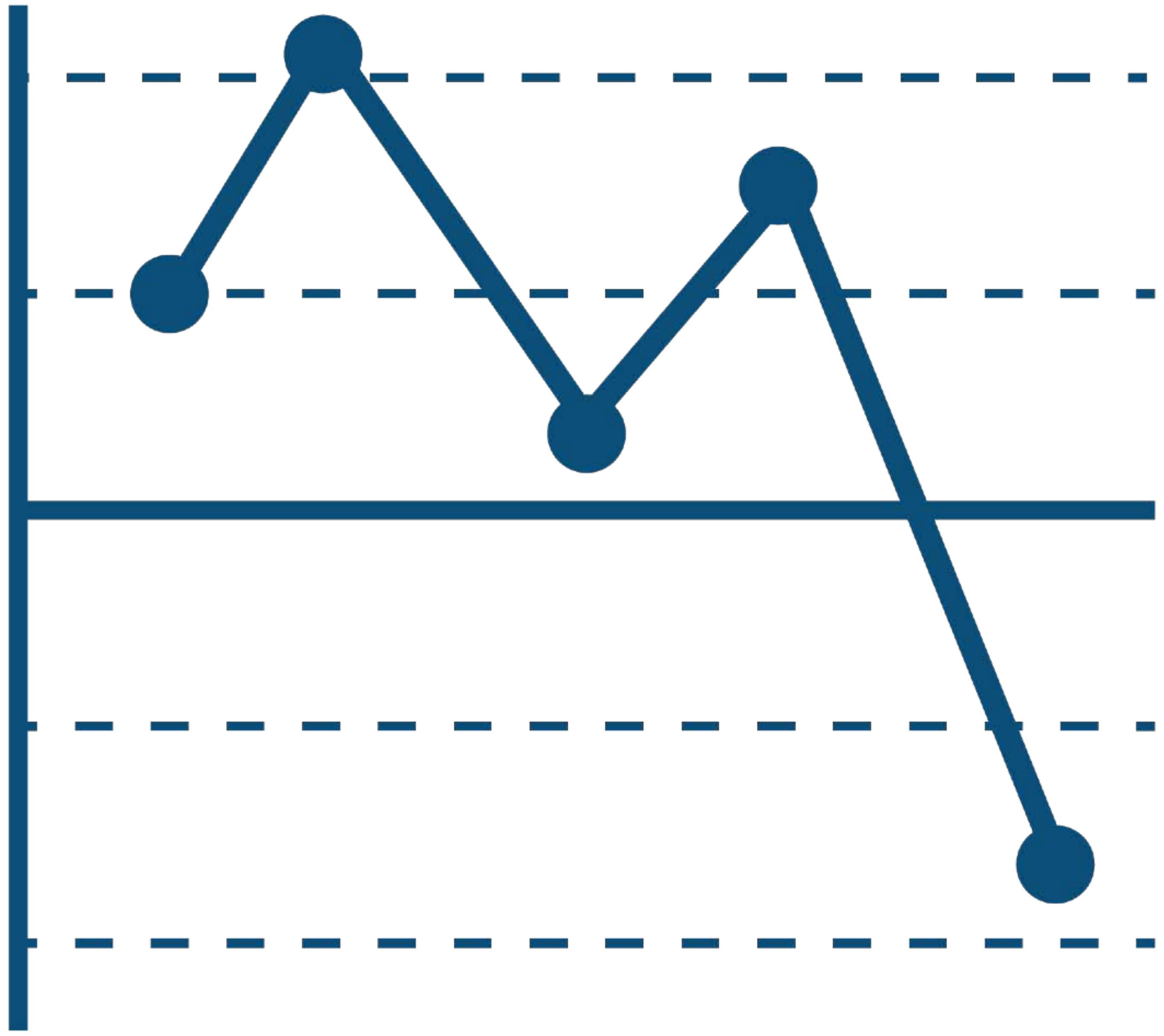
Variation and Quality

Statistical Process Control



SPC

2.008 Topic Coverage

Variations

Statistical Representation

Process **Capability**

Process **Control**

Accuracy vs Precision

Quality Loss

Variation and Quality

