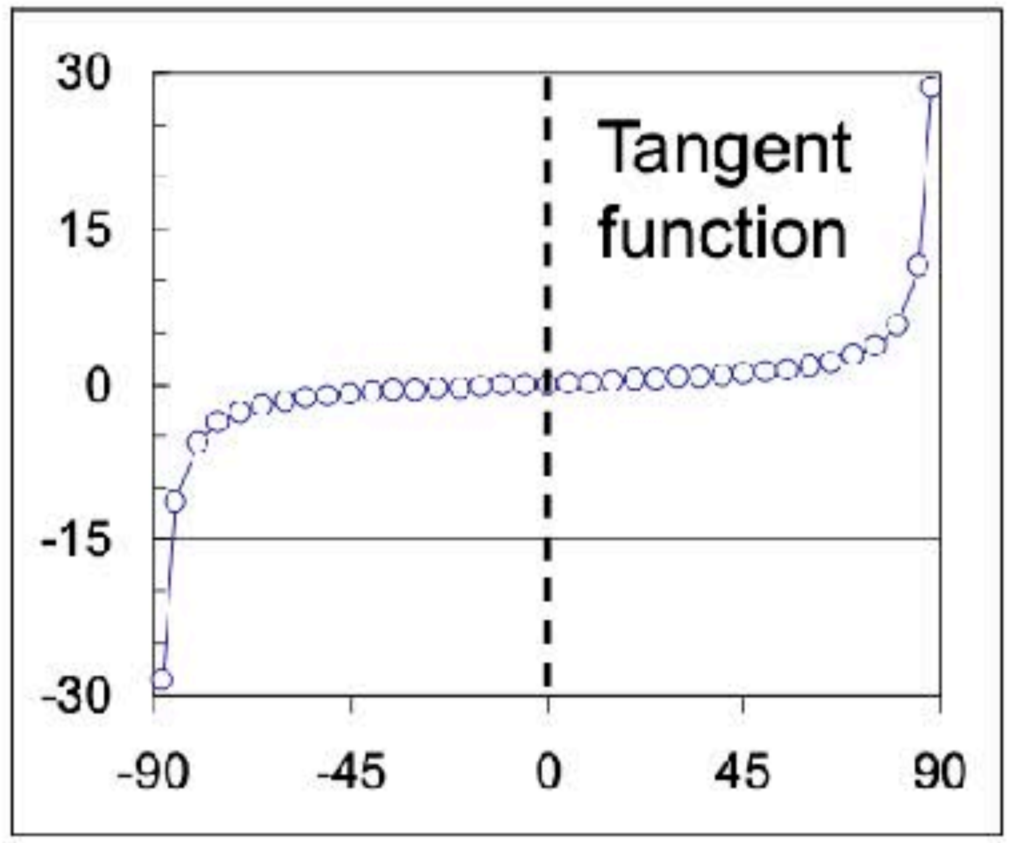
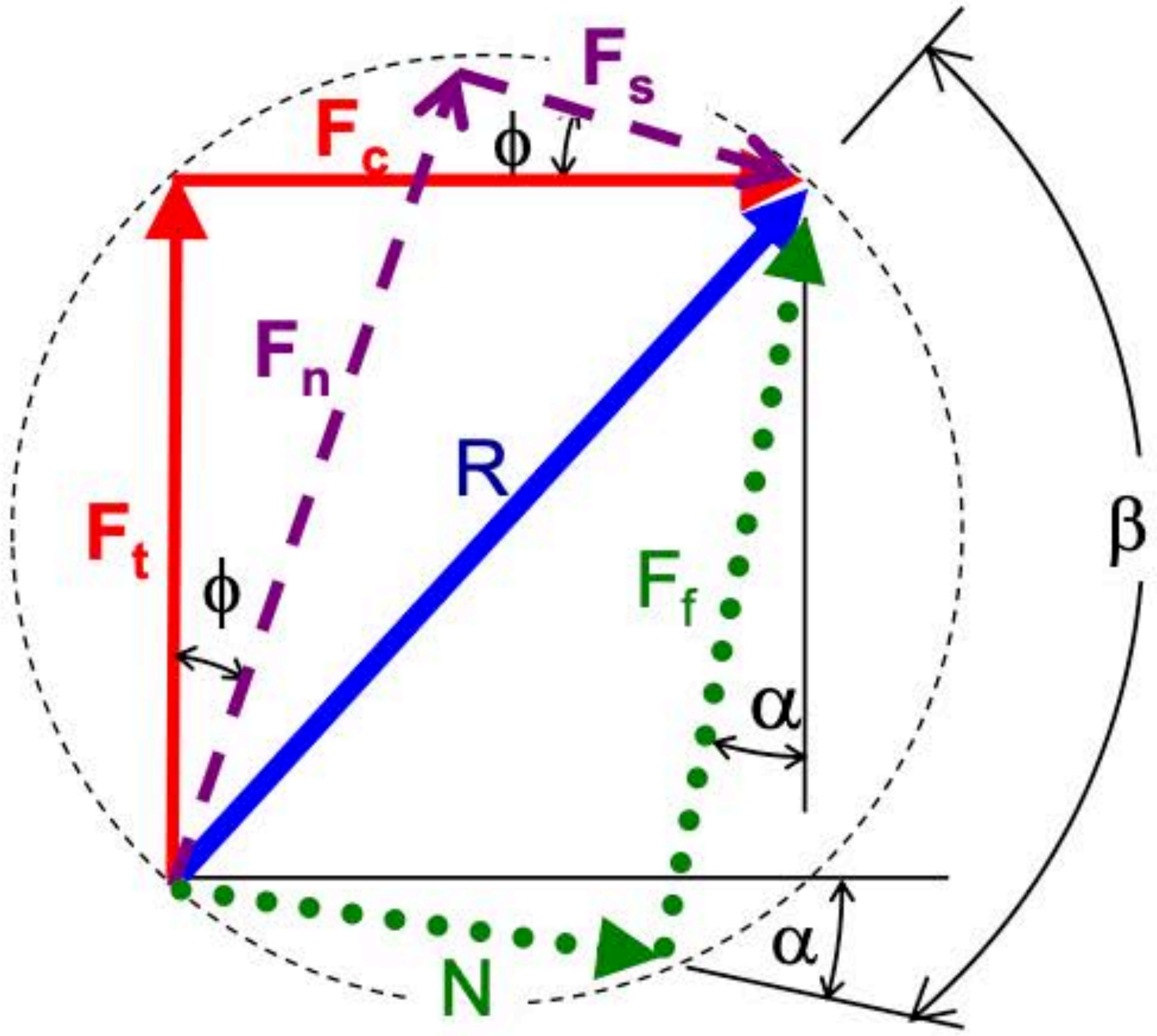
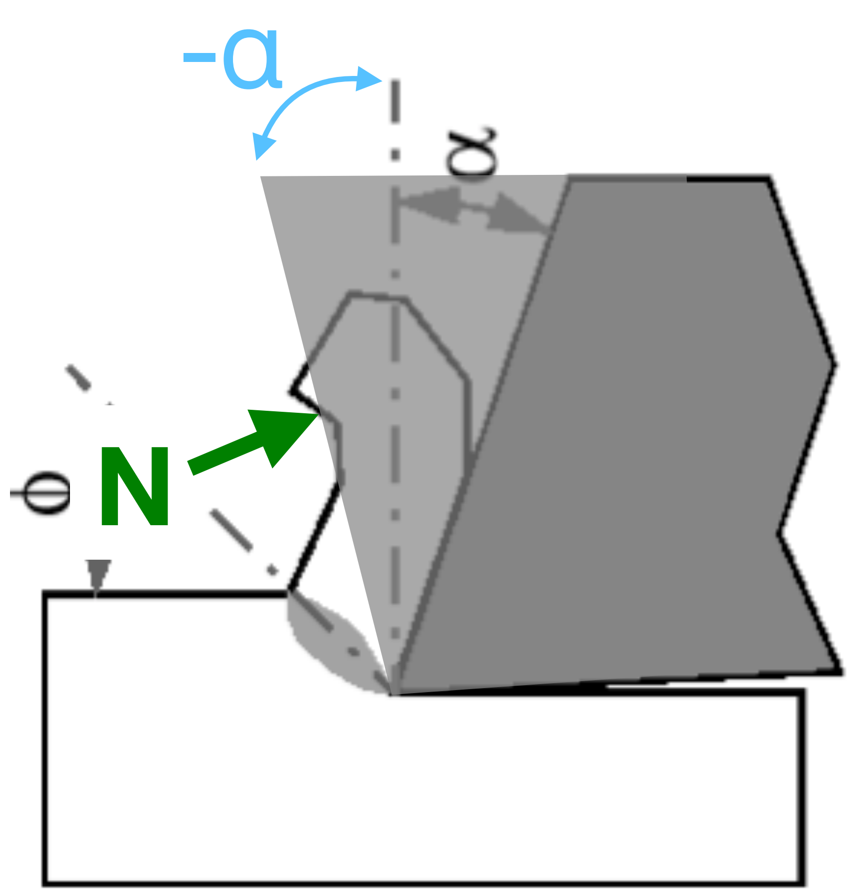
Thrust Force

$$F_t = F_c \tan(\beta - \alpha)$$

- ⊙ $\beta < \alpha$ tool is pulled into part
- ⊙ $\beta > \alpha$ tool is pushed away
- ⊙ $\beta = \alpha$ no thrust force







why negative rake angles?

