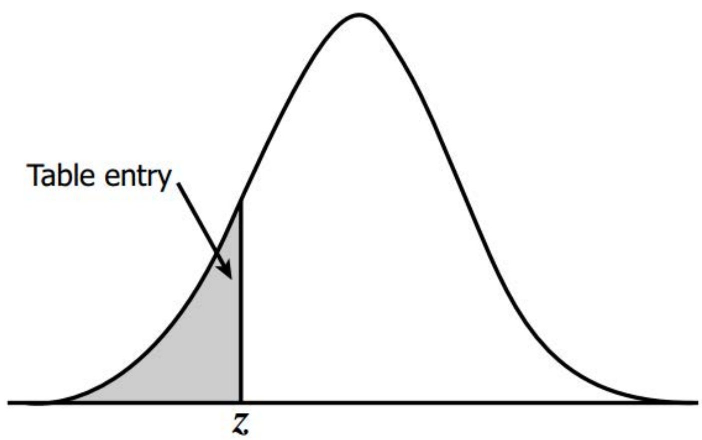
Notes on Variation and Quality

Z-Scores
For the purposes of this class, we assume our processes can be modeled by a normal distribution. This assumption is supported by years of research and also allows us to use tools such as z-scores etc to understand how the process is performing.

A z-score is a value that tells us how far away an observation is from the mean. More specifically, it tells us this distance in terms of how many standard deviations. The z-score can be calculated with the following expression

$$z = \frac{(x - \mu)}{\sigma}$$

With a z-score, you can use a z table to find the percentage of values below a specific observation (There are other conventions but this is the one observed by the z table in the appendix).



Let's take this example. Say we have a process that can drill a hole with a mean of 5mm and a standard deviation of 1.5mm. What is the probability that a rod we pull from a batch of parts is 3 mm.

Start with the z score

$$z = \frac{(x - \mu)}{\sigma} = \frac{(3mm - 5mm)}{1.5mm} = -1.33$$

Let’s find this value on the z-table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

This gives us the value for the shaded area under the bell curve, which here is .0918. Recall that the area under a bell curve is = 1. With this in mind we can multiply this value by 100 to get a percentage.

This means 9.18% holes will be 3mm or less.

You can use this process and work in reverse, so say you’re given a percentage value, you can convert it to the area value, find the z score and back out the nominal diameter observed or other features such as the mean or standard deviation of the distribution depending on what information you have access to.

Combining distribution for part fits

Questions you have seen with the yo-yo ring and body, or the cylinder and groove on the practice exam can be generally reframed as follows. *What is the probability of a successful assembly?*

Now you have two distributions to consider: the one of the proverbial peg and the other of the hole. It’s useful to think of these values by defining a new critical dimension, which is the gap or the clearance between the two parts. This is detailed below.

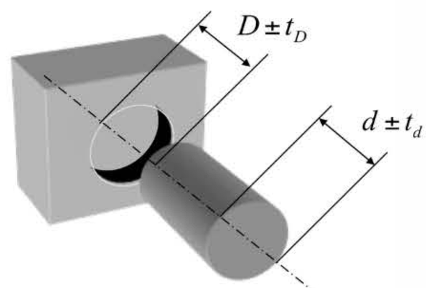
The new critical dimension is the clearance (*c*):

$$c = D - d$$

The distribution of clearances is defined by:

$$\bar{c} = \bar{D} - \bar{d}$$

$$\sigma_c = \sqrt{\sigma_D^2 + \sigma_d^2}$$

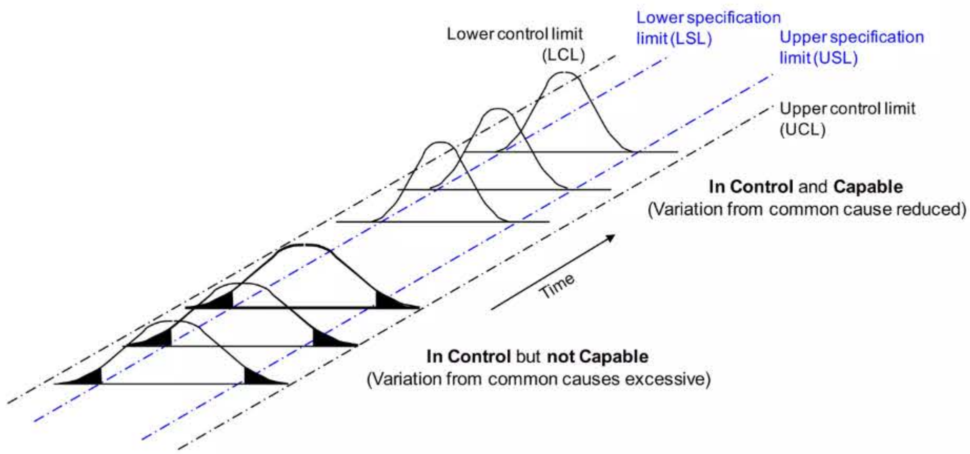


You can now use the z-scores/z table to draw conclusions about the probability of a desired fit, with the values for the clearance distribution and your knowledge about what the clearance should be to meet the fit needs. Hopefully this framing is helpful as you review the variation and quality challenge and the practice quiz.

Process Control and Capability

Process control vs. capability

→ *Even if a process is <u>in control</u> (i.e., constant mean and variation), it may not be <u>capable</u> (i.e., giving what we want as set by the specifications a.k.a. the tolerances)*



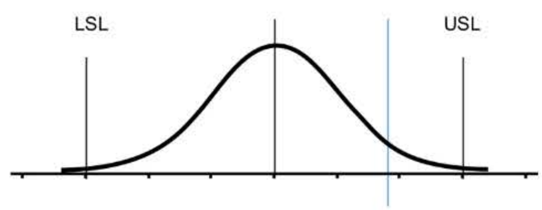
Process capability: compares process variation to tolerances

$$C_p = \frac{USL - LSL}{6\sigma_x}$$

General rule: C_p should be at least 1.33

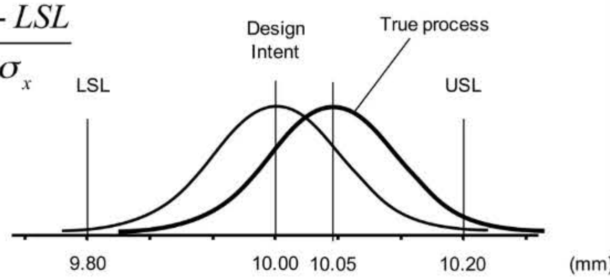
LSL, USL = tolerance limits

σ_x = process stdev



$$C_{pk} = \frac{USL - \mu_x}{3\sigma_x} \quad or \quad C_{pk} = \frac{\mu_x - LSL}{3\sigma_x}$$

use whichever is smaller, because →

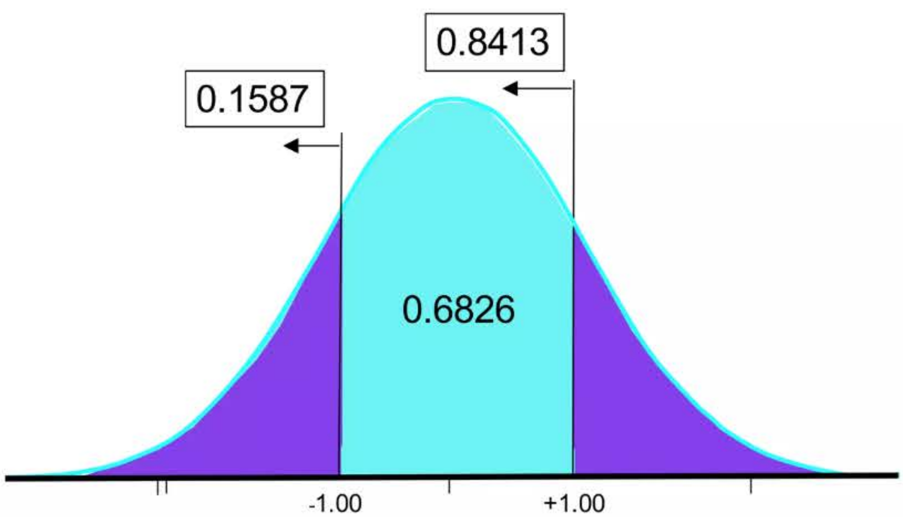


Example: manipulating the normal distribution

Car tires have a lifetime that can be modeled using a normal distribution with a mean of 80,000 km and a standard deviation of 4,000 km.

→ What fraction of tires can be expected to wear out within ±4,000 miles of the average?

Solution: how many wear out between 76,000 and 84,000 miles?



→ Area under the curve between these points
z(1) – z(-1) = 0.8413 – 0.1587 = 0.6826
= 68% will wear out

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