Defining, Measuring, and Controlling Quality in Manufacturing

Statistical Distributions

Probability

$$P\{a \le x \le b\} = \int_a^b f(x)\ dx$$

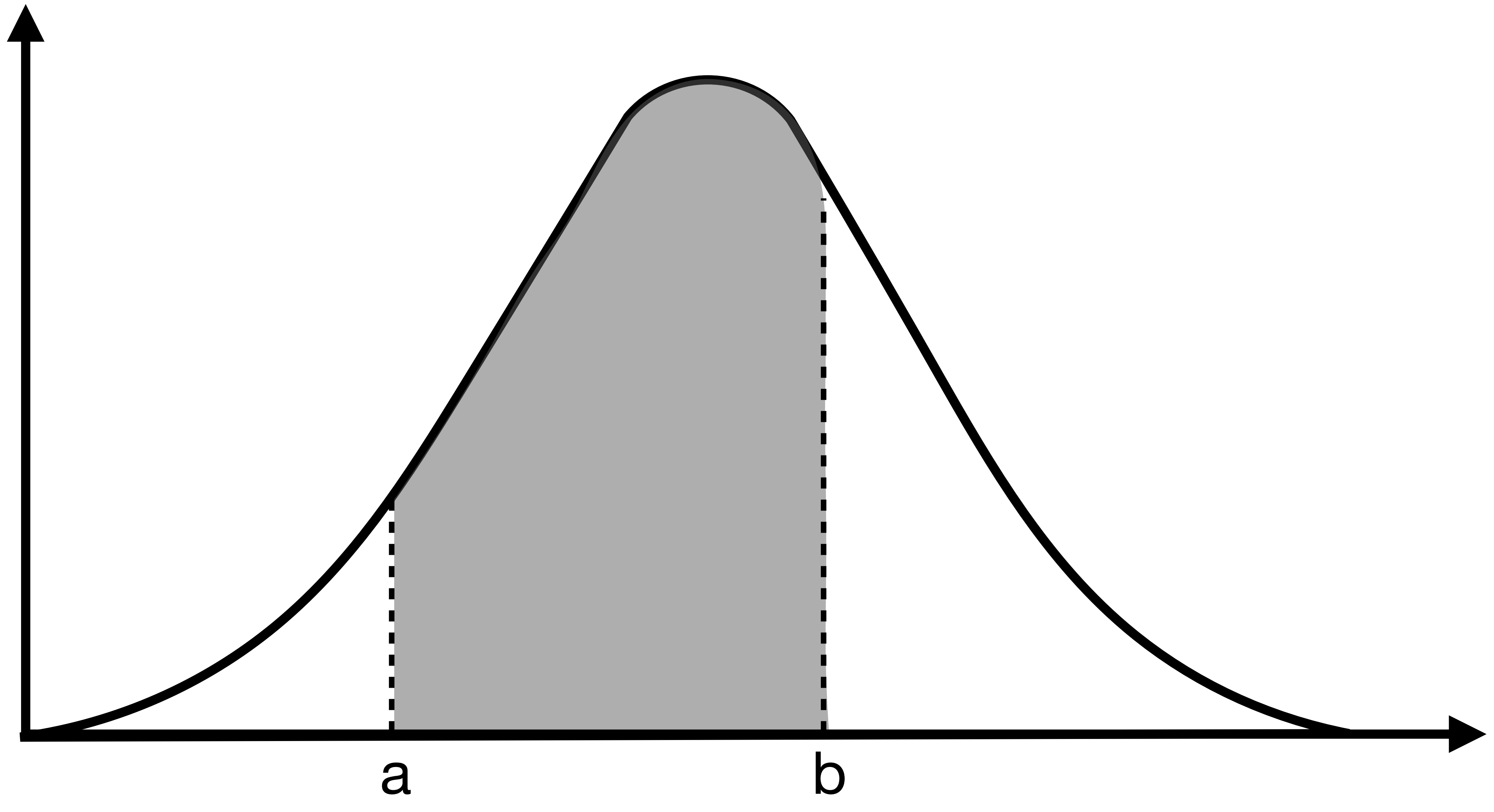
$$P\{-\infty \le x \le \infty\} = \int_{-\infty}^{\infty} f(x)\ dx = 1\ for\ all\ \bar{x},\ s$$