- higher force, but less wear on the “point” of the cutting edge
- better for removing more material quickly: roughing

Merchant's Relationship

where is ϕ? what plane does shear take place in?

Merchant's energy assumption: ϕ adjusts to a plane that minimizes energy

$$\phi = \frac{\pi}{4} + \frac{\alpha}{2} - \frac{\beta}{2}$$

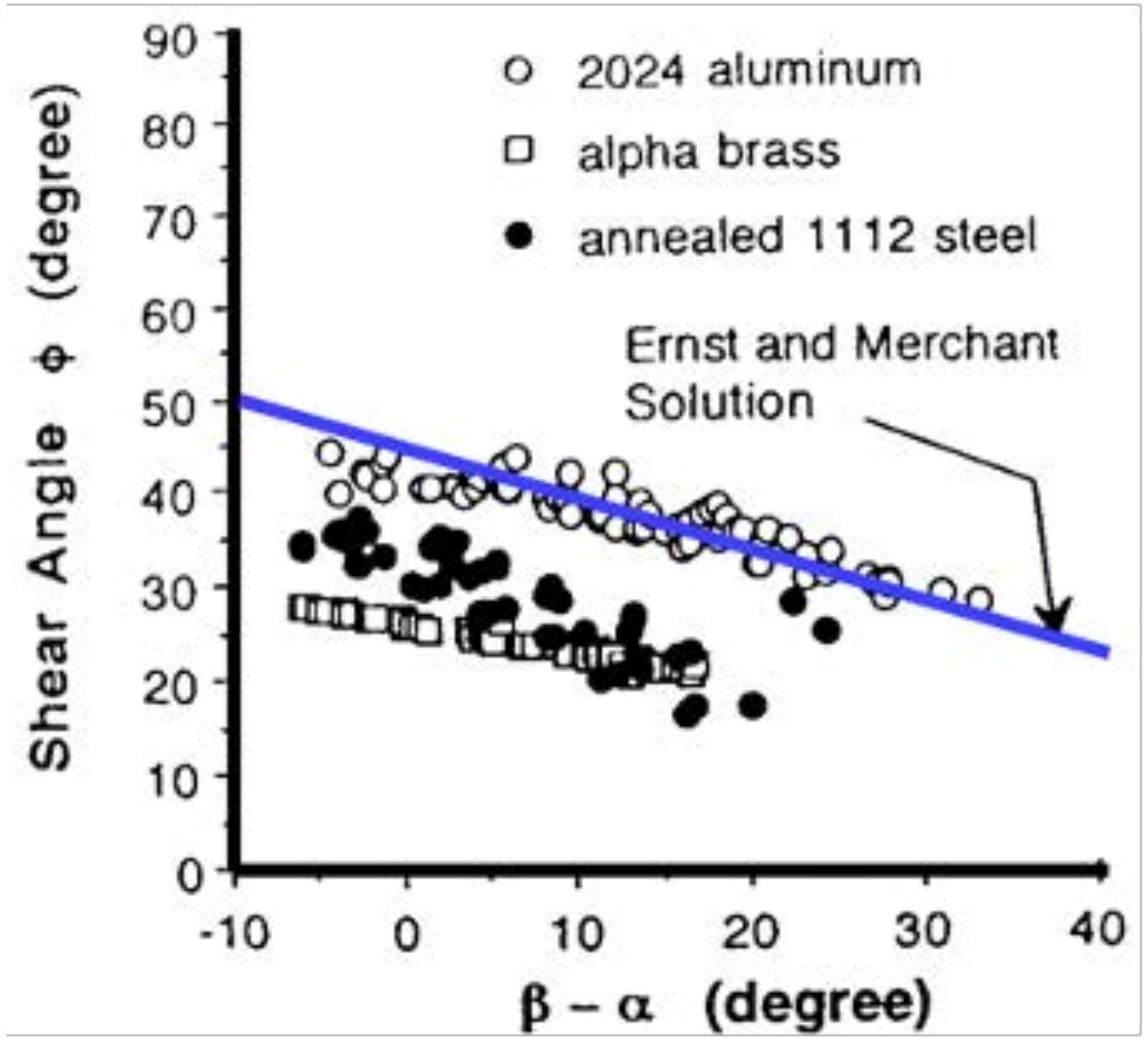


Chart adapted from: Metal Cutting Theory and Practice, Stephenson and Agapiou

assumptions:
- oblique vs orthogonal
- a shear plane vs whole area
- constant friction coefficient
- no strain hardening

Merchant's Relationship

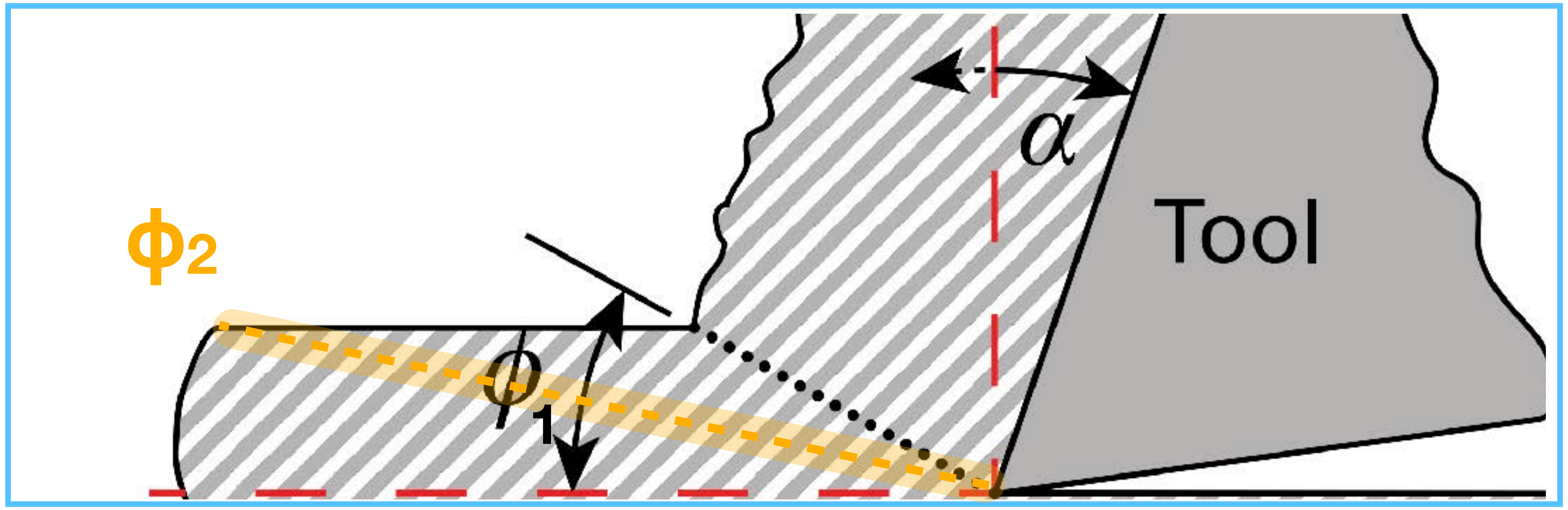
where is ϕ? what plane does shear take place in?

Merchant's energy assumption: ϕ adjusts to a plane that minimizes energy

$$\phi = \frac{\pi}{4} + \frac{\alpha}{2} - \frac{\beta}{2}$$

consequences of a smaller shear angle:

- chip thickness ⬆
- energy dissipation via shear ⬆
- heat generation ⬆
- temperature ⬆



