Variation and Quality

Defining, Measuring, and Controlling Quality in Manufacturing

Here's what Subway had to say:

> "As you know, all of our sandwiches are made to order, and our bread is baked daily in every one of our more than 38,000 restaurants in 100 countries worldwide. We have policies and procedures in place to ensure that our products are consistent and have the same great taste no matter which Subway restaurant you visit.
>
> We have seen the photo you referenced of a Subway sandwich that looks like it doesn't meet our standards. We always strive for our customers to have the most positive experience possible, and we believe this was an isolated case in which the bread preparation procedures were unfortunately not followed."

There were many theories out there, ranging from toasting shrinkage to faulty bread to a fake tape measure.

While it denies the claims, Subway announced a proposed settlement Monday. According to court documents, franchisees would be required to have a measurement tool in stores and adhere to regular compliance inspections that would include measuring a sampling of baked bread to make sure loaves are 12-inches.

Subway also said it would amend training materials and other communication that had "allowed for a small tolerance in the size of a Footlong sandwich."

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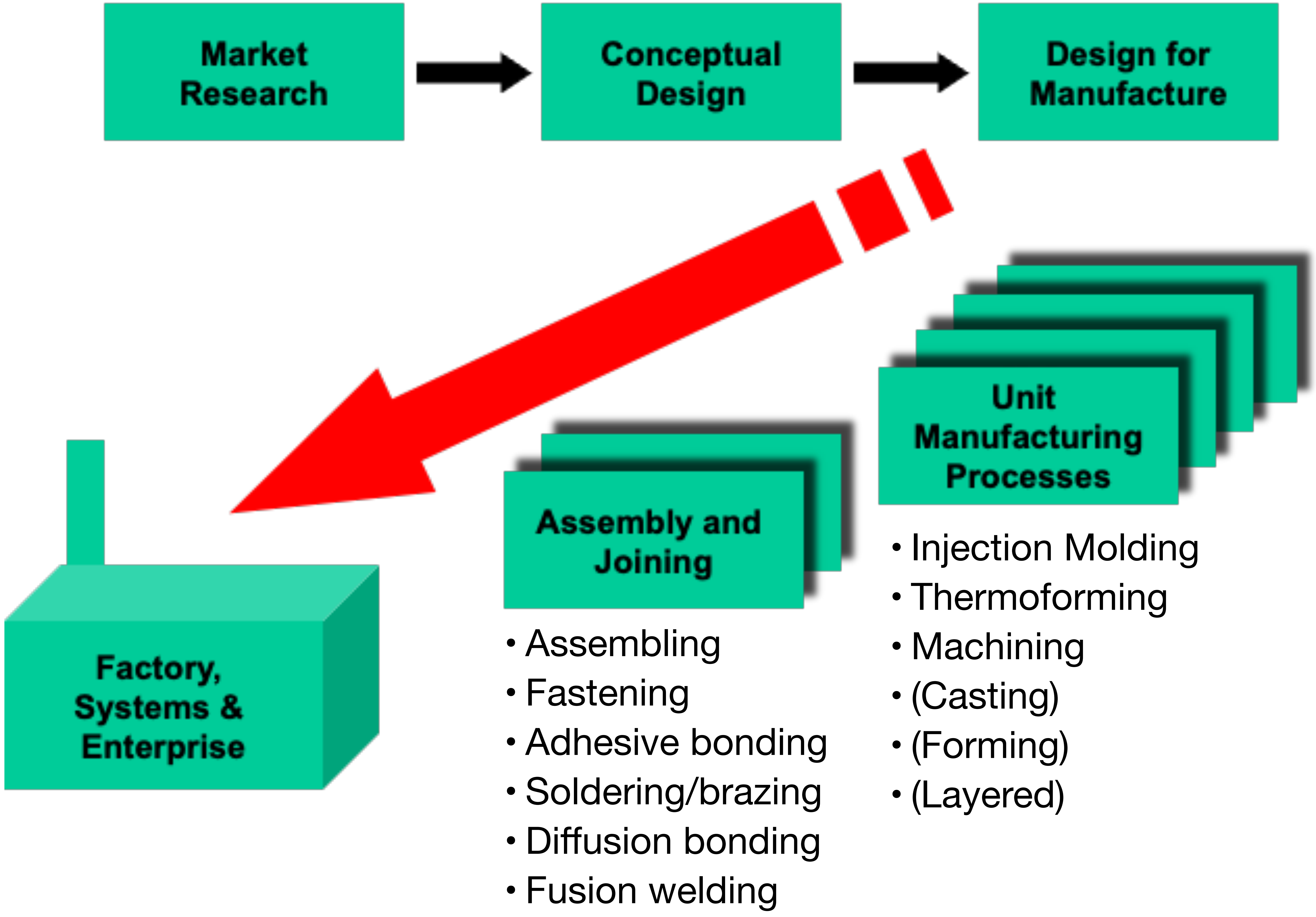


2.008 Manufacturing Context

what have we covered so far:

- some unit processes (more to come)
- assemble to make useful products
- (covered in **Review Session** and **Quiz**)
    - closed book, 1 page cheat sheet

what is still coming up

- **variations and quality control**
- manufacturing systems
- process planning
- cost

2.008 Objectives

Internalize the rate, quality, cost, and flexibility as manufacturing attributes

e.g. injection molding: cooling

machining: MRR

multiple processes: systems

Apply physics to understand the factors that influence the rate, quality, cost, and flexibility of processes

mostly qualitative so far

today: how to define, measure and control it

Apply an understanding of variation to the factor that influence rate, quality, cost, and flexibility of processes and systems

Understand the impact of manufacturing constraints on product design and process planning

What is Quality?

can we agree on what it is?

to **control it** you need to be able to **measure it**

what is the metric and the specification?

“within spec” = 👍, otherwise 👎

2.008 Topic Coverage

Variations

Statistical Representation

Process **Capability**

Process **Control**

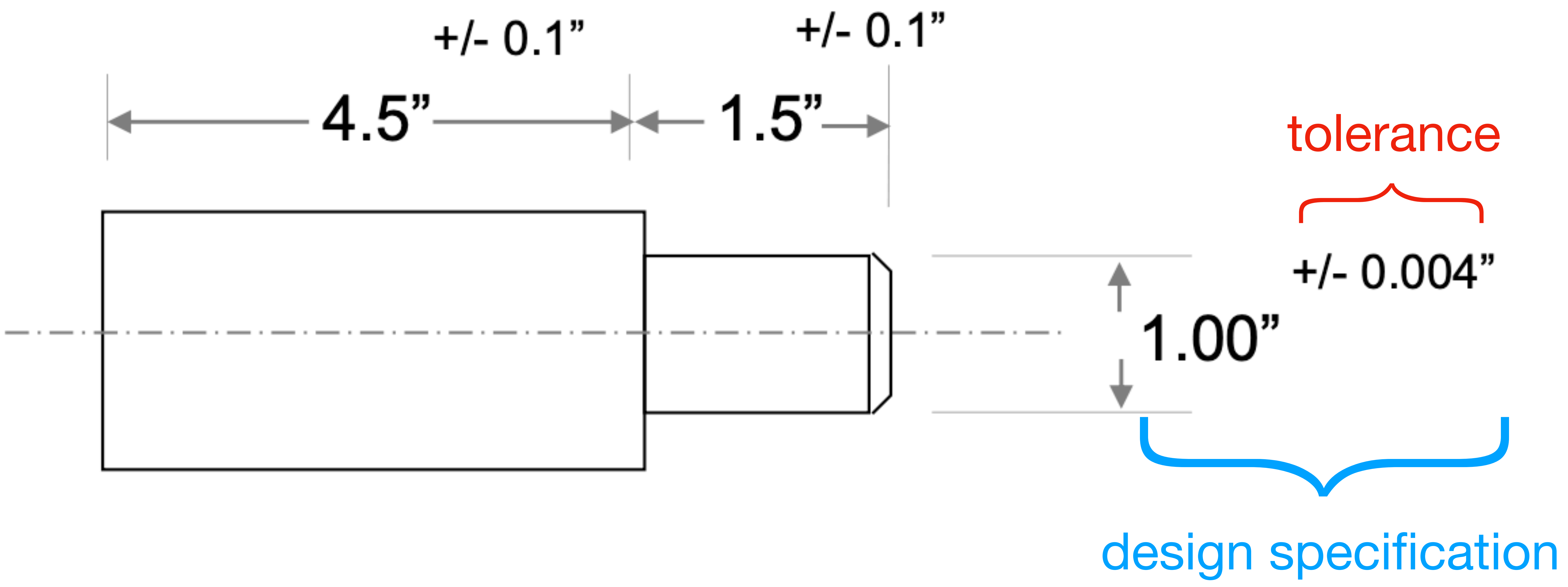
Accuracy vs Precision

Quality Loss

What is our **yield**? % that meets quality.

What can we do about it?

Variations in Engineered Part



how much variation can your design tolerate?

“within spec” = 👍, otherwise 👎

compare your **process outcome** (single measurement for a part) to your specification (range)

Variation and Quality

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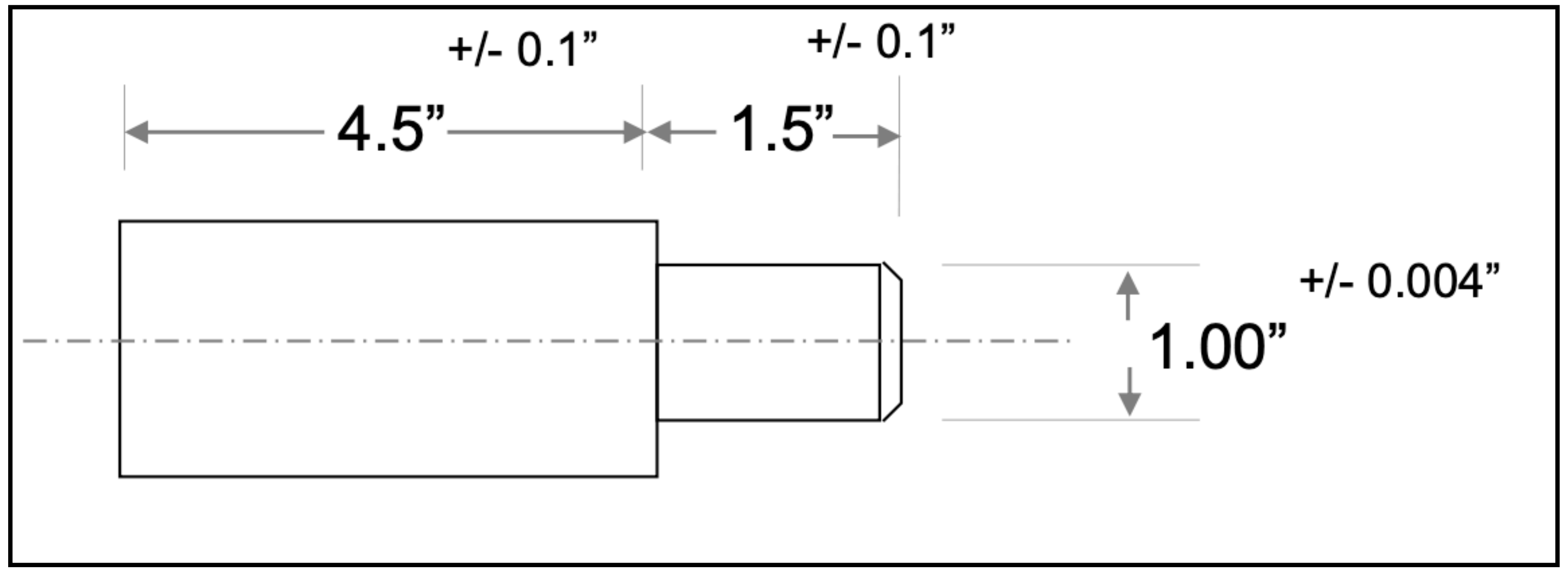
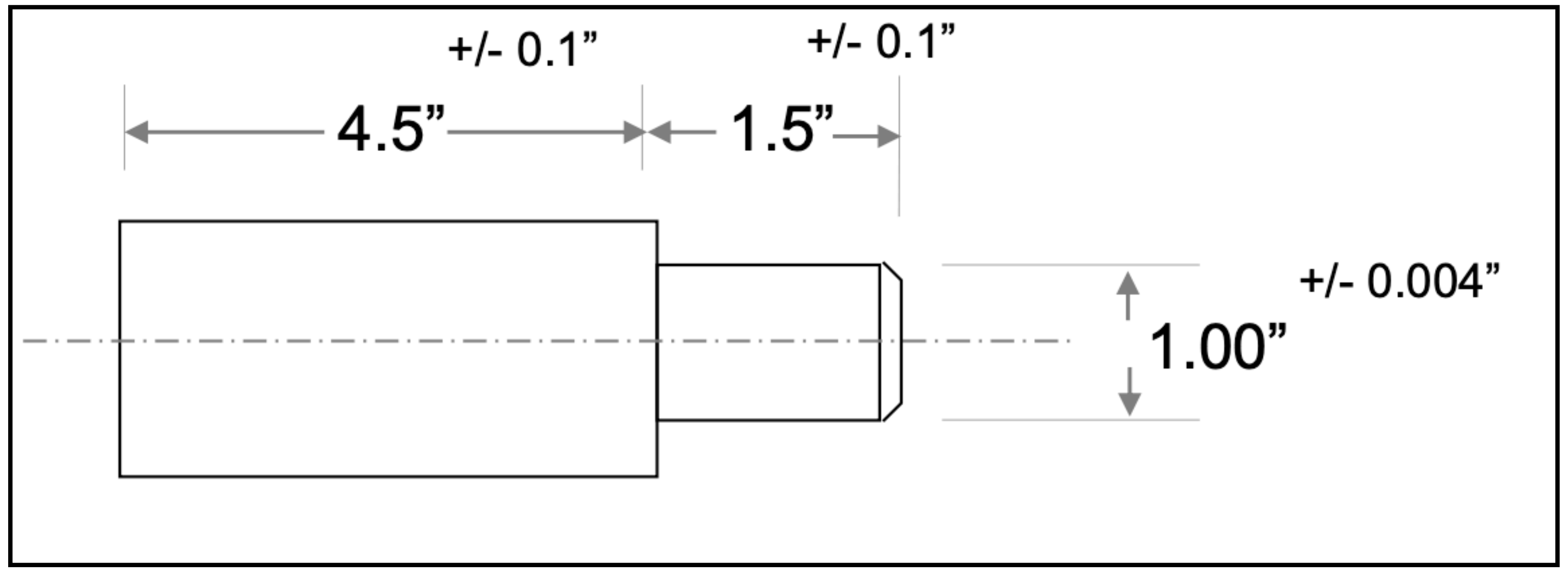
Variations in Critical Diameter