Normalized

$$z = \frac{x - \bar{x}}{s}$$ ("number of std devs")

$$P\{z_1 \le x \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{z^2}{2}\right)}$$

$$f(x) = \frac{1}{2\pi s} e^{\left(-\frac{(x-\bar{x})^2}{2s^2}\right)}$$

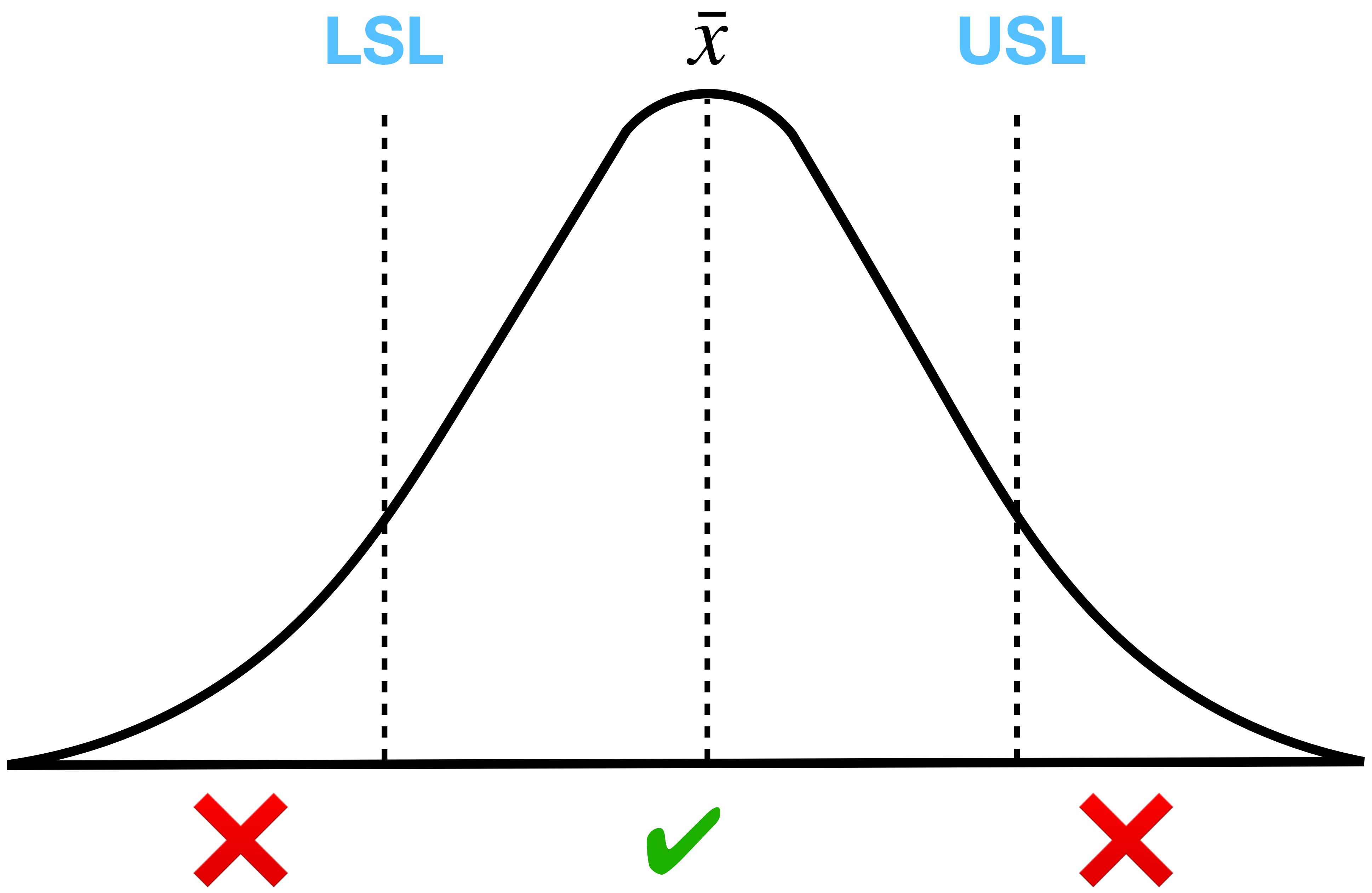


Process Capability!

Is the process **capable** of meeting the design requirements?

$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$

	Recommended process capability for two-sided specifications	Defects (parts out of spec) per million operations
Existing (stable) process	1.33	63
New process	1.50	8
Existing process, safety-critical	1.50	8
New process, safety-critical	1.67	1
Six-sigma quality	2.00	0.002