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3. Energy and Power

Forces ➡ Power

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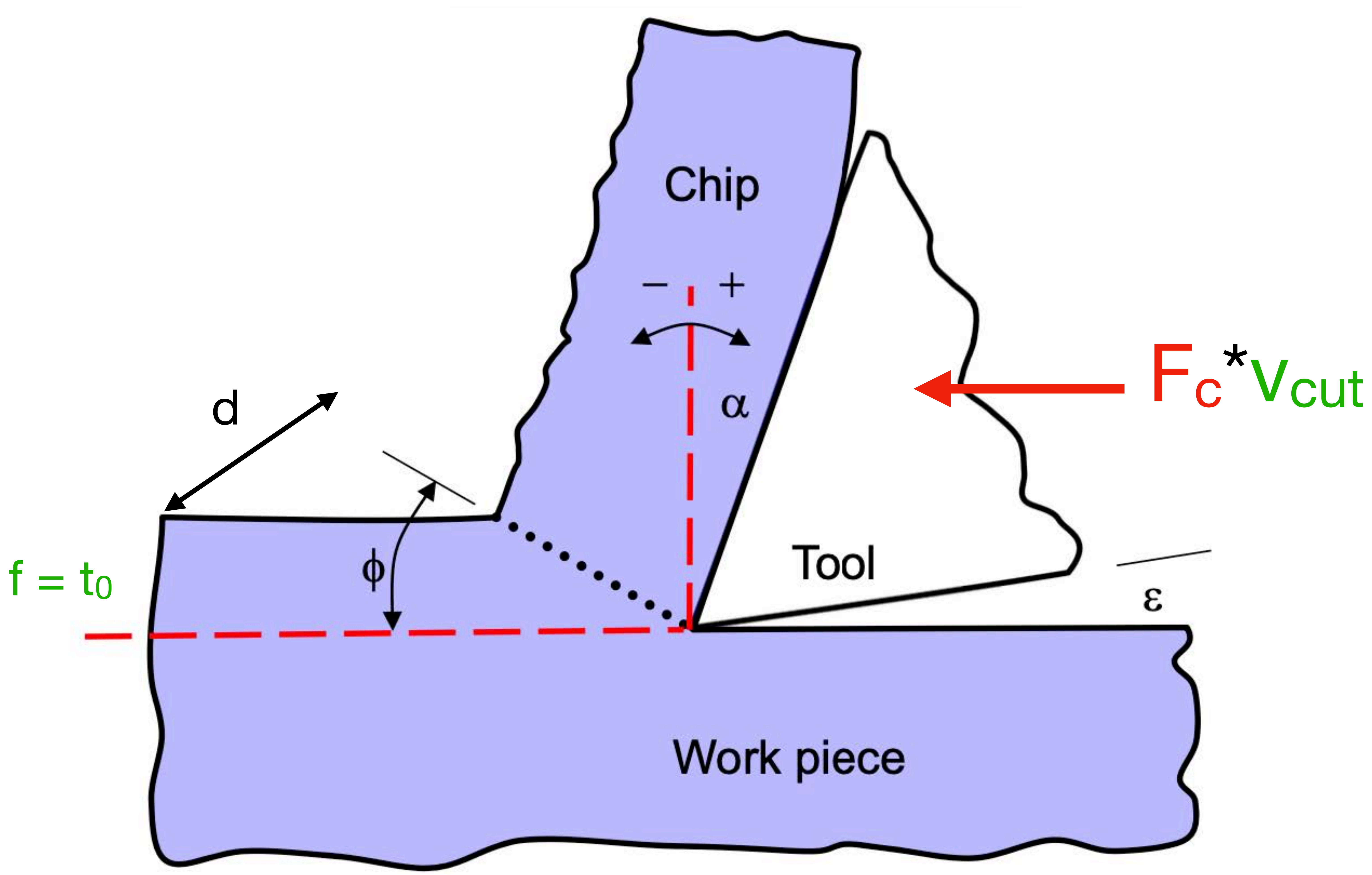


Cutting Power

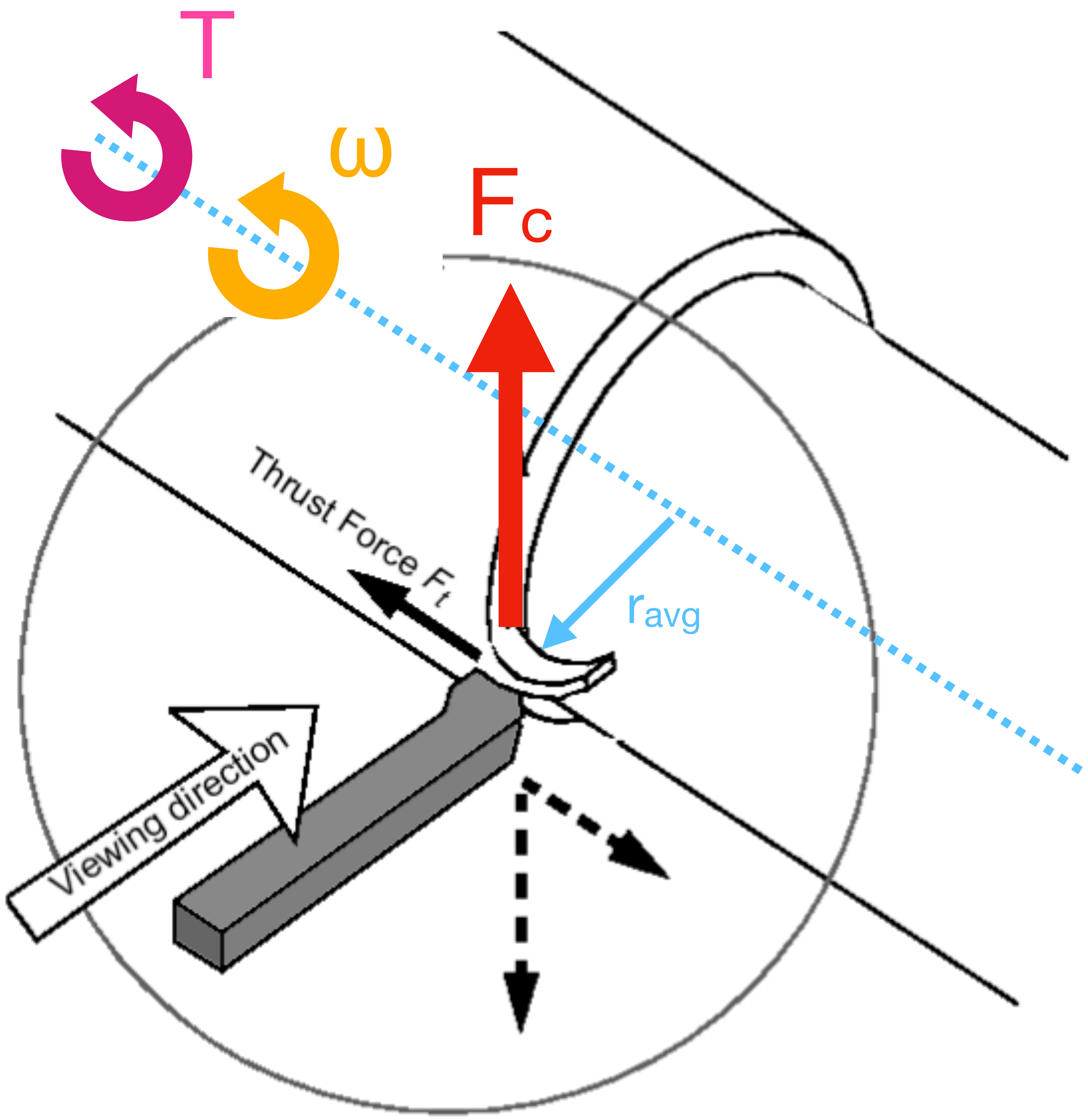
benchmarks:

- Bridgeport Milling Machine: 2 HP (1.5kW)
- HAAS VF2: 30HP (22 kW)

$$T = r_{avg} F_c$$

$$P = T\omega$$

T: torque
ω: rotational velocity
P: power input from machine



Cutting Power

power in = ~~power out~~ + power dissipation

(times efficiency) chips are small

power in: from machine $P_{in} = F_c v_c$

power dissipated: shear + friction

$P_{shear} = F_s v_{shear}$ ~75%

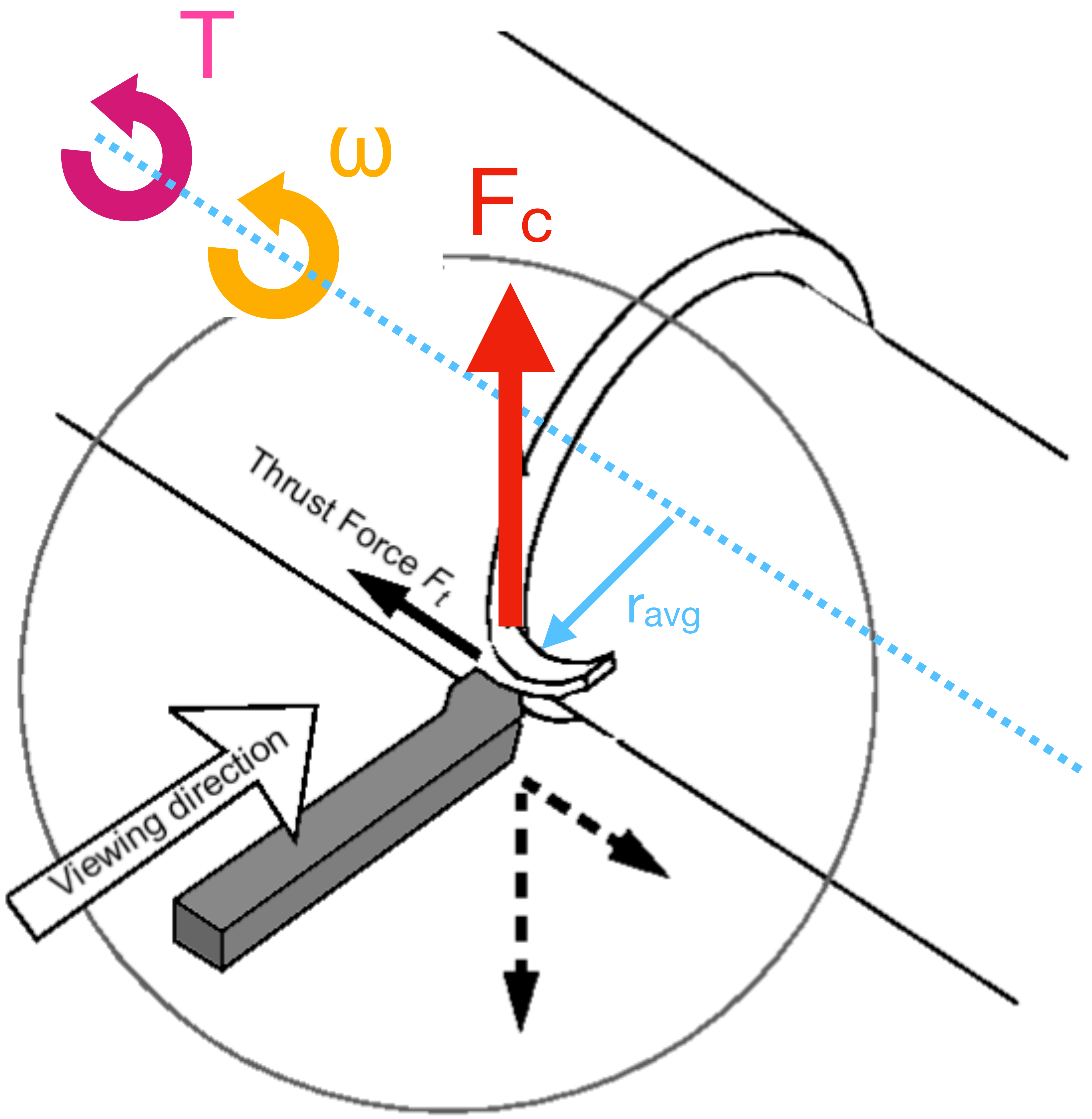
$P_{friction} = F_f v_{chip}$ ~20%

other: ~5%

$T = r_{avg} F_c$

$P = T\omega$

T: torque
ω: rotational velocity
P: power input from machine



Specific Energy

how much energy does it take to cut different materials? there's an empirical chart for that!