Raw data, n = 20

1.0013	0.9986	1.0015	0.9996
1.0060	0.9997	1.0029	0.9977
1.0042	0.9955	1.0019	0.9970
0.9992	1.0034	0.9995	1.0022
1.0020	0.9960	1.0013	1.0020

Variations in Critical Diameter



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1.0013	0.9986	1.0015	0.9996
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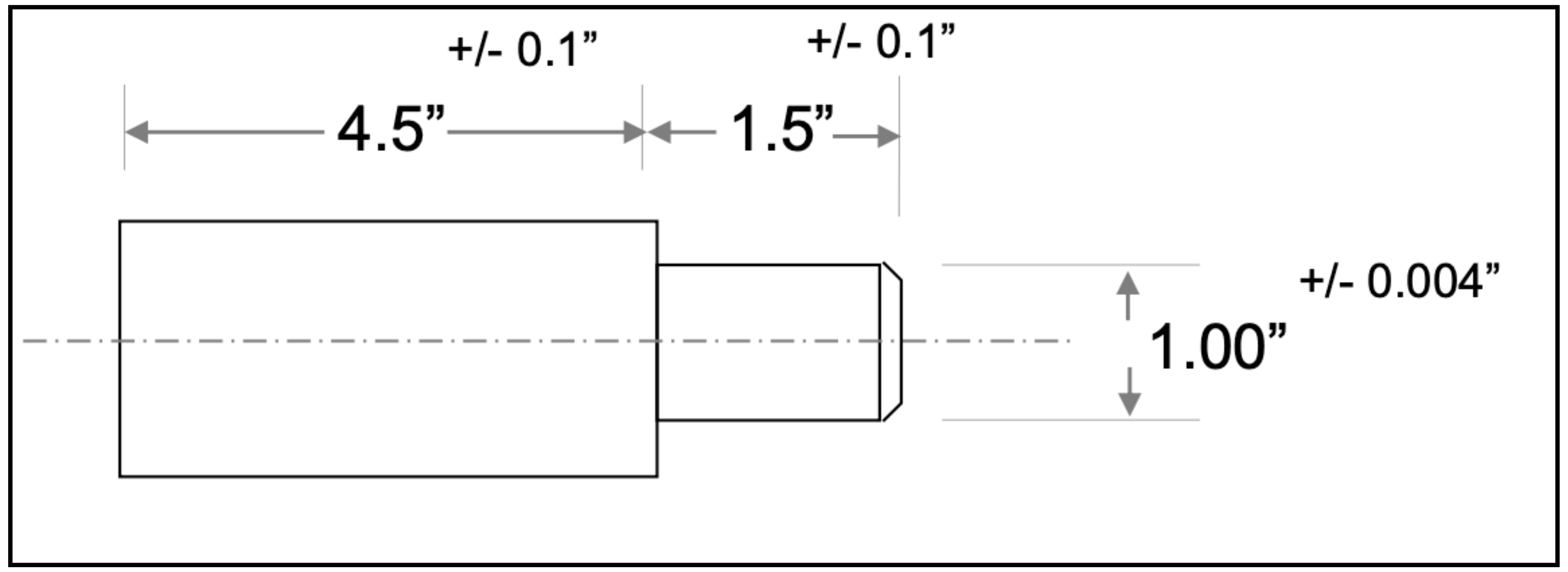
1.0042	0.9955	1.0019	0.9970
0.9992	1.0034	0.9995	1.0022
1.0020	0.9960	1.0013	1.0020

6 Buckets

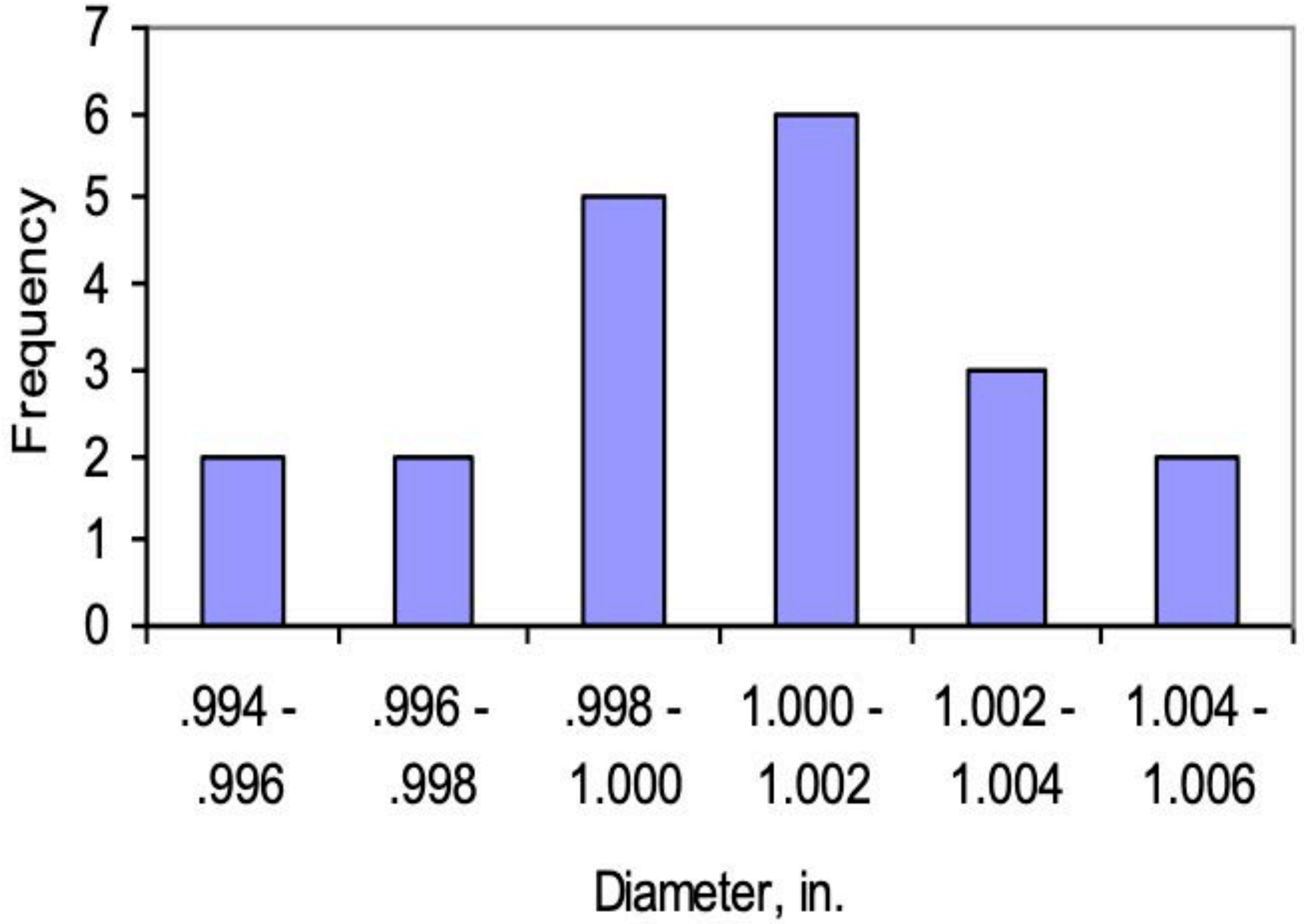
.994 - .996	2	✗
.996 - .998	2	✓
.998 - 1.000	5	✓
1.000 - 1.002	6	✓
1.002 - 1.004	3	✓
1.004 - 1.006	2	✗

bucketing helps you understand the data

Variations in Critical Diameter



get statistical: **process outcome distribution**



Causes of Variations for the Consumer

Manufacturing:

- part variations from unit manufacturing processes
- assembly variations

Use:

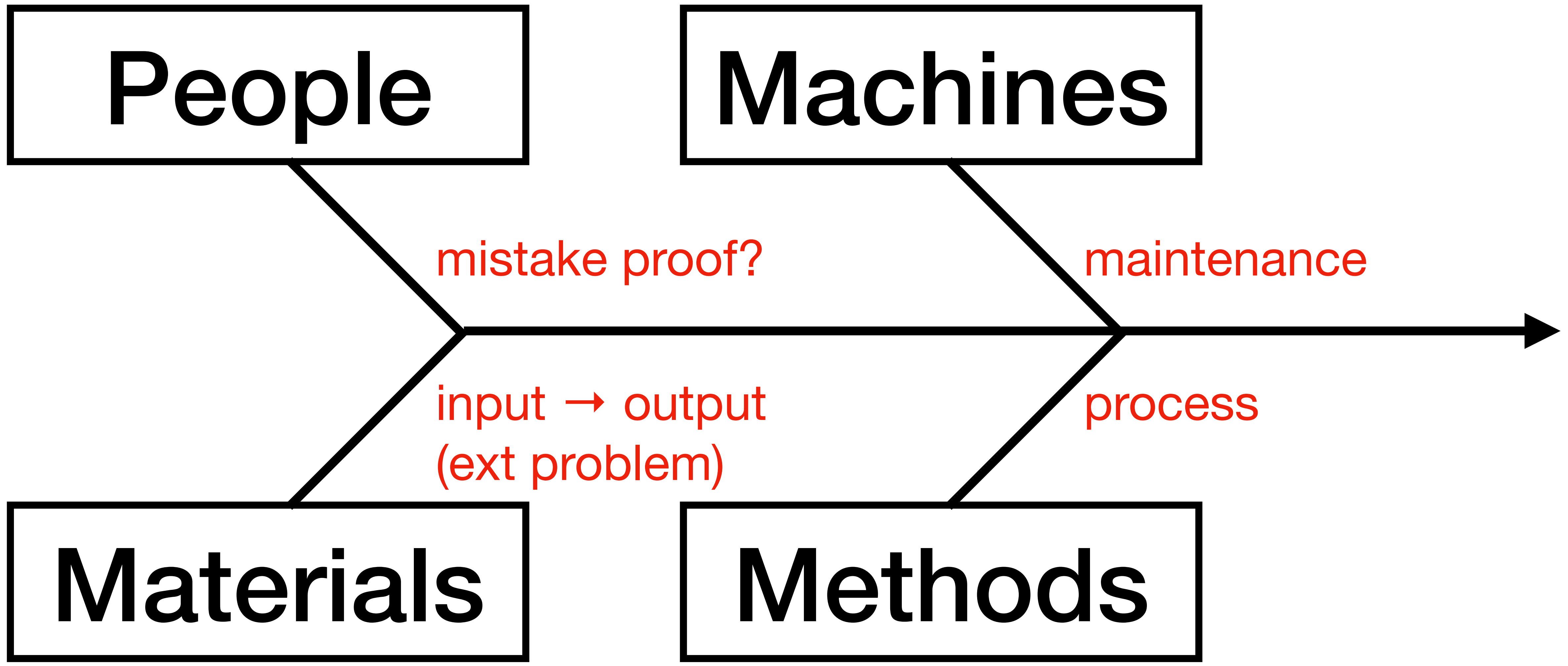
- variations in conditions of use
- deterioration

cars: tested in high elevations, arid, humid, cold - needs to be addressed in design stage and turned into **specifications**

Variation and Quality

Defining, Measuring, and Controlling Quality in Manufacturing

Part and Assembly Variation



Outcome Examples
- shaft O.D. (inches)
- hole distance from reference surface (mm)
- circuit resistance (ohms)
- heat treat temperature (degrees)
- engineering change processing time (hours)

Outcome is **measured**

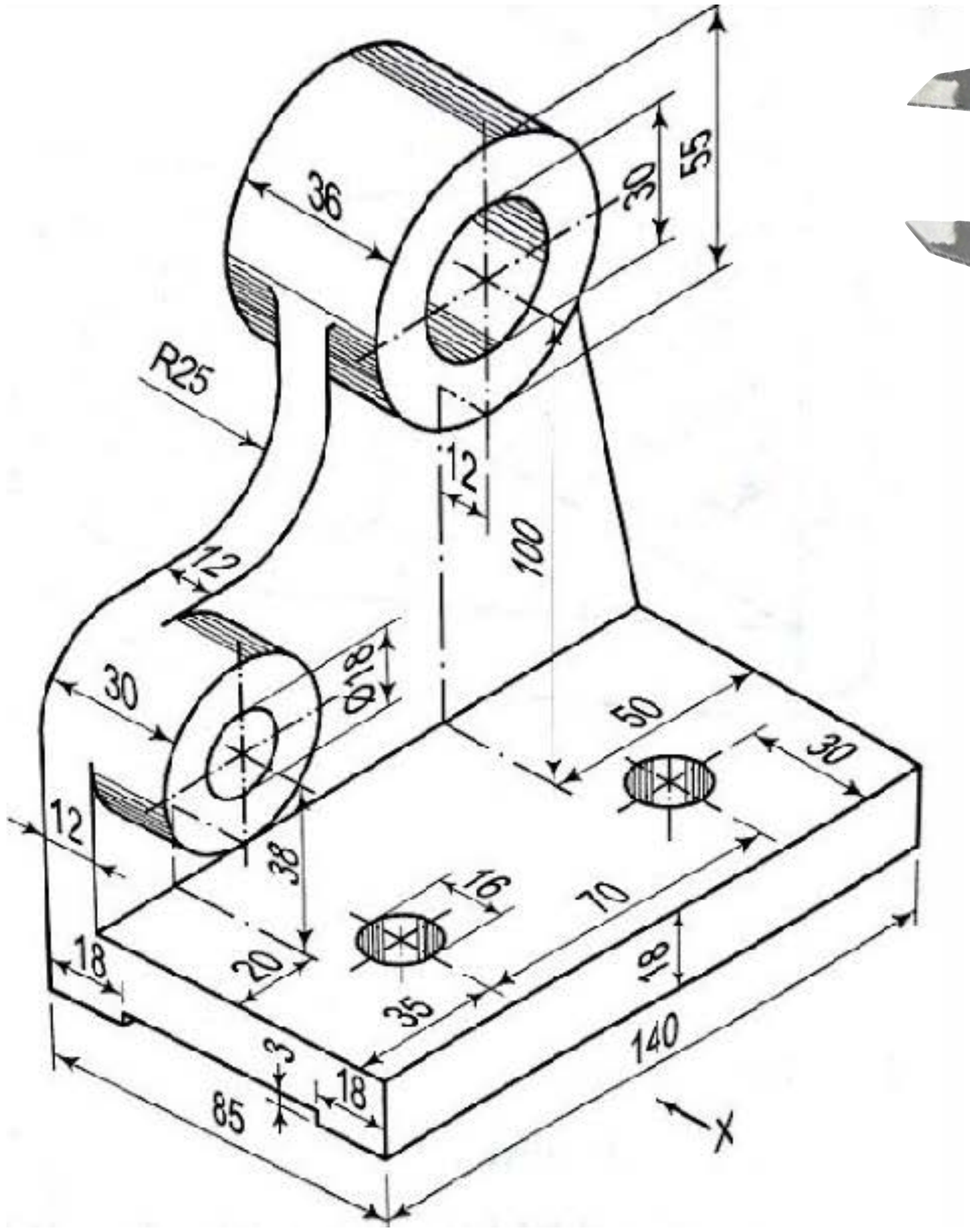
unit of measure (mm, kg, etc.)