LSL: lower specification limit
USL: upper specification limit

Yield

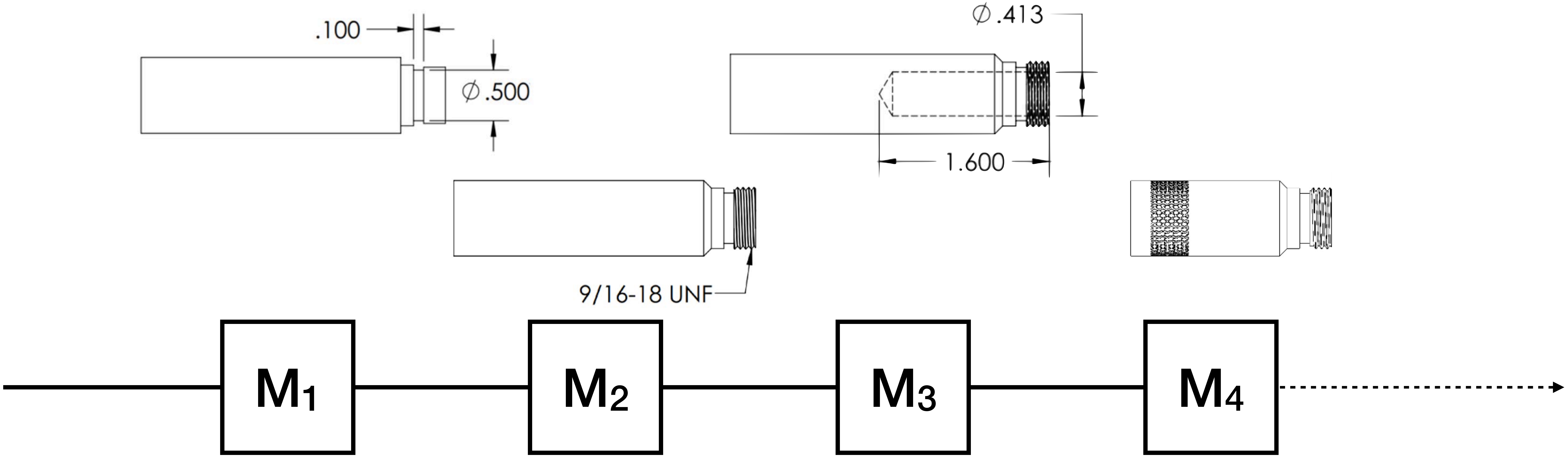
$$Yield = \%\ of\ acceptable\ parts$$