$$u = \frac{Enegry}{Volume}\Big|_{certain\ conditions}$$

volume → volume flow "Material Removal Rate"

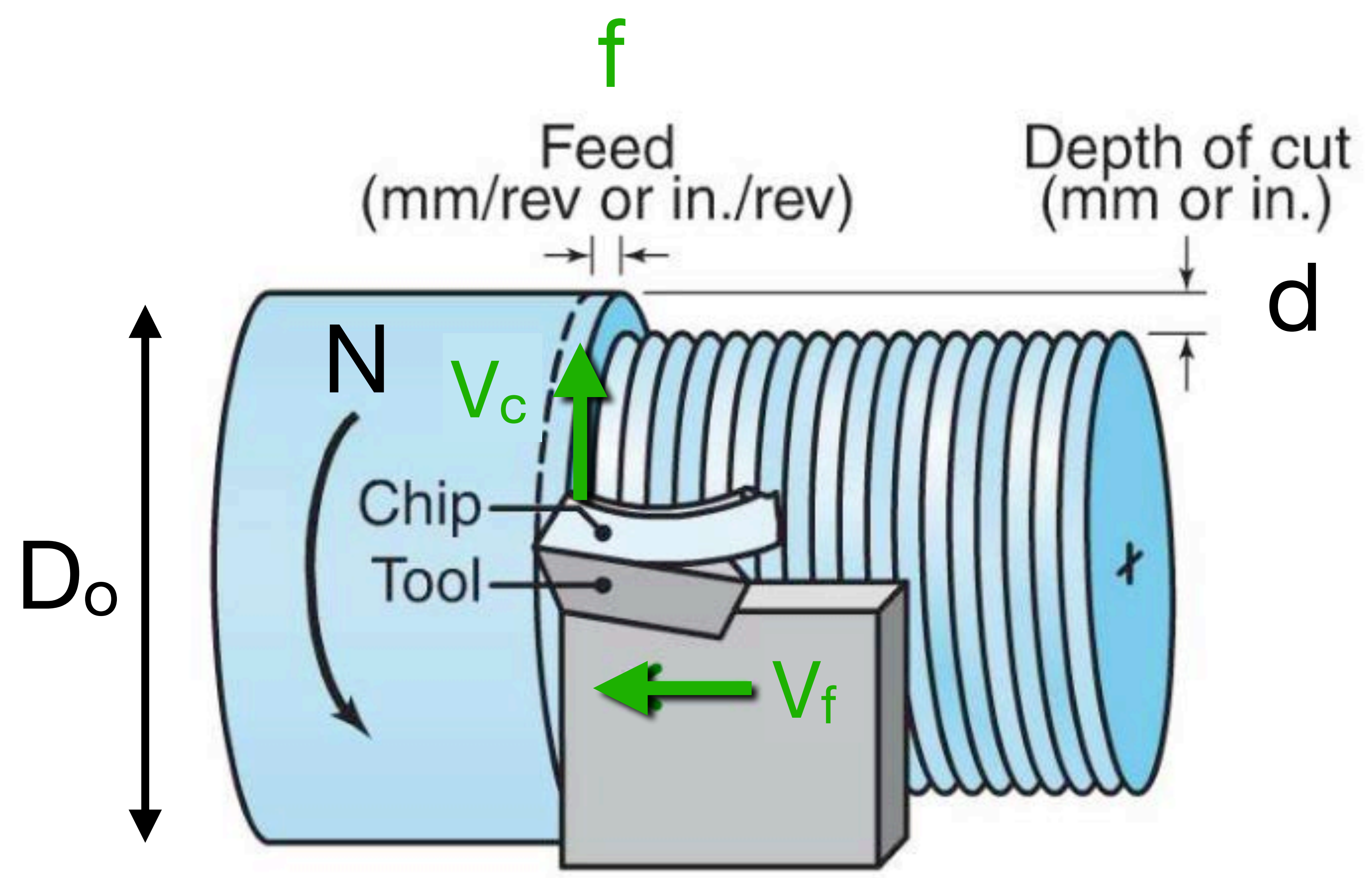
energy → power how much power is needed?

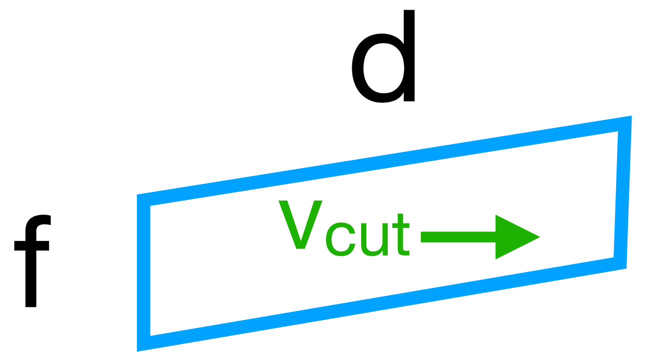
TABLE 21.2

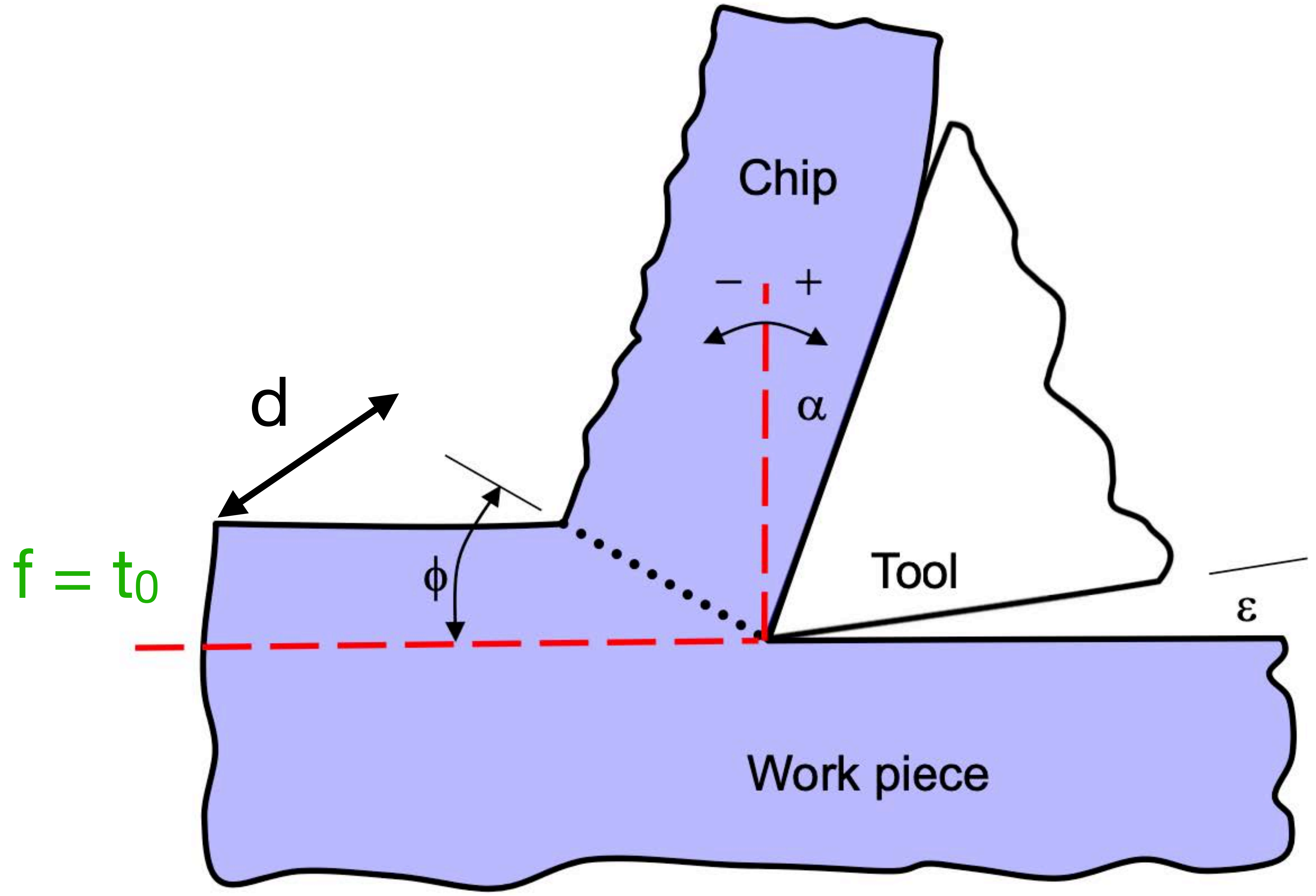
Approximate Range of Energy Requirements in Cutting Operations at the Drive Motor of the Machine Tool (for Dull Tools, Multiply by 1.25)