measurement method: accurate and precise over time

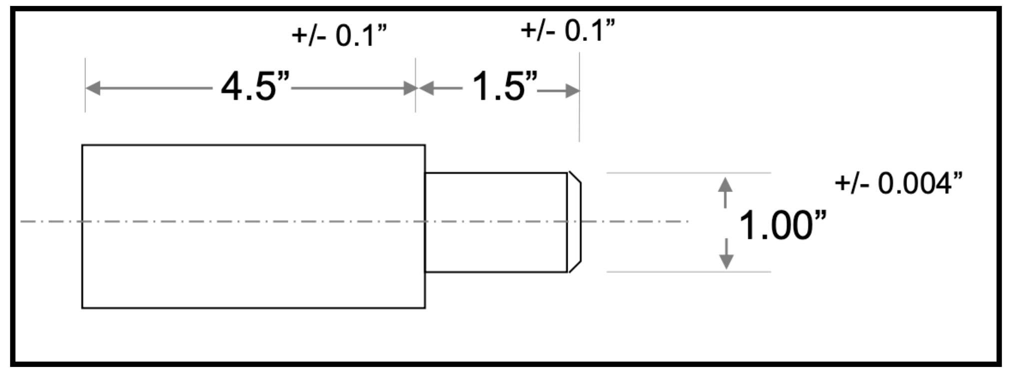
Control of Variations: Technological Development

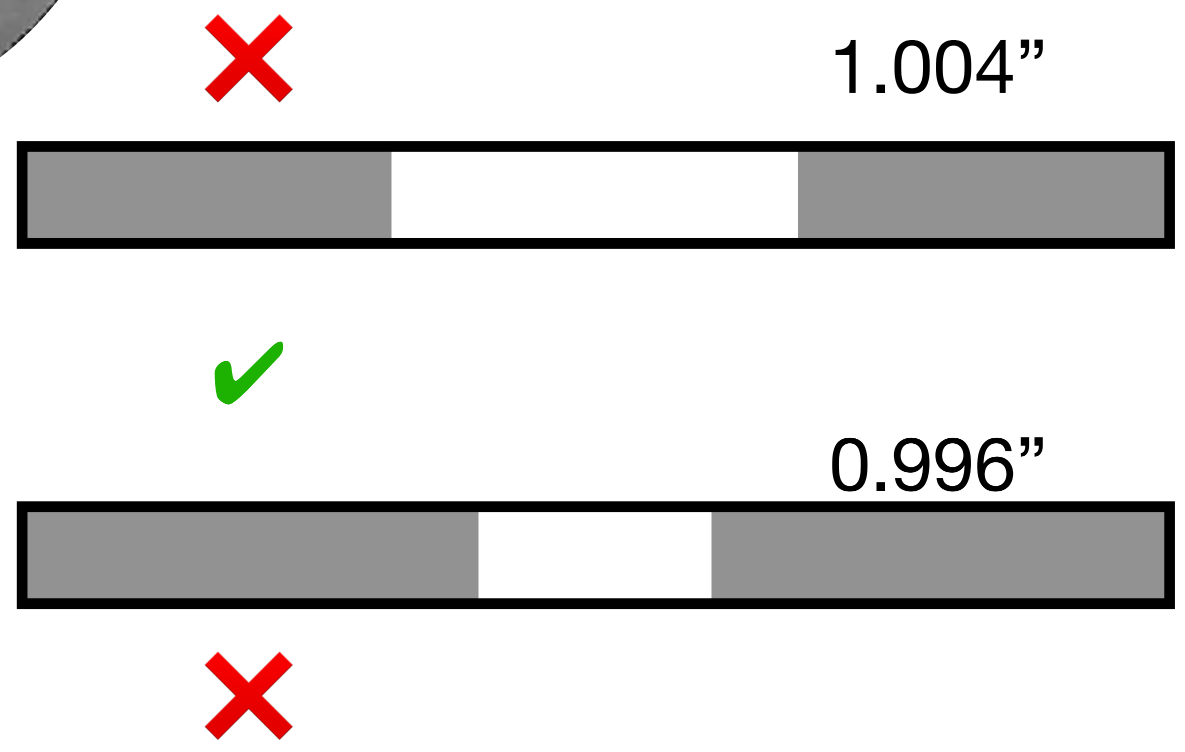
physical masters

engineering drawings



go/no-go gauge





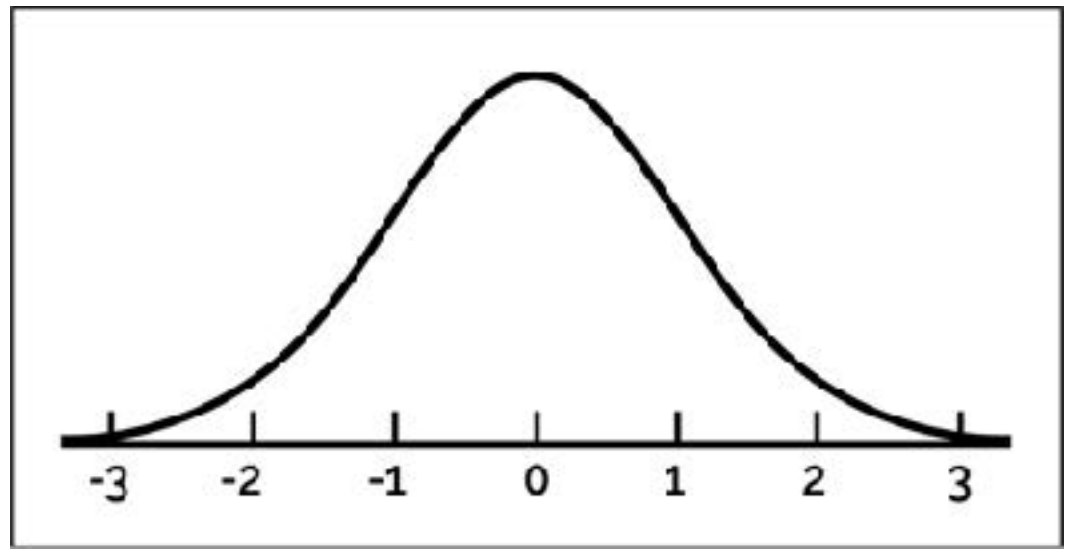
go

no go

Control of Variations: Technological Development

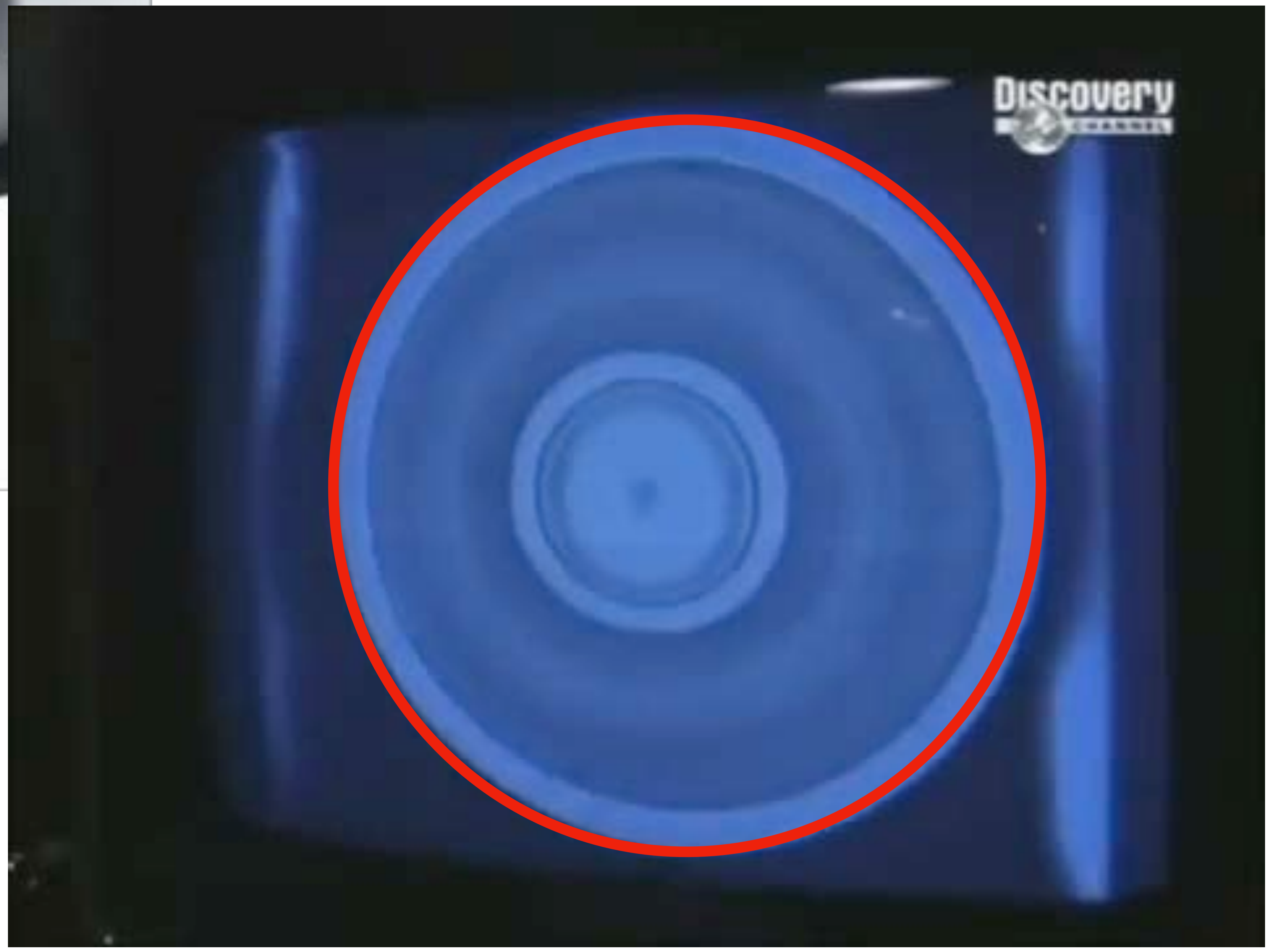
measurement is good, but costly

statistical representation

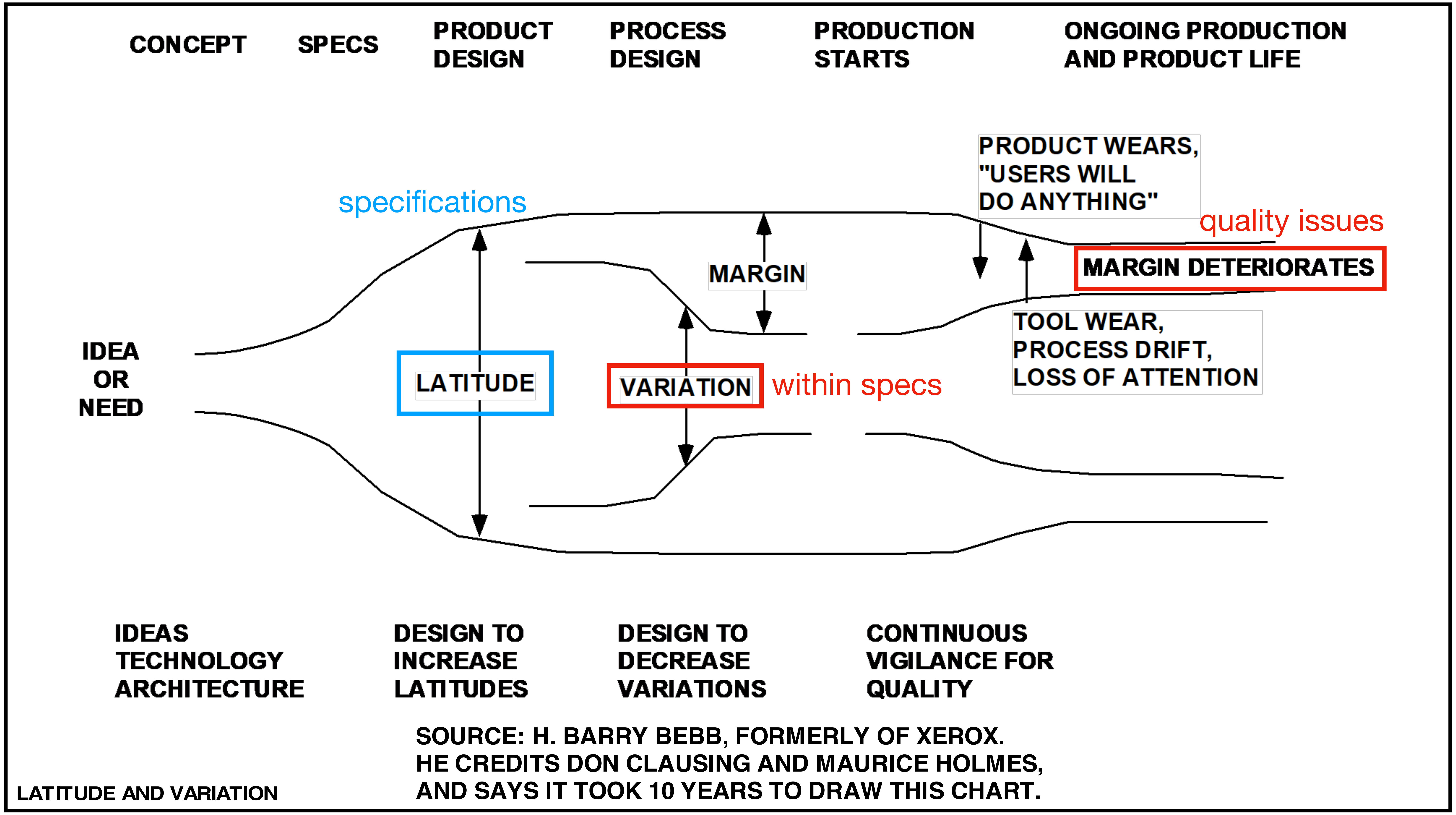


continuous on-line measurement

optical measurement demo



Process Management Over Time



Types of Variations

Systematic / Assignable / Special Cause

(you know or can figure out what is happening)

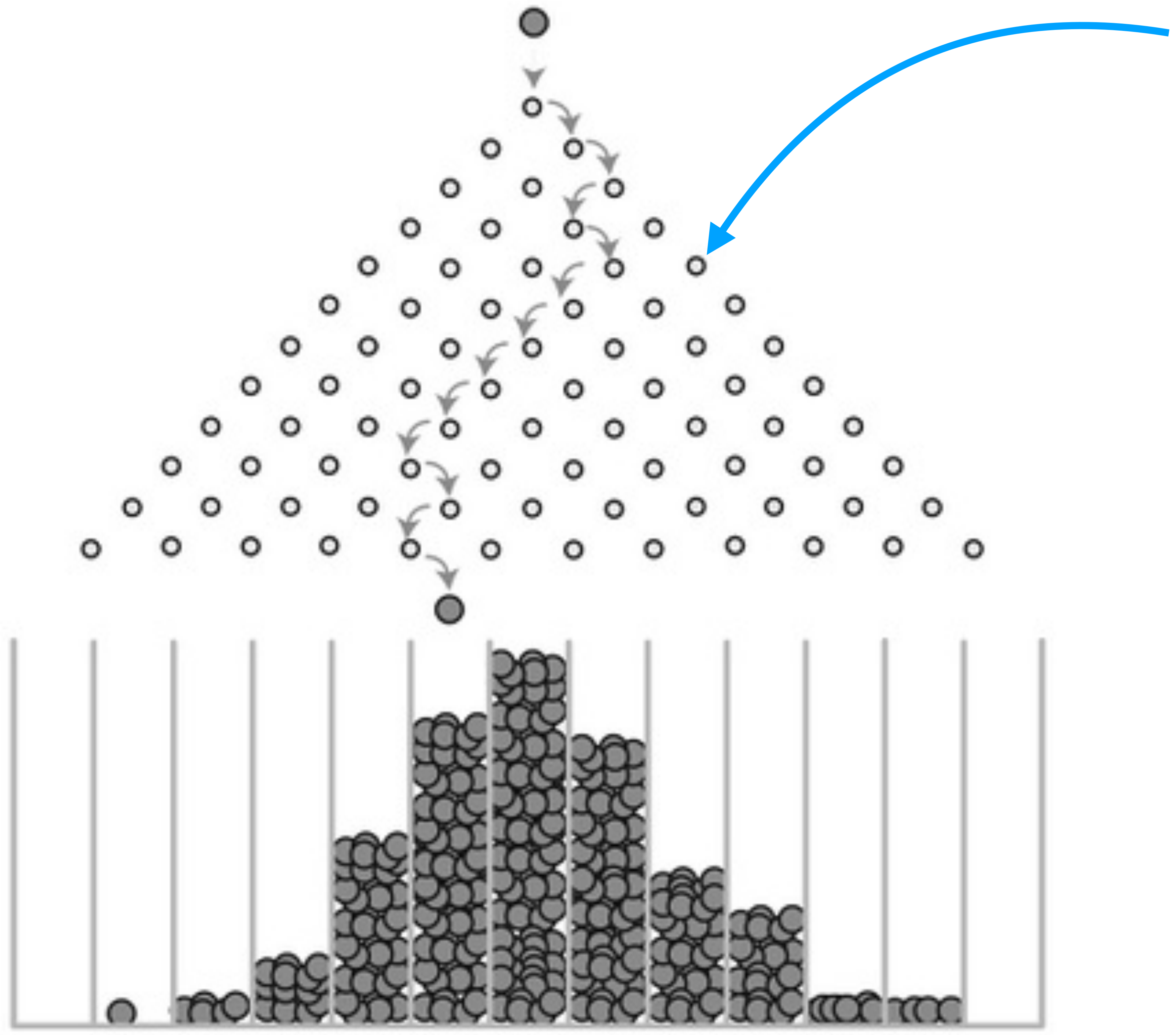
- tool is wearing out
- operator used the wrong depth of cut
- typically a “single direction” shift

Random / Un-assignable / Common Cause

(there’s always some amount of randomness)

- there’s some natural variation due to vibrations, nonlinearities, etc.
- i.e. a truck passed by and caused extra vibration or there was a solar flare
- “positive and negative” shifts

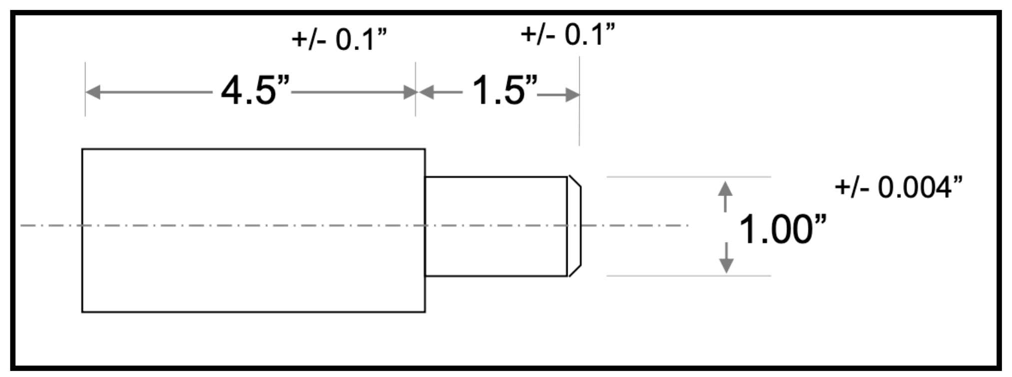
Manufacturing Outcomes

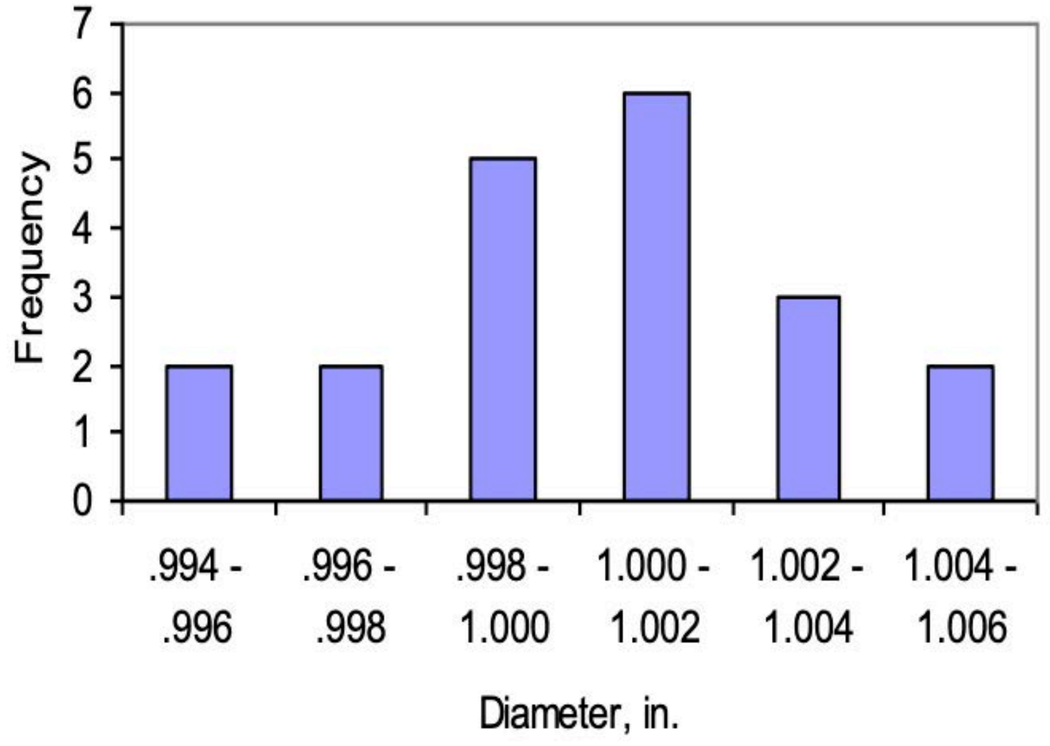


each row is **independent**

just like a manufacturing process with multiple inputs

Outcomes are **normally distributed**





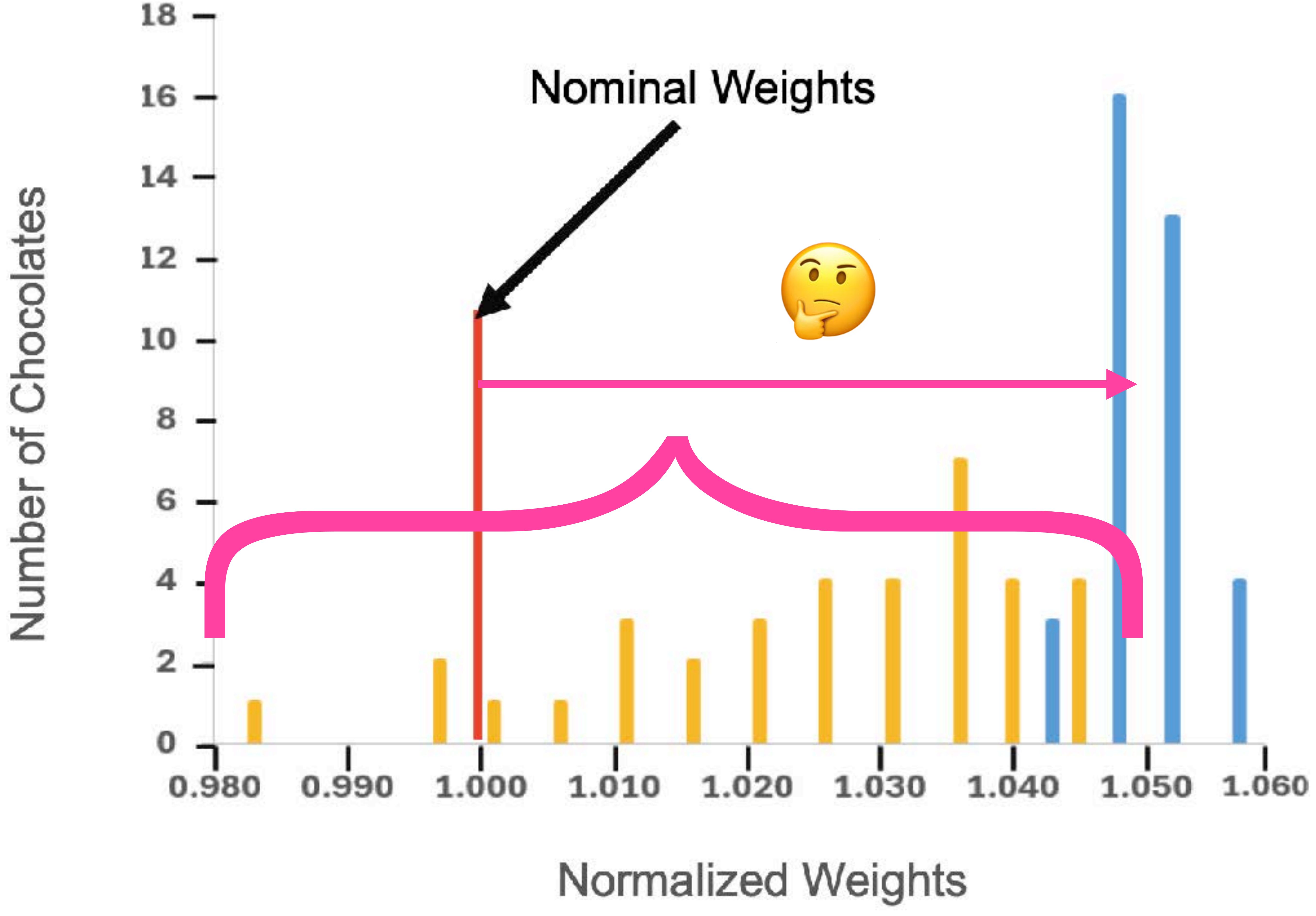
Chocolate Bar Distributions

Which bar do you expect to have lower variation?

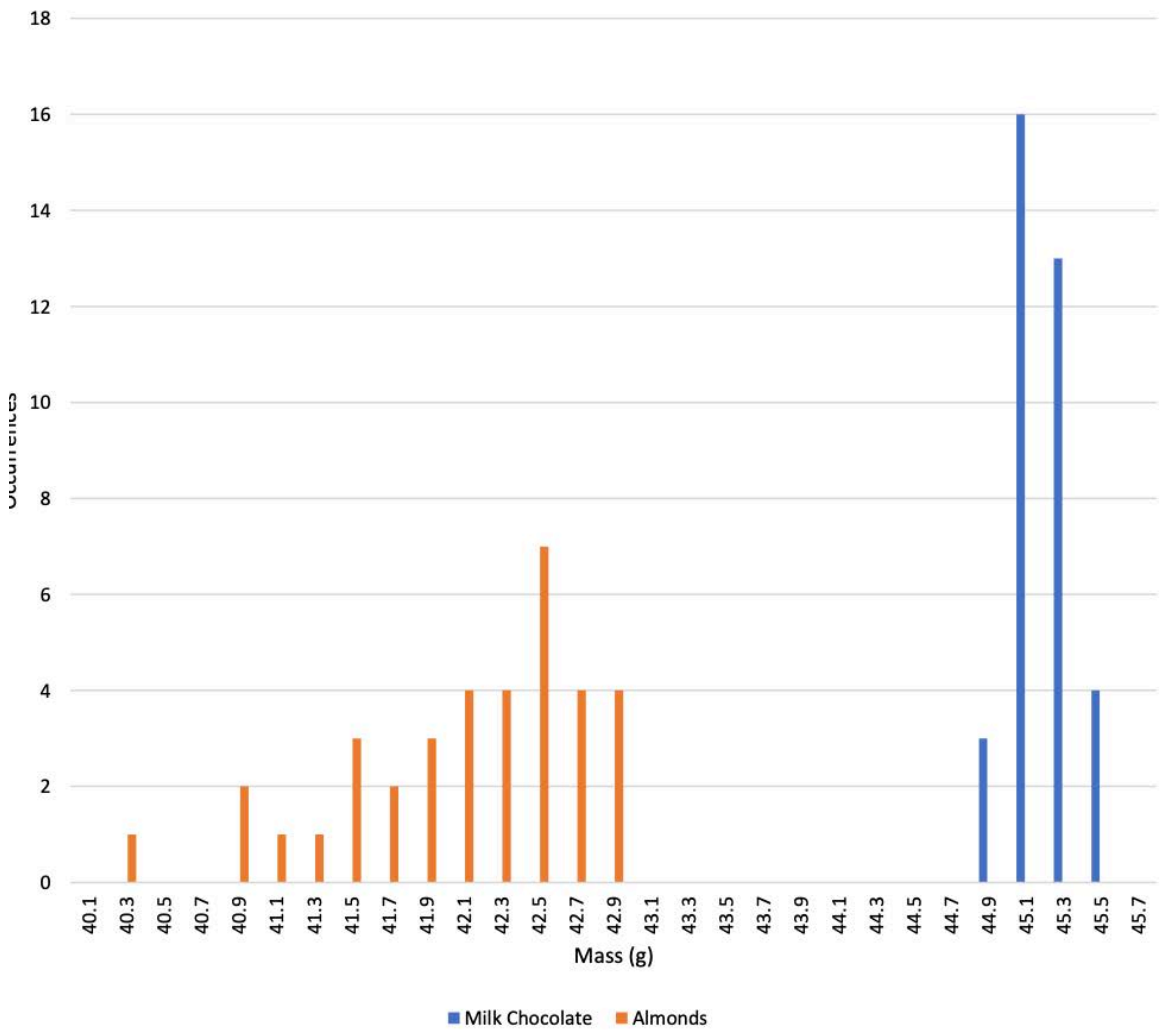
- investigate mean and standard deviation of chocolate bar weight distributions

Weight Distribution of Hershey's Milk Chocolate and Milk Chocolate with Almond Bars

?