Material	Specific energy W-s/mm^3	Specific energy hp-min/in^3
Aluminum alloys	0.4–1	0.15–0.4
Cast irons	1.1–5.4	0.4–2
Copper alloys	1.4–3.2	0.5–1.2
High-temperature alloys	3.2–8	1.2–3
Magnesium alloys	0.3–0.6	0.1–0.2
Nickel alloys	4.8–6.7	1.8–2.5
Refractory alloys	3–9	1.1–3.5
Stainless steels	2–5	0.8–1.9
Steels	2–9	0.7–3.4
Titanium alloys	2–5	0.7–2

Material Removal Rate: Lathe







d: depth of cut [in]
f or t_0: feed [in/rev]
N: spindle speed [rev/min]
Do: original diameter [in]
u: specific cutting energy [Ws/mm³]

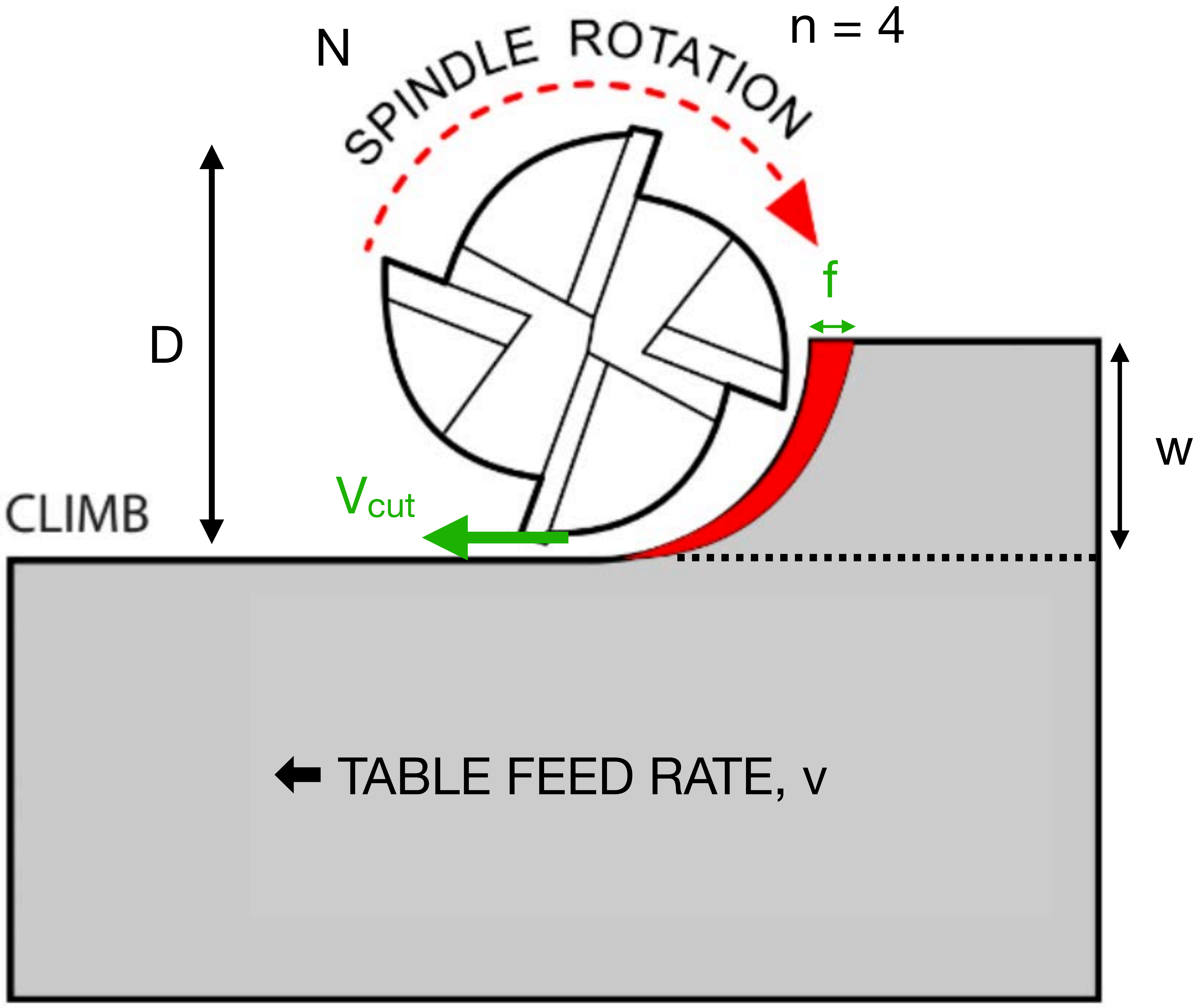
V_f: feed rate = f*N [in/min]

V_c: cutting velocity = π*D*N [in/min]

$$MRR_{turning} = fdv_{cut} = fd\pi D_{avg}N$$

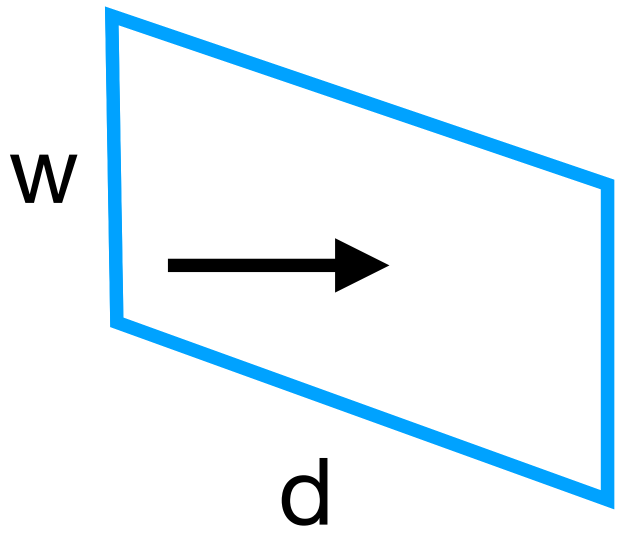
$$P_{turning} = uMRR_{turning}$$

Material Removal Rate in Milling



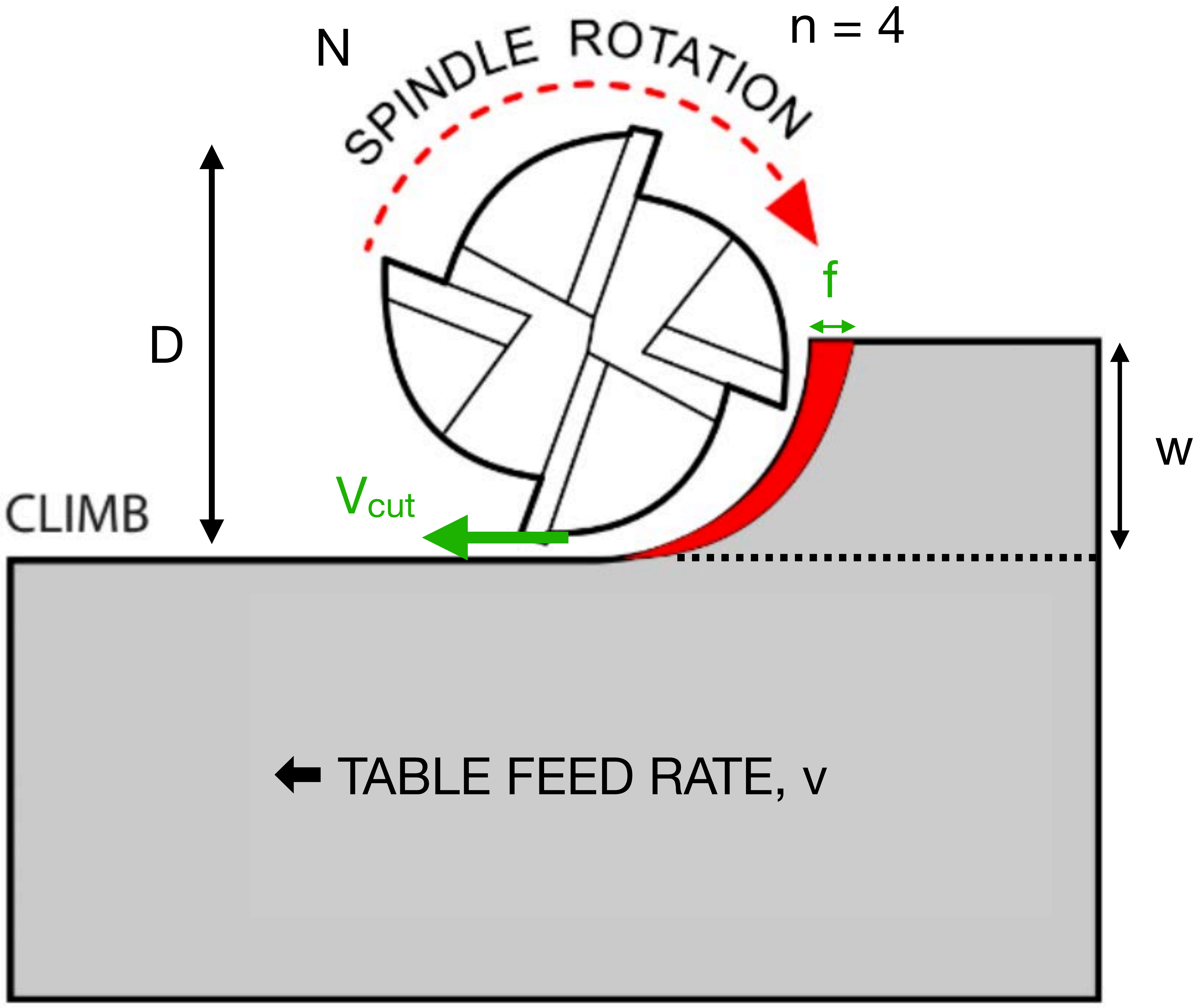
$$f = \frac{v}{Nn}$$

$$v_{cut} = \pi D N$$



- f: feed per tooth [in/tooth]
- n: number of teeth [#]
- N: spindle speed [rpm]
- v: feed rate, velocity of tool (center) relative to workpiece [in/min]
- w: width of cut [in]
- d: depth of cut [in]
- D: cutter diameter [in]