Distribution of Hershey's Milk Chocolate and Almond Bar Masses



Milk Chocolate Analysis (n=36)	
Nominal Weight (g)	43
Median	45.08
Mean	45.10
Min	44.72
Max	45.41
Standard Deviation	0.16

Almond Bar Anaylsis (n=36)	
Nominal Weight (g)	41
Median	42.16
Mean	42.01
Min	40.13
Max	42.83
Standard Deviation	0.64

Statistical Distributions

central tendency: more in the middle

- sample mean (arithmetic)
- sample median when do we choose median?

measures of **dispersion**

- variance
- standard deviation ("average deviation from the mean")
- range

$$mean: \ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$variance: \ s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$$std\ dev: \ s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

x: sample variable
n: number of values

Statistical Distributions

central tendency: more in the middle

- sample mean (arithmetic)
- sample median

measures of **dispersion**

- variance
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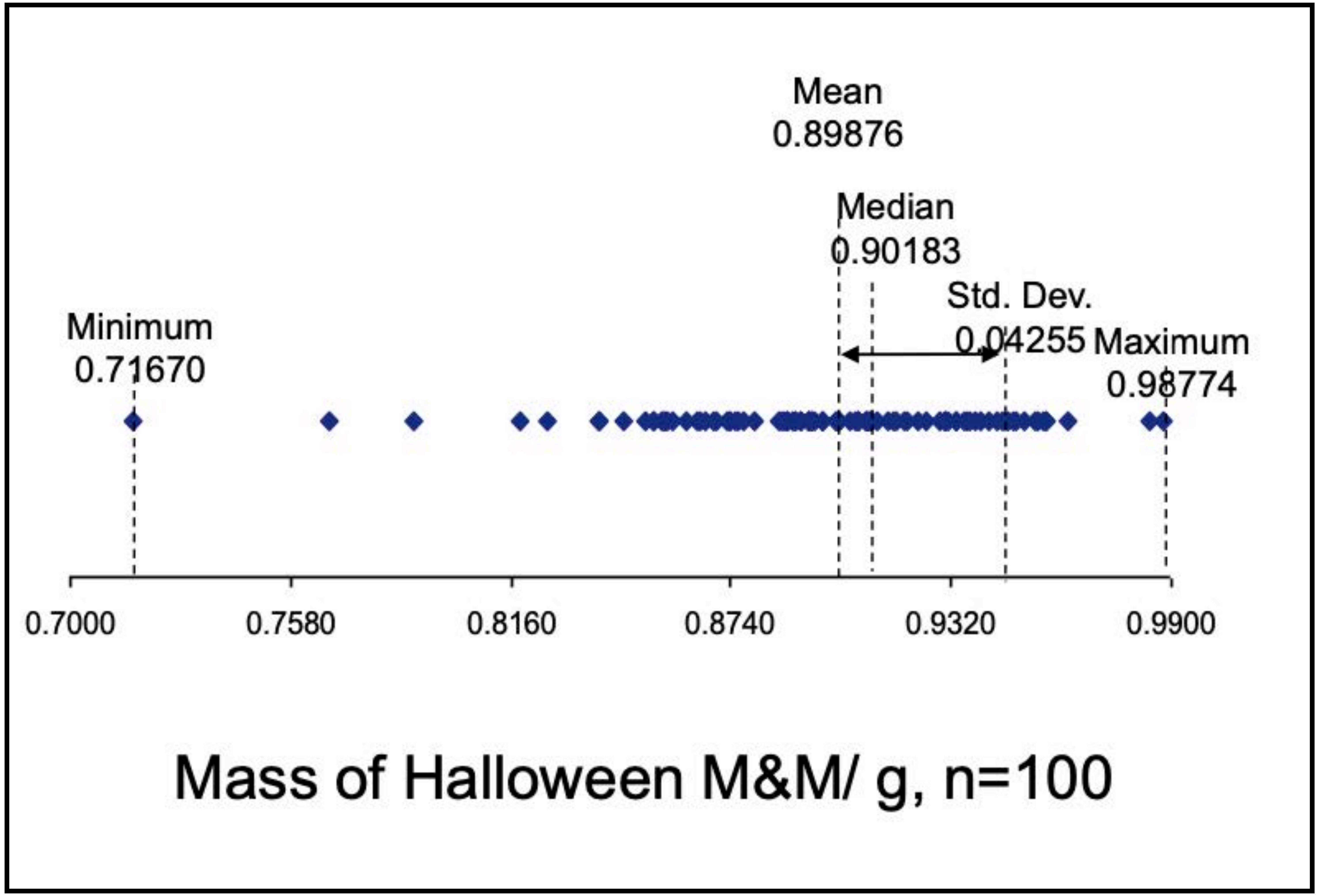


Statistical Distributions

Probability

$$P\{a \le x \le b\} = \int_a^b f(x)\ dx$$

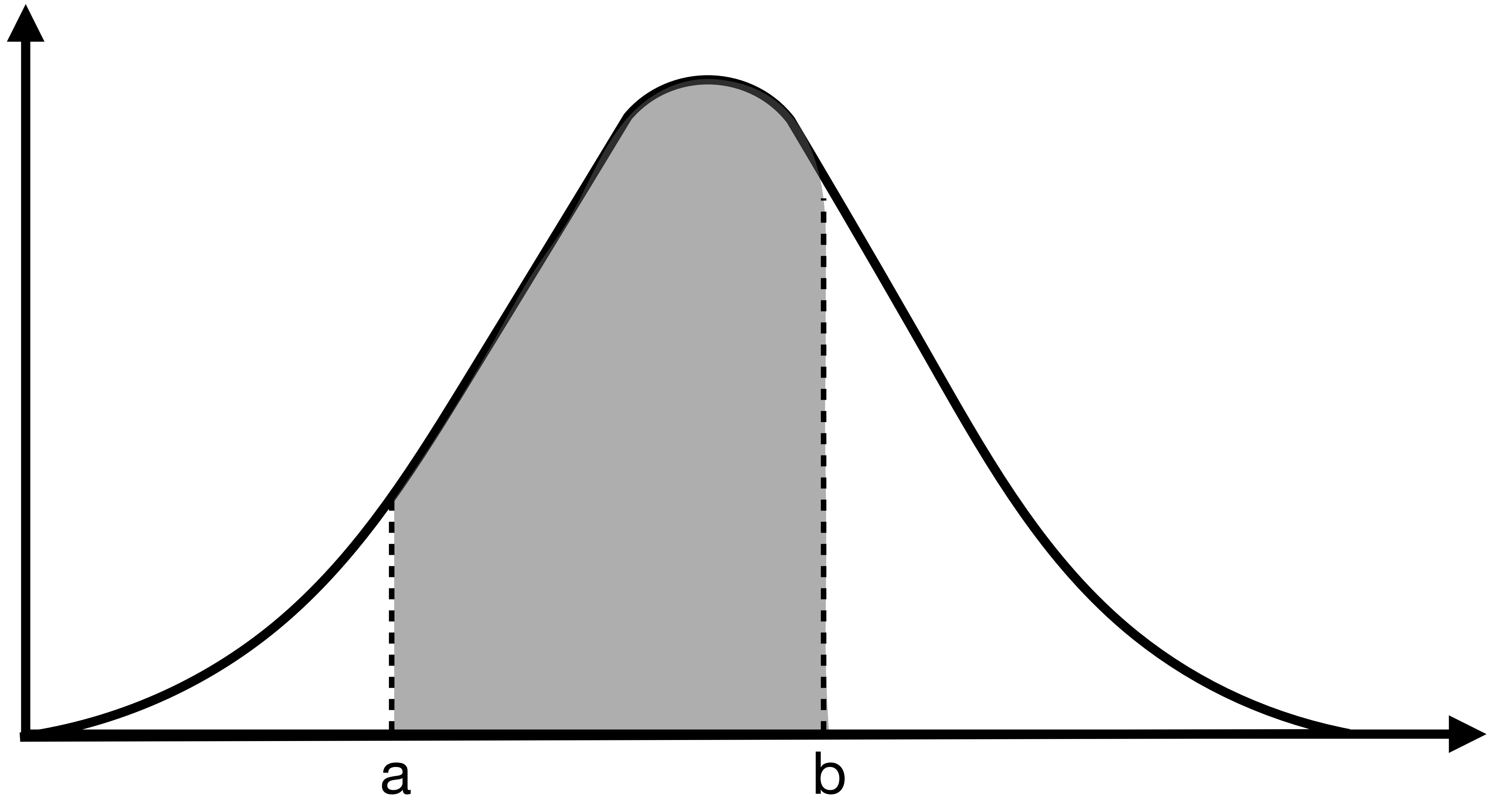
$$P\{-\infty \le x \le \infty\} = \int_{-\infty}^{\infty} f(x)\ dx = 1\ for\ all\ \bar{x},\ s$$

Normalized

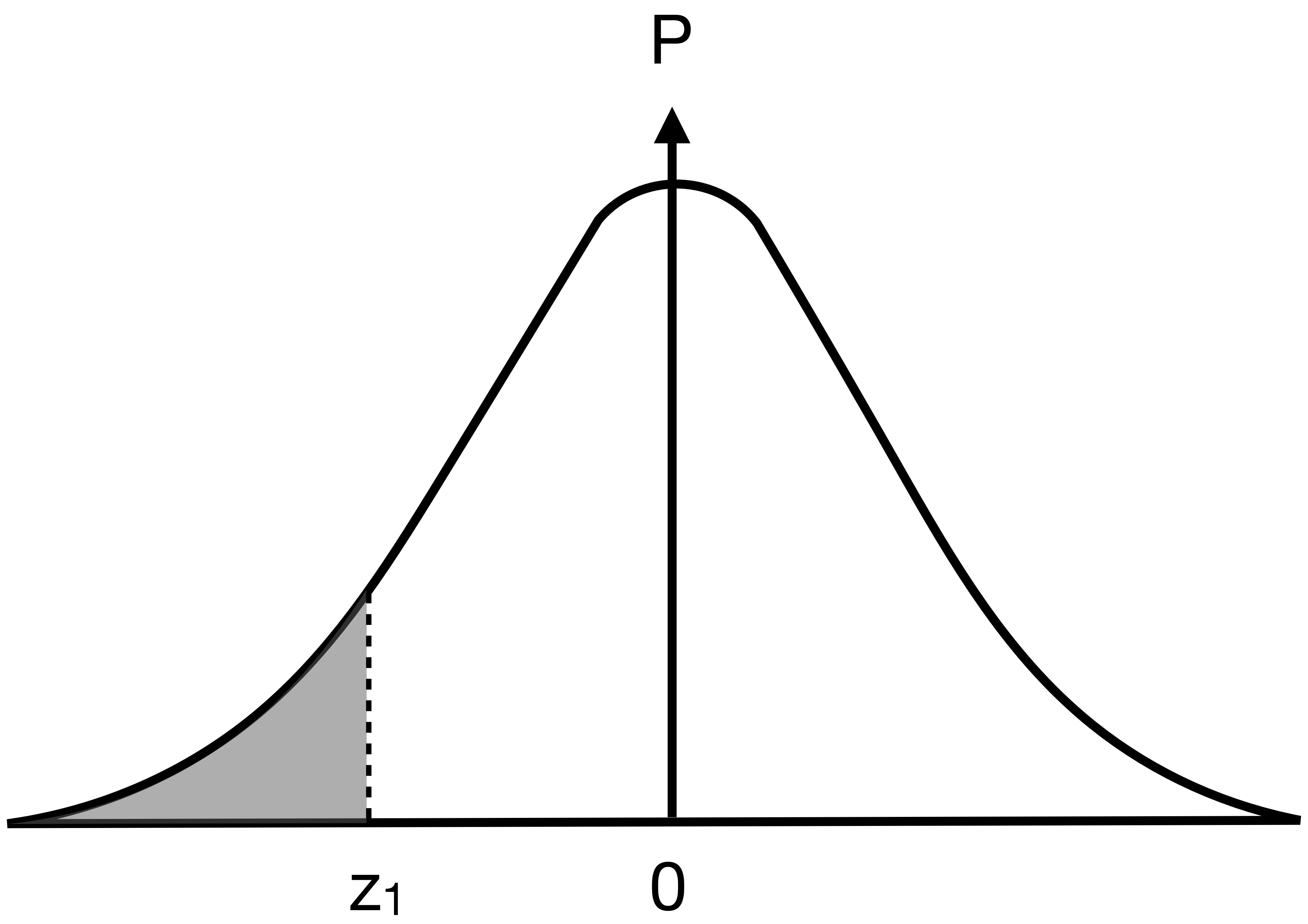
$$z = \frac{x - \bar{x}}{s}$$ ("number of std devs")

$$P\{z_1 \le x \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{z^2}{2}\right)}$$