Material Removal Rate in Milling



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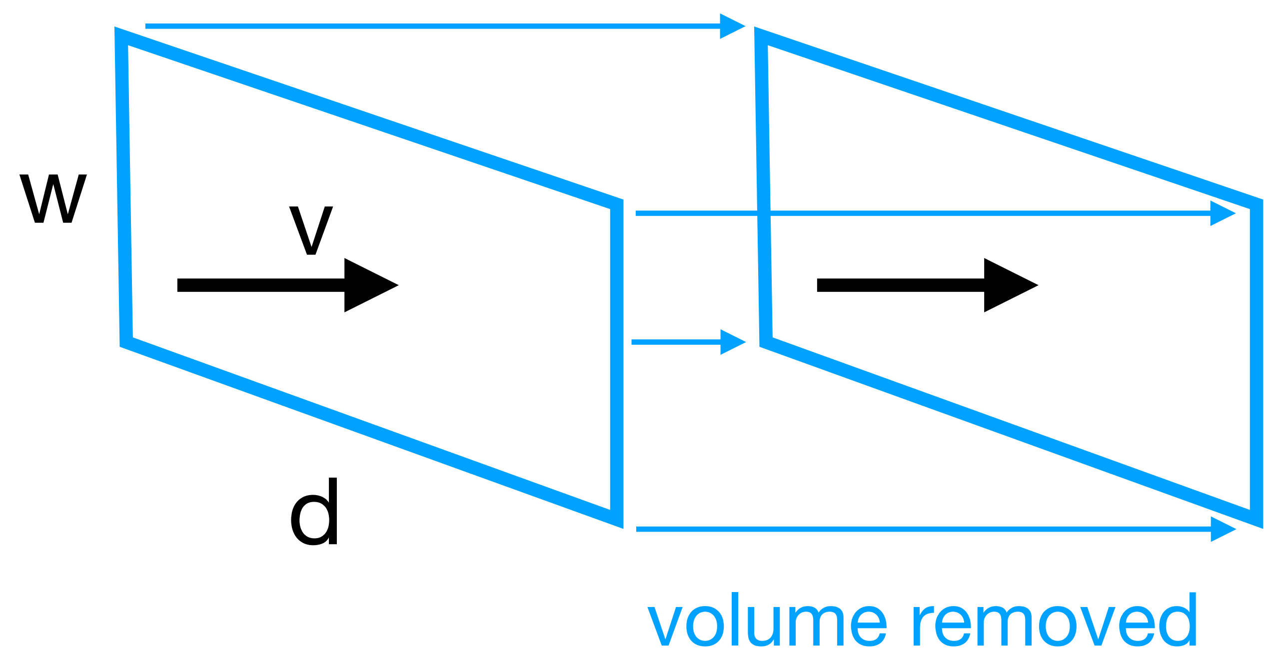


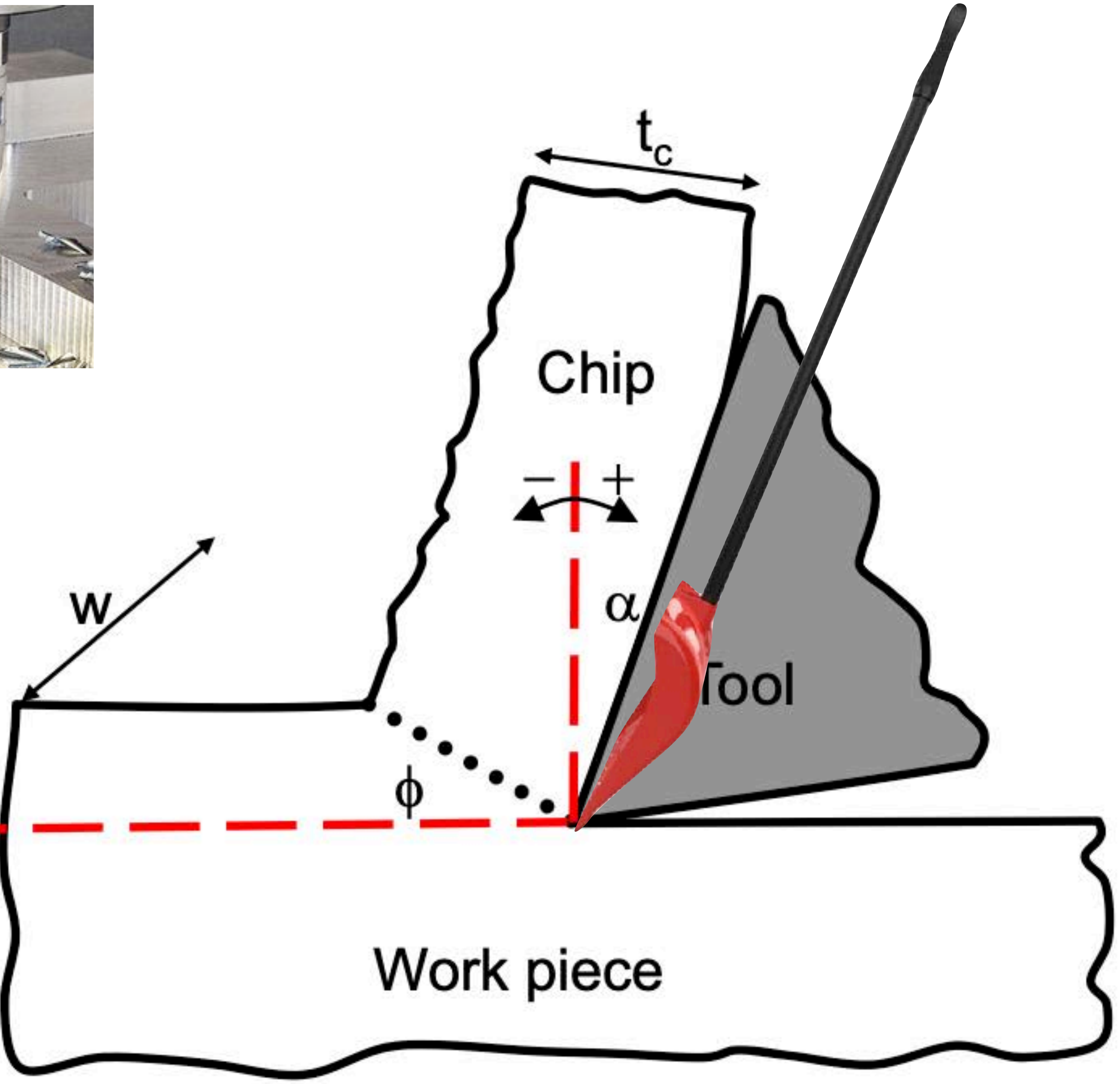
$$MRR_{milling} = wdv$$

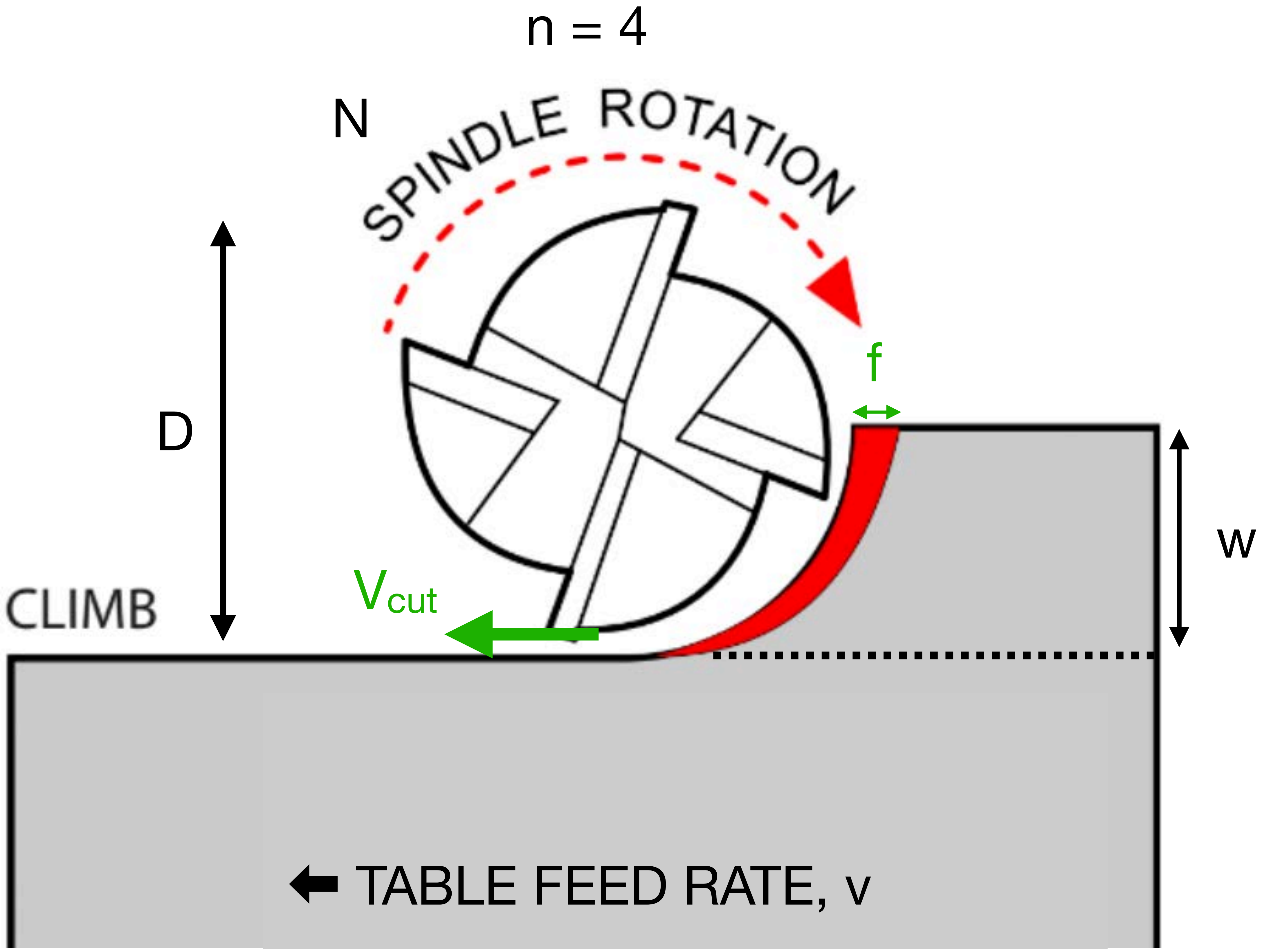
$$Power = u \cdot MRR_{milling}$$

- Power: total cutting power from machine [W]
- u: specific energy of the material [W-s/mm^3]
- MRR: material removal rate [mm^3/s]