$$f(x) = \frac{1}{2\pi s} e^{\left(-\frac{(x-\bar{x})^2}{2s^2}\right)}$$

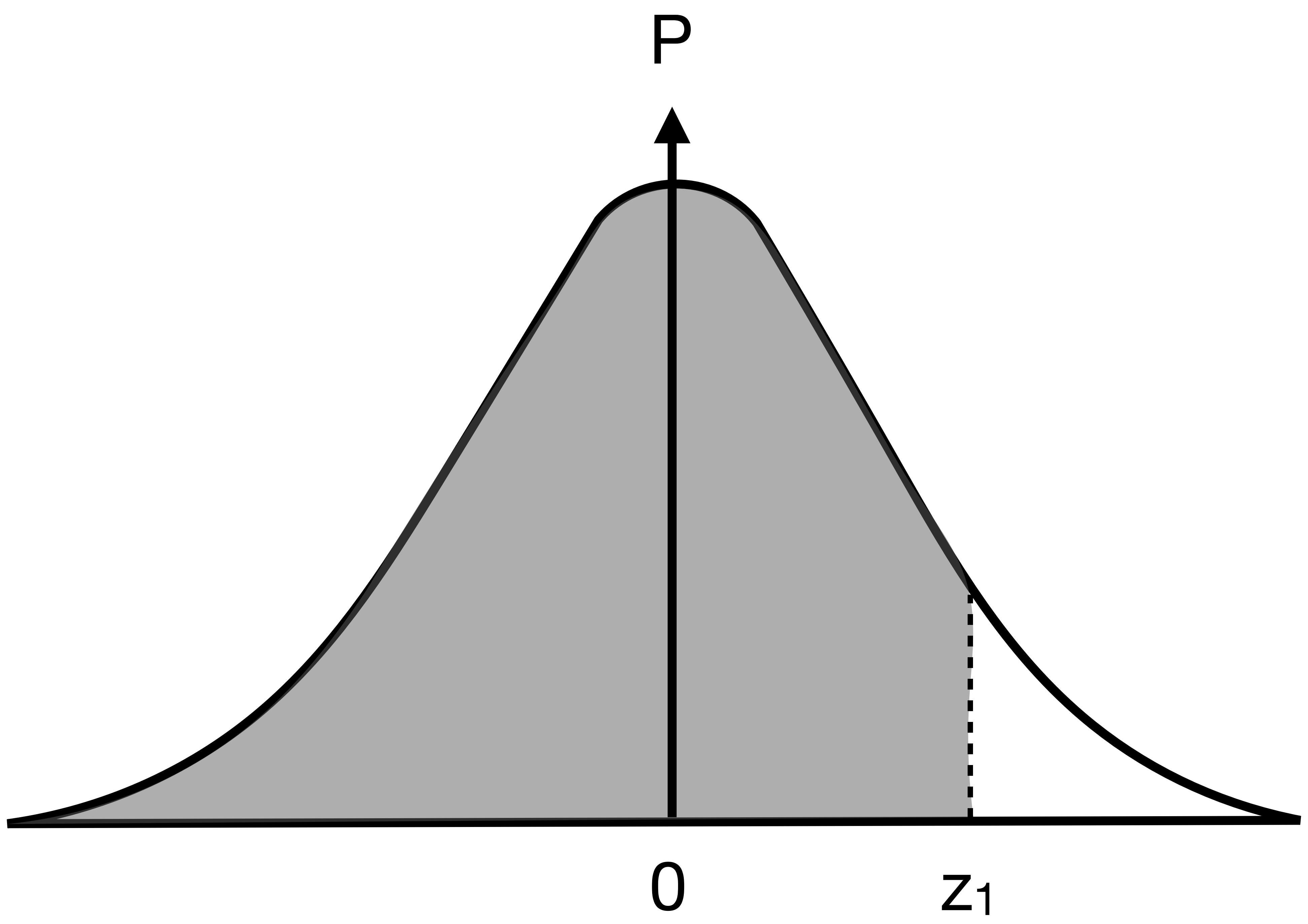


Areas under the Normal Distribution Curve



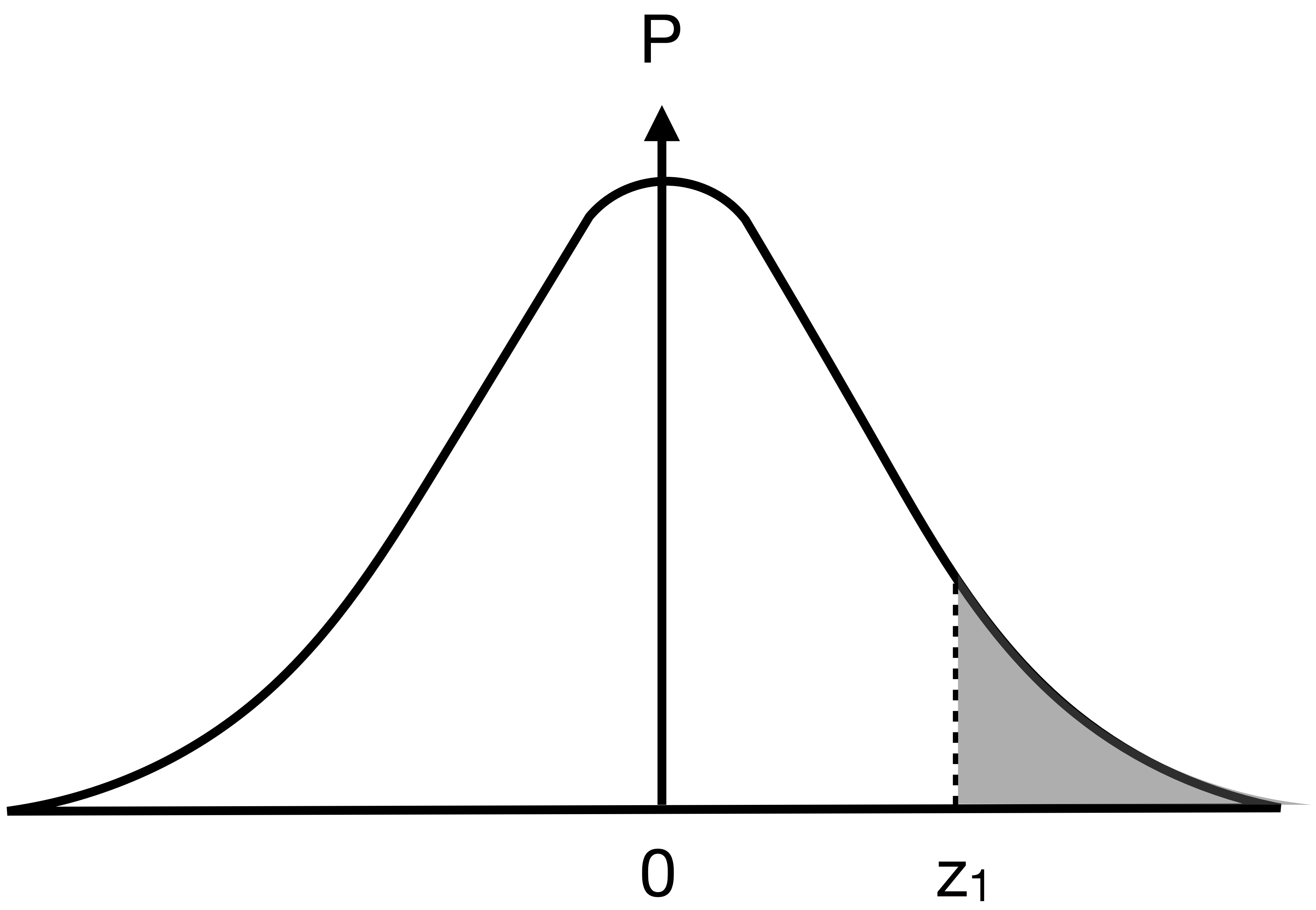
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359

Areas under the Normal Distribution Curve



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Normal Distribution Example



Take an M&M with mass = 0.9g. Based on our normal curve, how many M&Ms in a package on average have a mass greater than 0.9g?

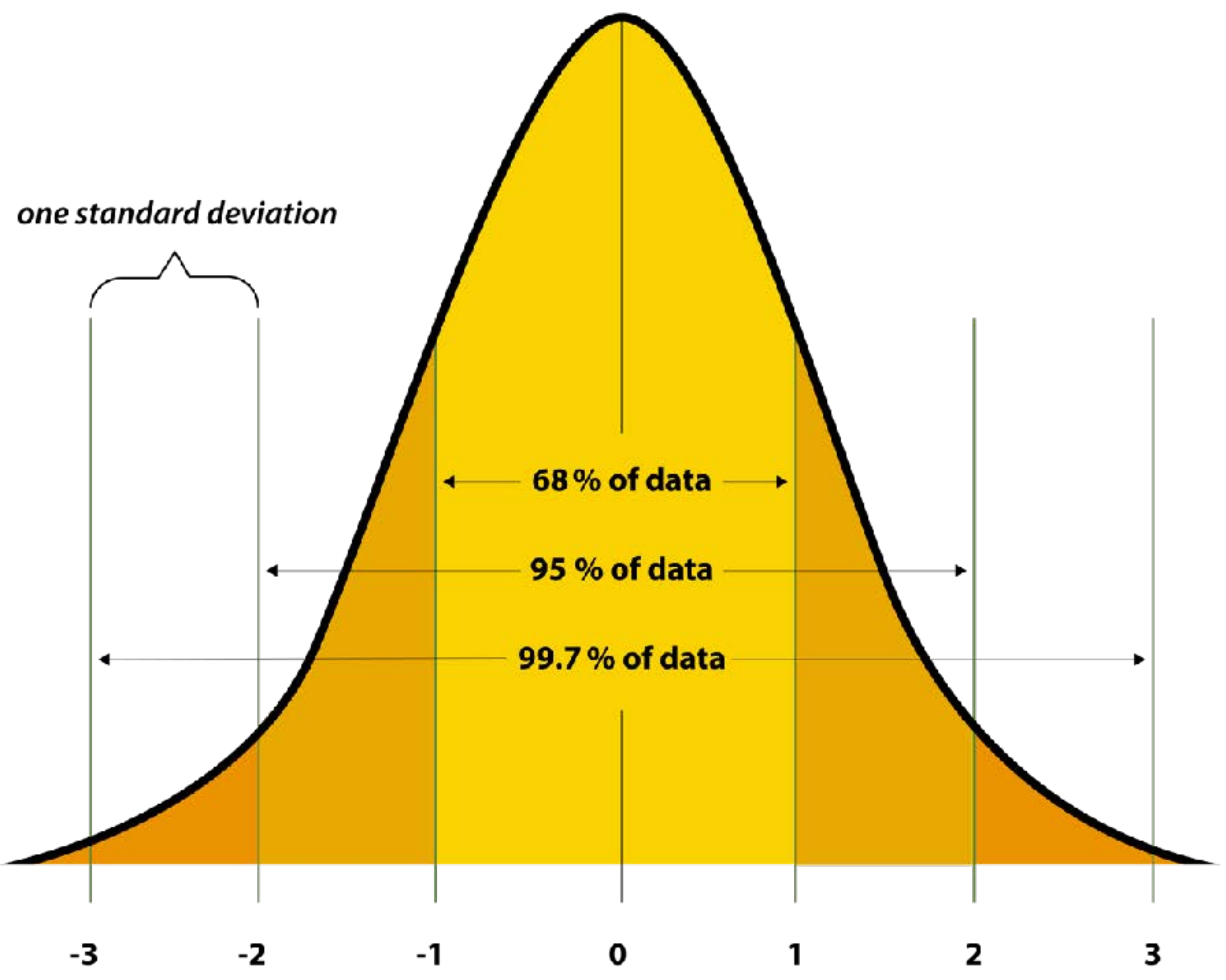
$$z = \frac{x - \bar{x}}{s} = \frac{0.9000 - 0.8988}{0.0425} = 0.29$$

The area to the right of z = 0.29 is 1 minus the area to the left of z = 0.29. Using the table:

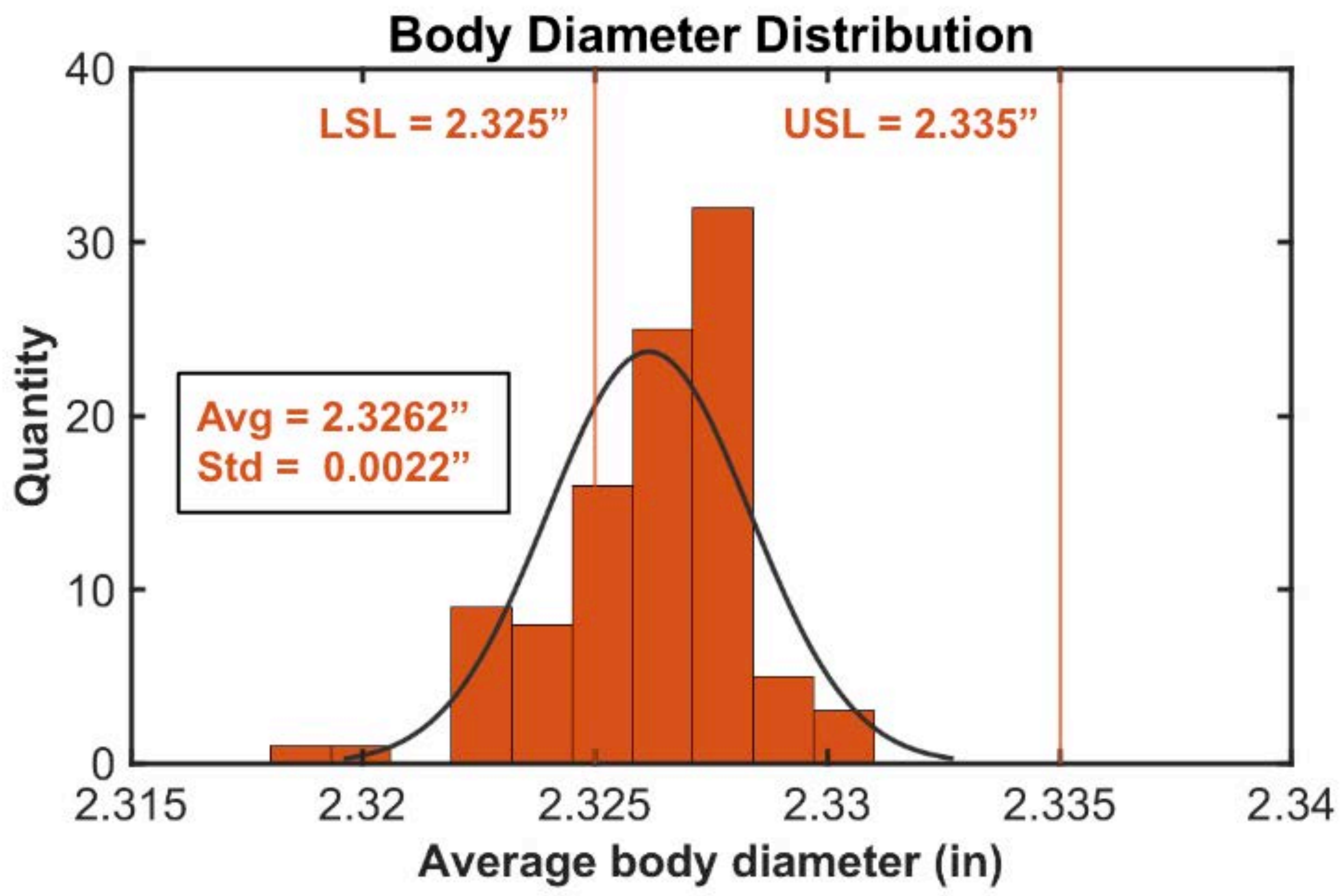
$$P = (1 - 0.6141) = 0.3859$$

So 38.59% of our 100 M&Ms will on average have a mass greater than 0.9g, or 39 M&Ms

Standard Deviation References



Yo-yo project distributions



Process Capability!