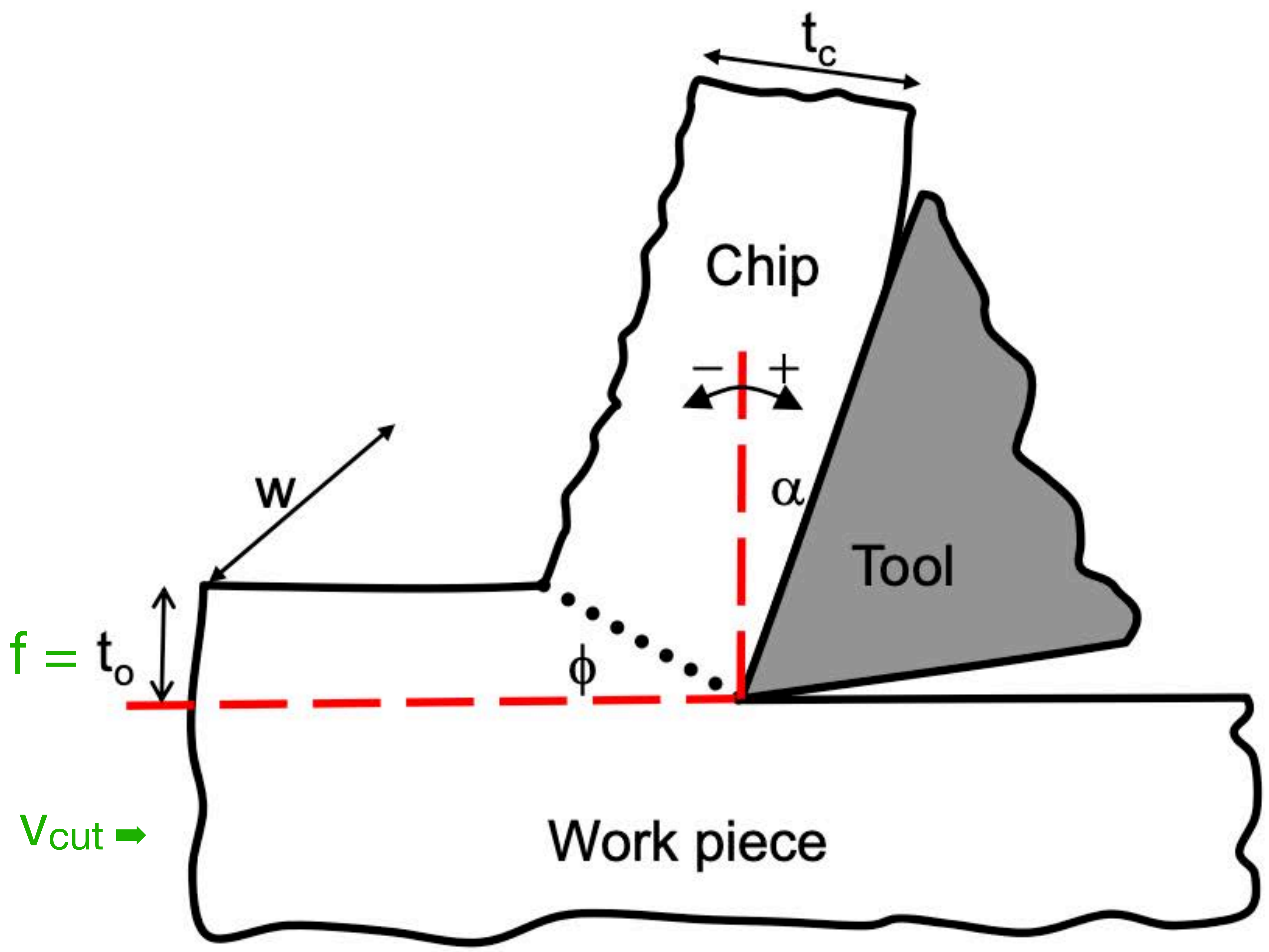
Milling

⚠️ looking down the spindle from above the machine

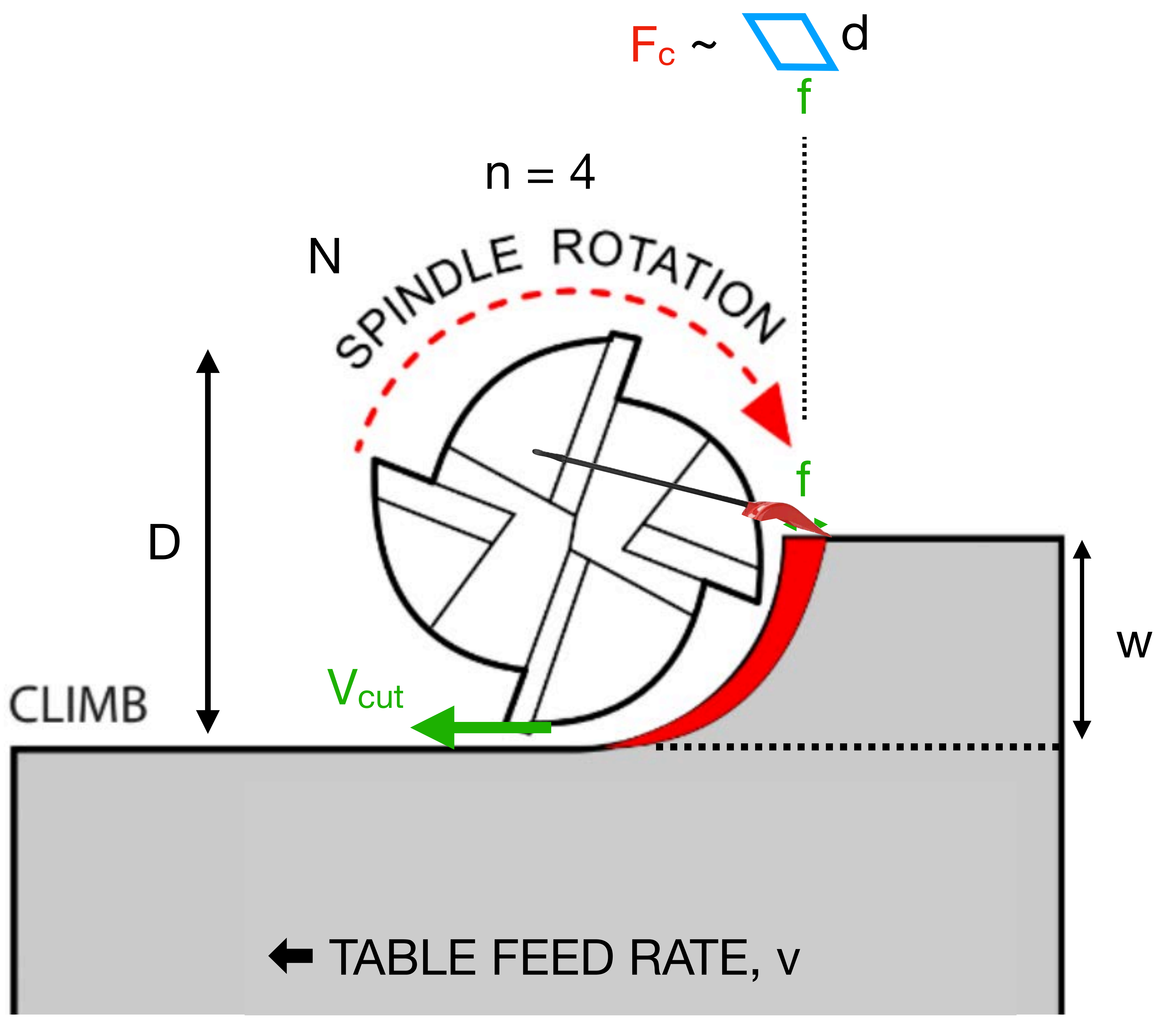




Milling



t_c
Chip
−
+
α
Tool
w
$f = t_o$
ϕ
Vcut ➡
Work piece



F_c ~
d
f
n = 4
N
SPINDLE ROTATION
f
D
w
CLIMB
V_{cut}
⬅ TABLE FEED RATE, v

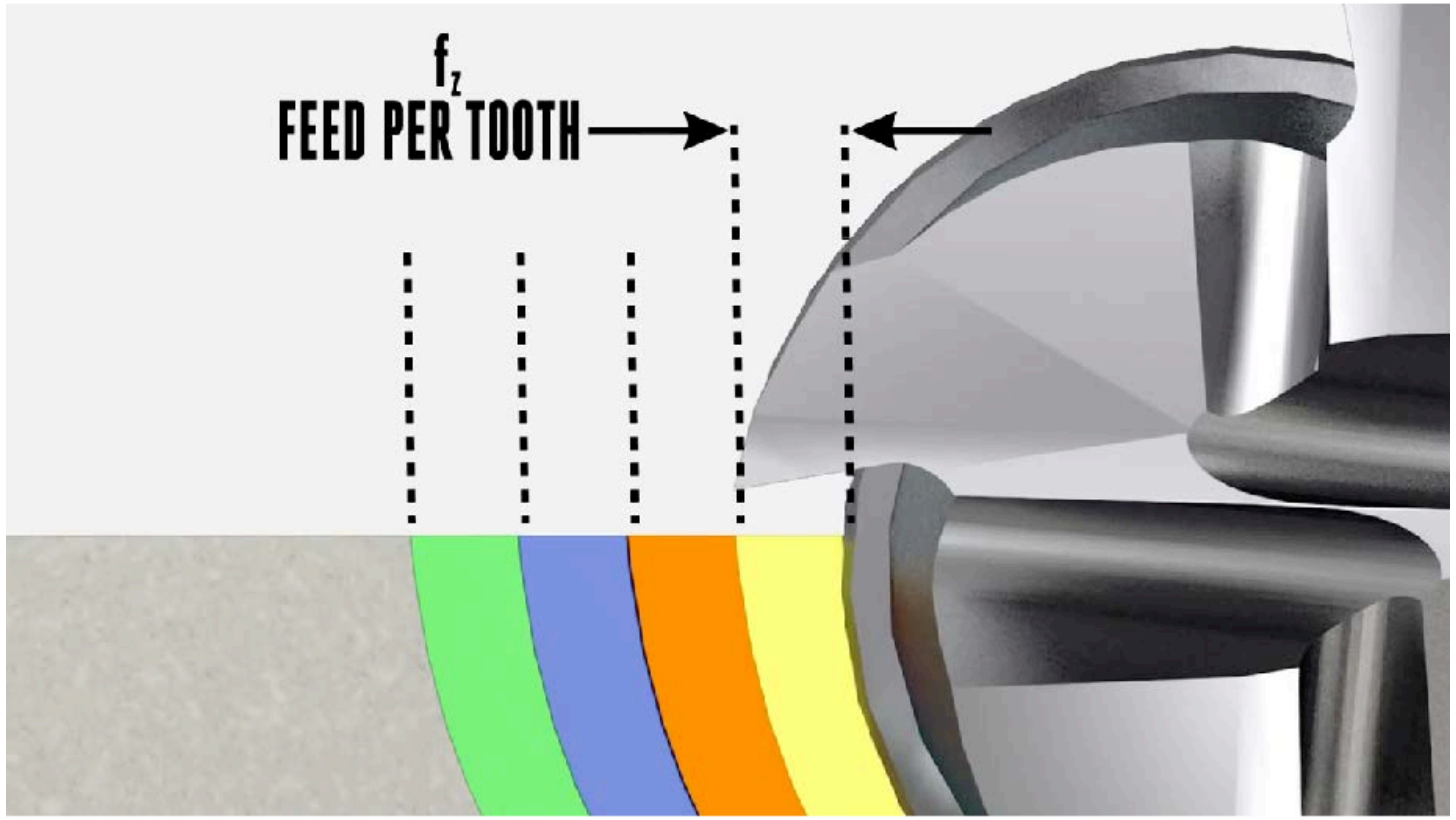
Cutting Forces in Milling

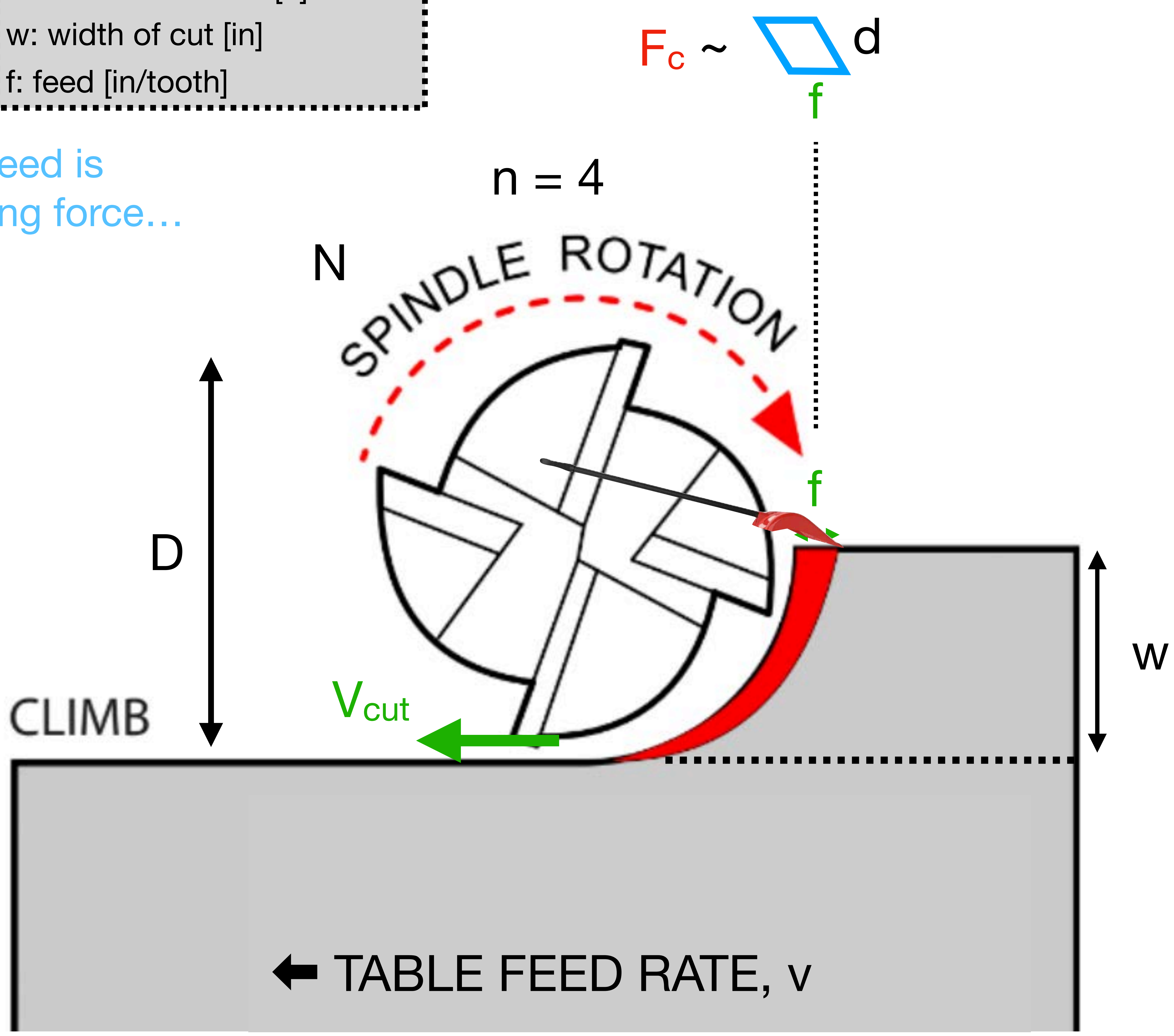
D: cutting tool diameter [in]
N: spindle speed [rev/min]
n: number of teeth [#]
w: width of cut [in]
f: feed [in/tooth]

⚠️ connection between spindle speed and feed makes this confusing

adjust N with v constant: f changes (v_{cut} also changes)

seems like speed is affecting cutting force…





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2.008 Design and Manufacturing II
Spring 2025