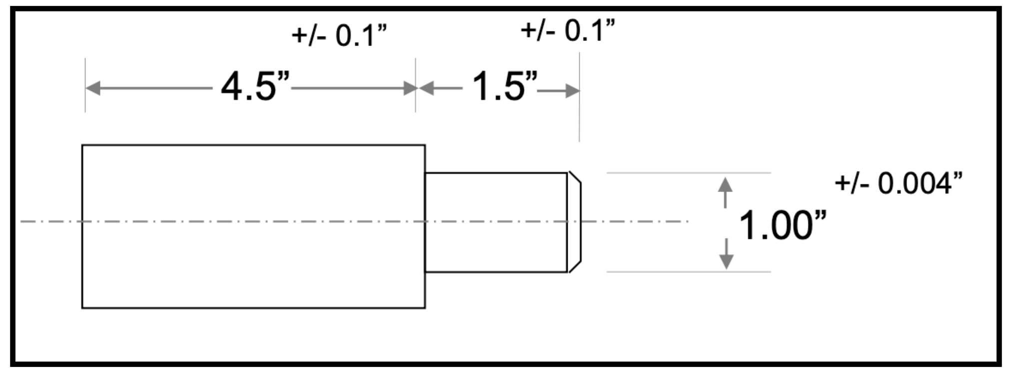
Is the process **capable** of meeting the design requirements?

Design and Manufacturing intersect

$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$

LSL: lower specification limit
USL: upper specification limit

Process Capability!



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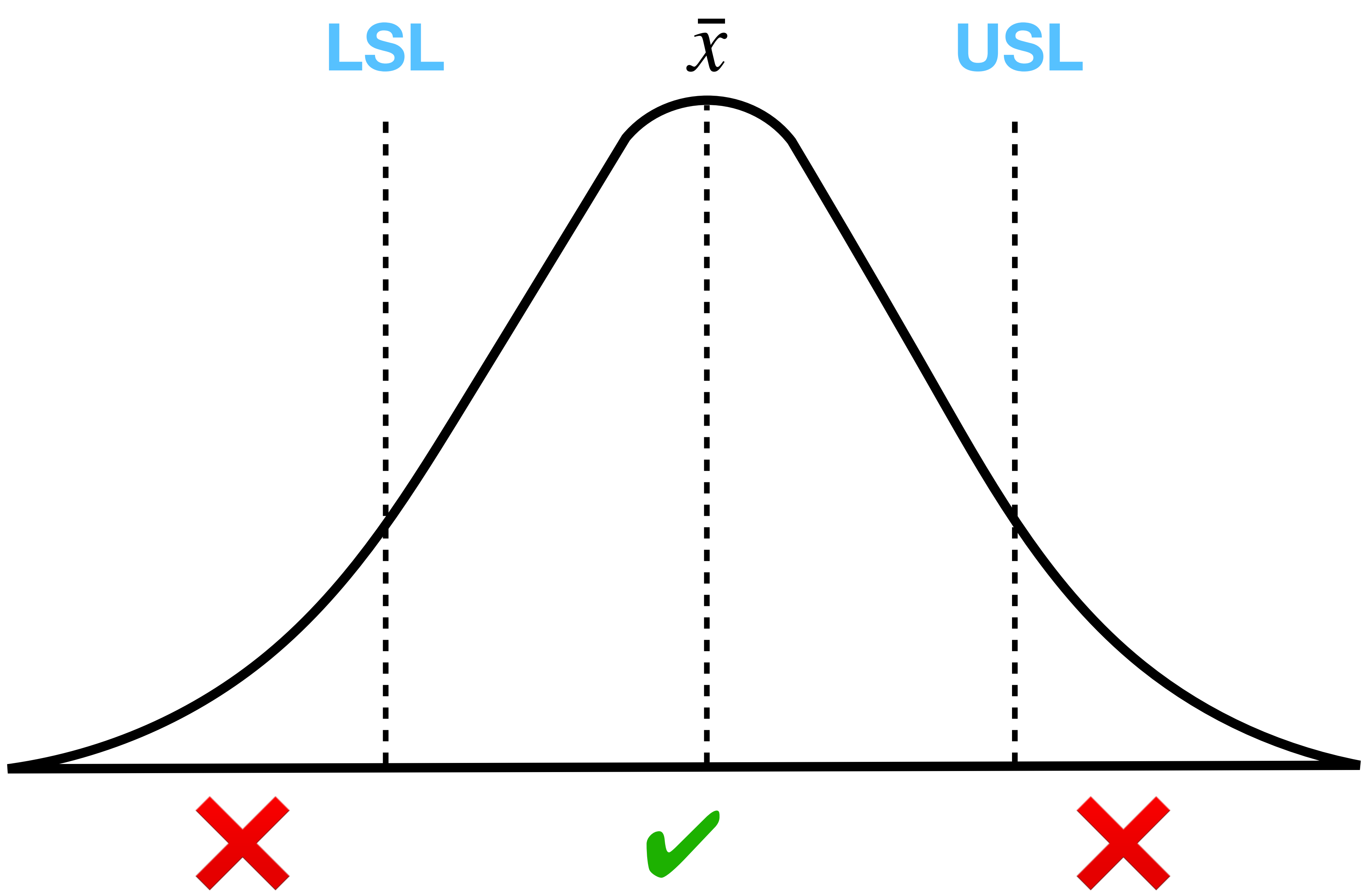

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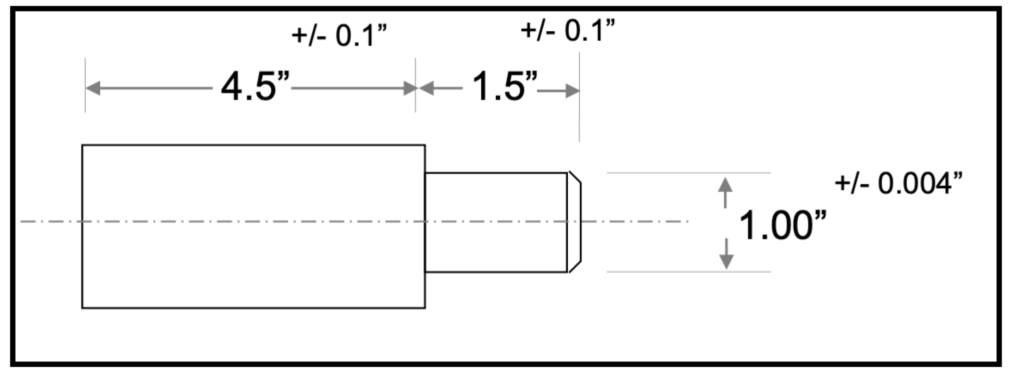
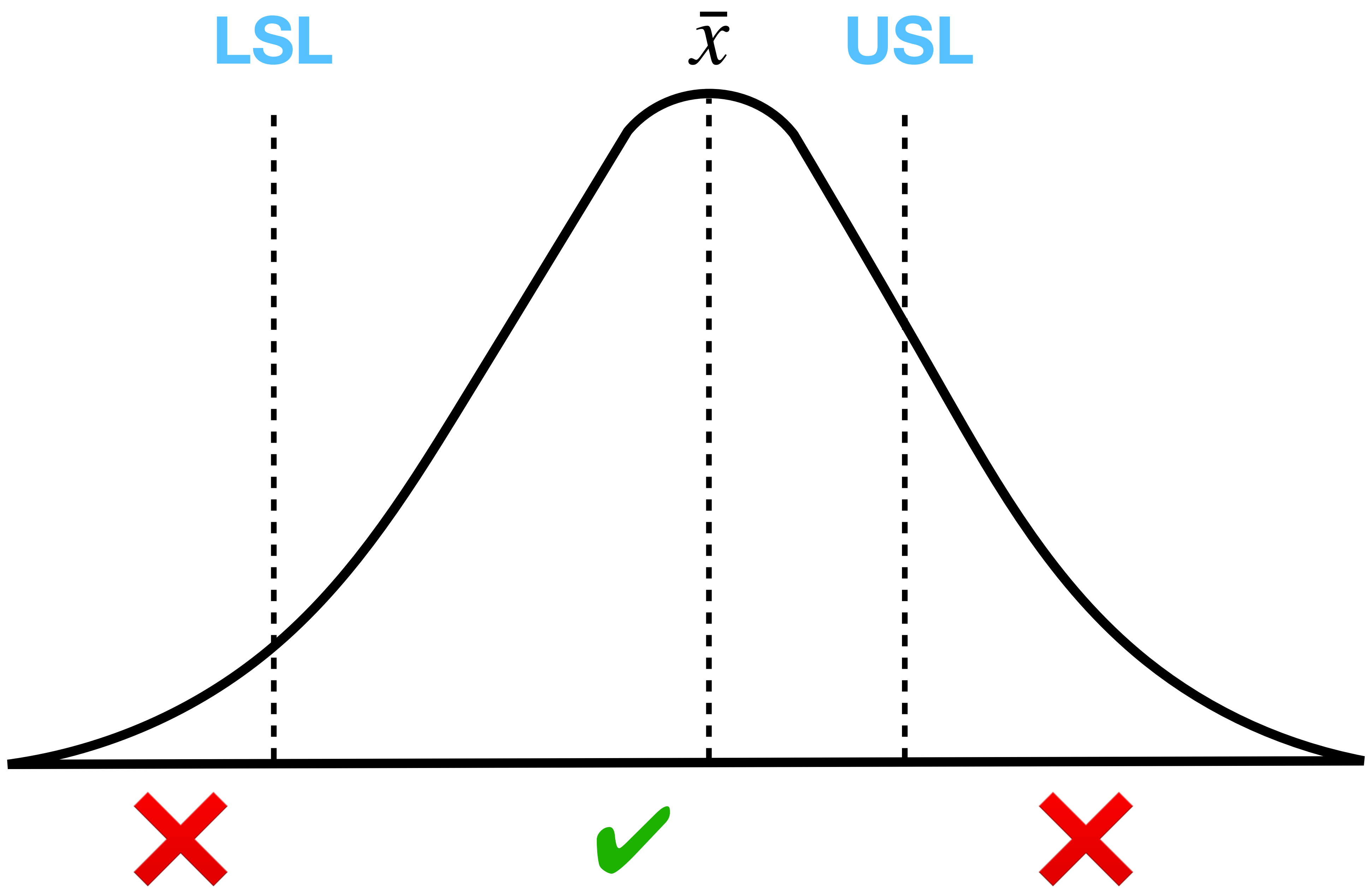


$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$



LSL: lower specification limit
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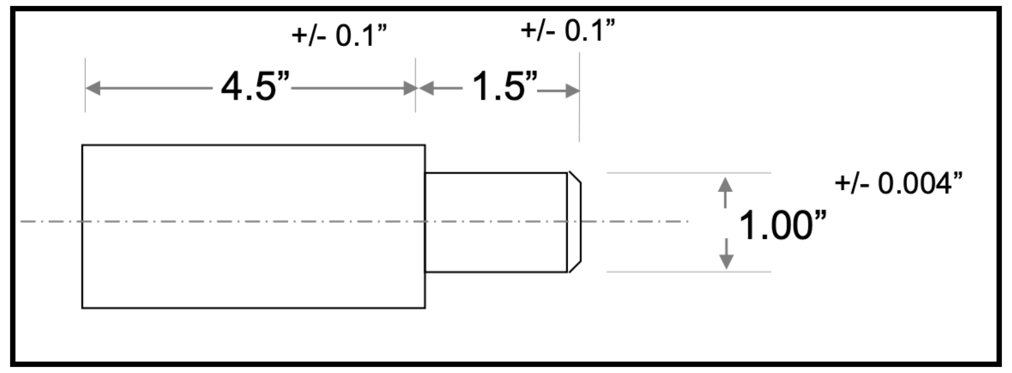
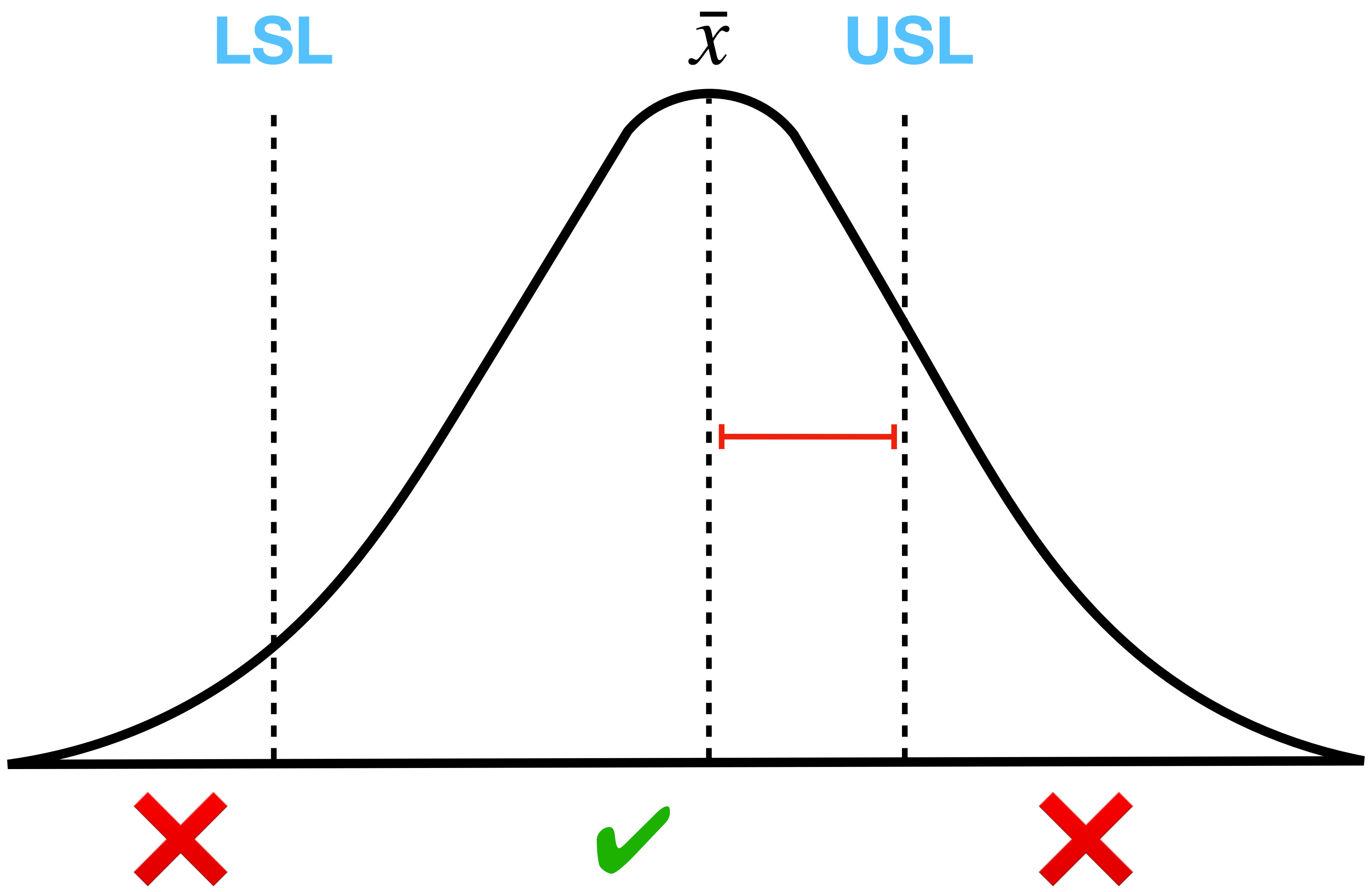
Process Capability!



Is the process **capable** of meeting the design requirements?

Design and Manufacturing intersect



$$C_{pk} = \frac{USL - \bar{x}}{3\sigma_{process}} \quad \text{or} \quad C_{pk} = \frac{\bar{x} - LSL}{3\sigma_{process}}$$

(whichever is smaller)

LSL: lower specification limit
USL: upper specification limit

C_{pk} is a more strict metric that is sensitive to shifts in the mean

Capability Example

Mean = 0.738

Standard Deviation = 0.0725

USL = 0.900

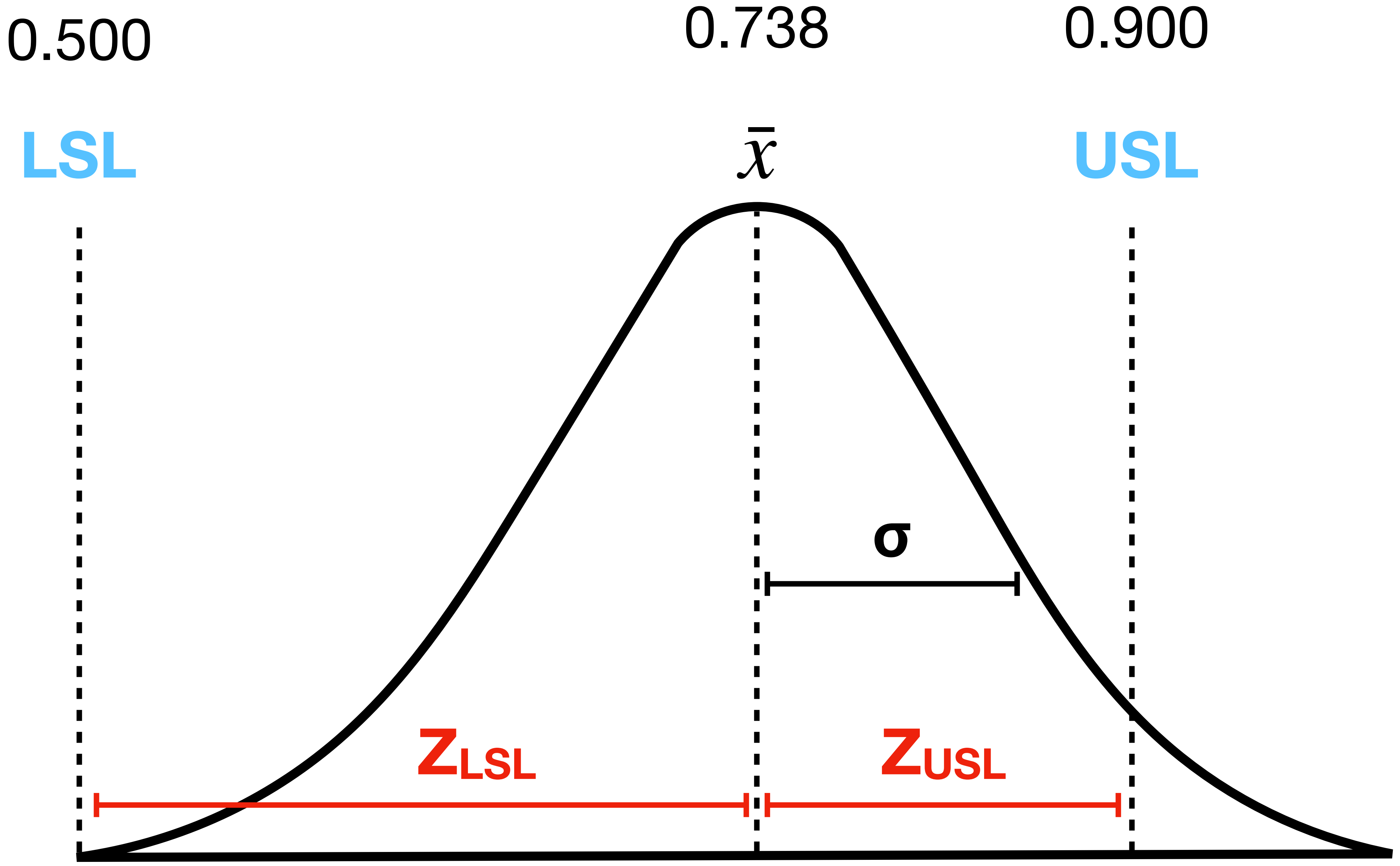
LSL = 0.500

Normalizing:

$$Z_{USL} = \frac{USL - \bar{x}}{\sigma} = \frac{0.900 - 0.738}{0.0725} = 2.23$$

$$Z_{LSL} = \frac{\bar{x} - LSL}{\sigma} = \frac{0.738 - 0.500}{0.0725} = 3.28$$

$$Z_{min} = 2.23$$



$$C_{pk} = 2.23/3 = 0.740$$

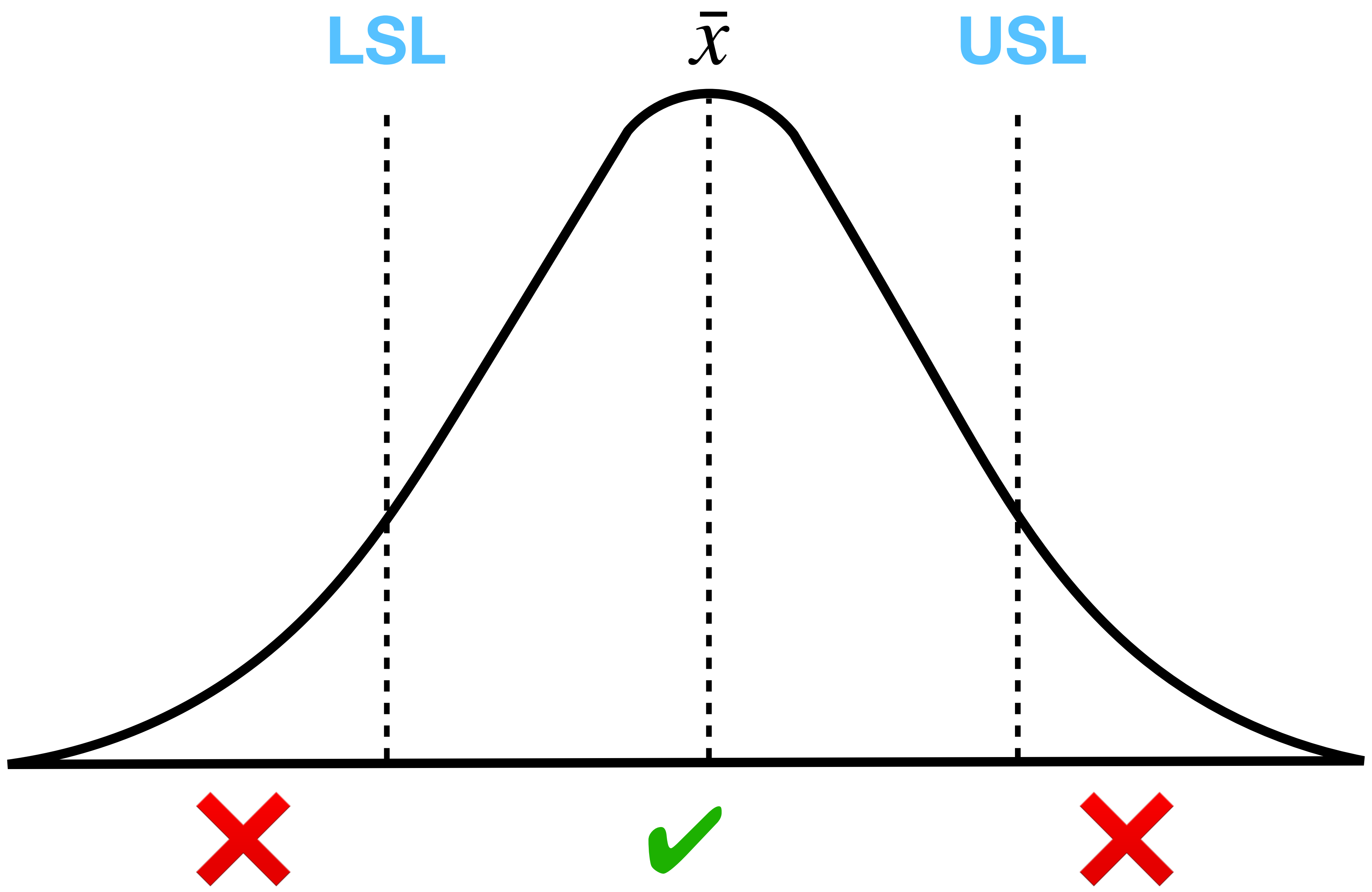
$$C_p = (0.900 - 0.500)/(6 * 0.0725) = 0.920$$

Process Capability!

How do we improve?

Design + Manufacturing

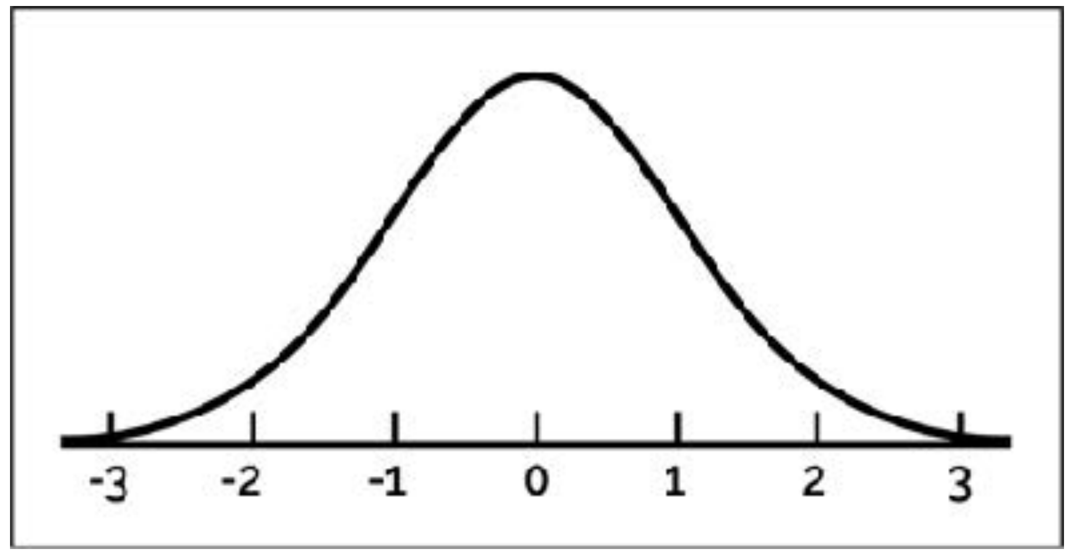
$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$



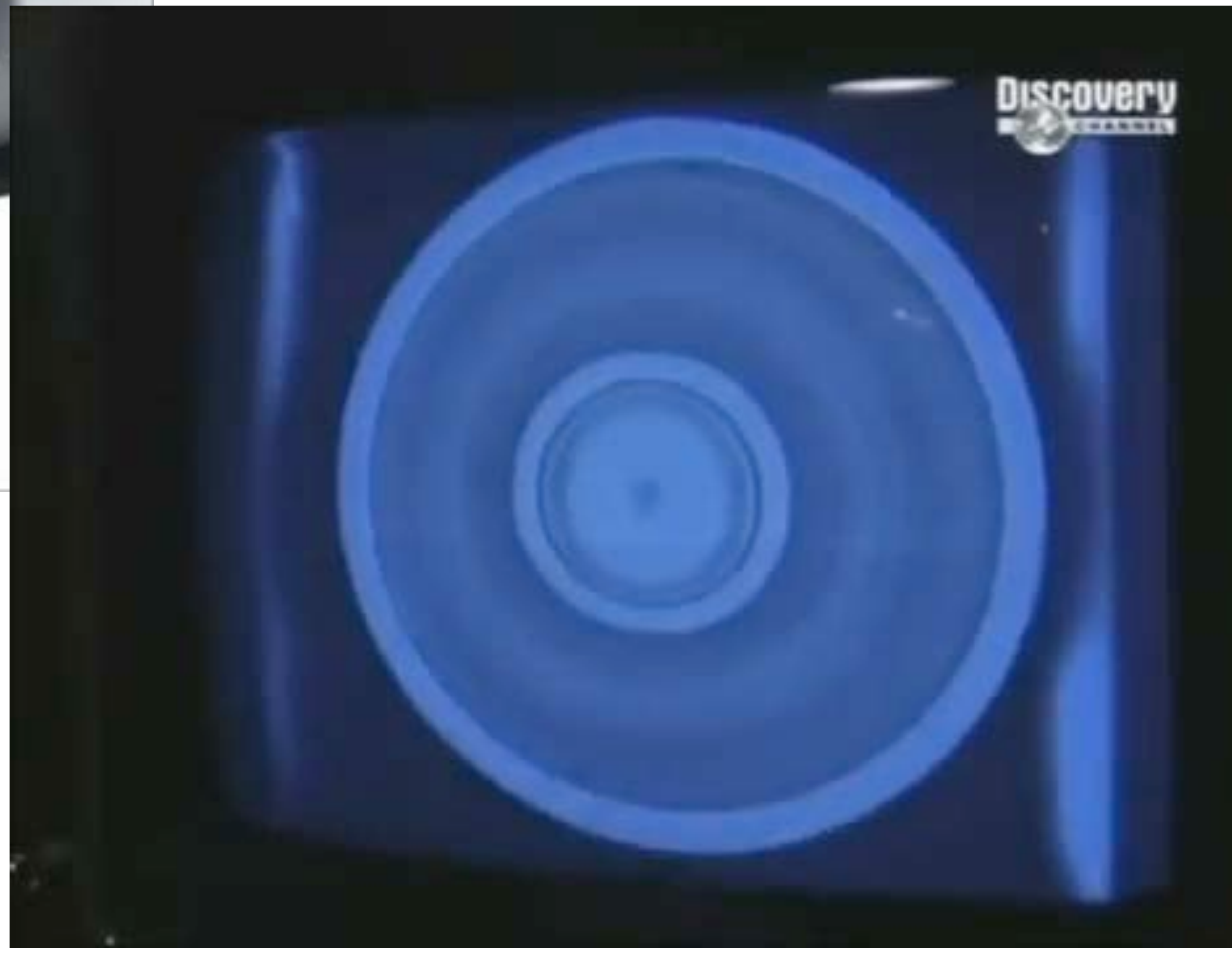
LSL: lower specification limit
USL: upper specification limit

Control of Variations: Technological Development

statistical representation



continuous on-line measurement



no statistical
representation needed

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