$$Total\ Yield = yield_{step1} * yield_{step2} * yield_{step3} \ldots$$

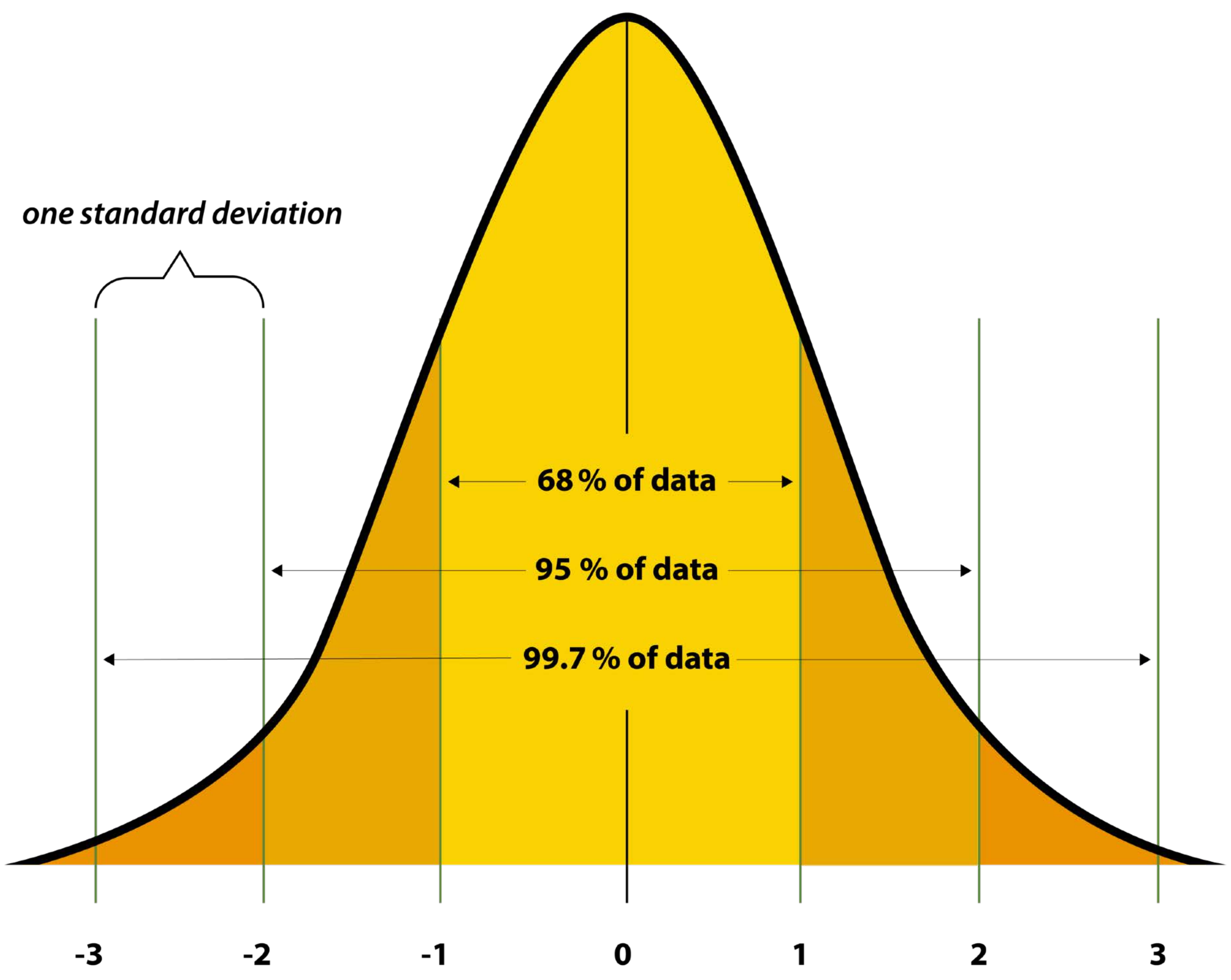
assuming each step is independent

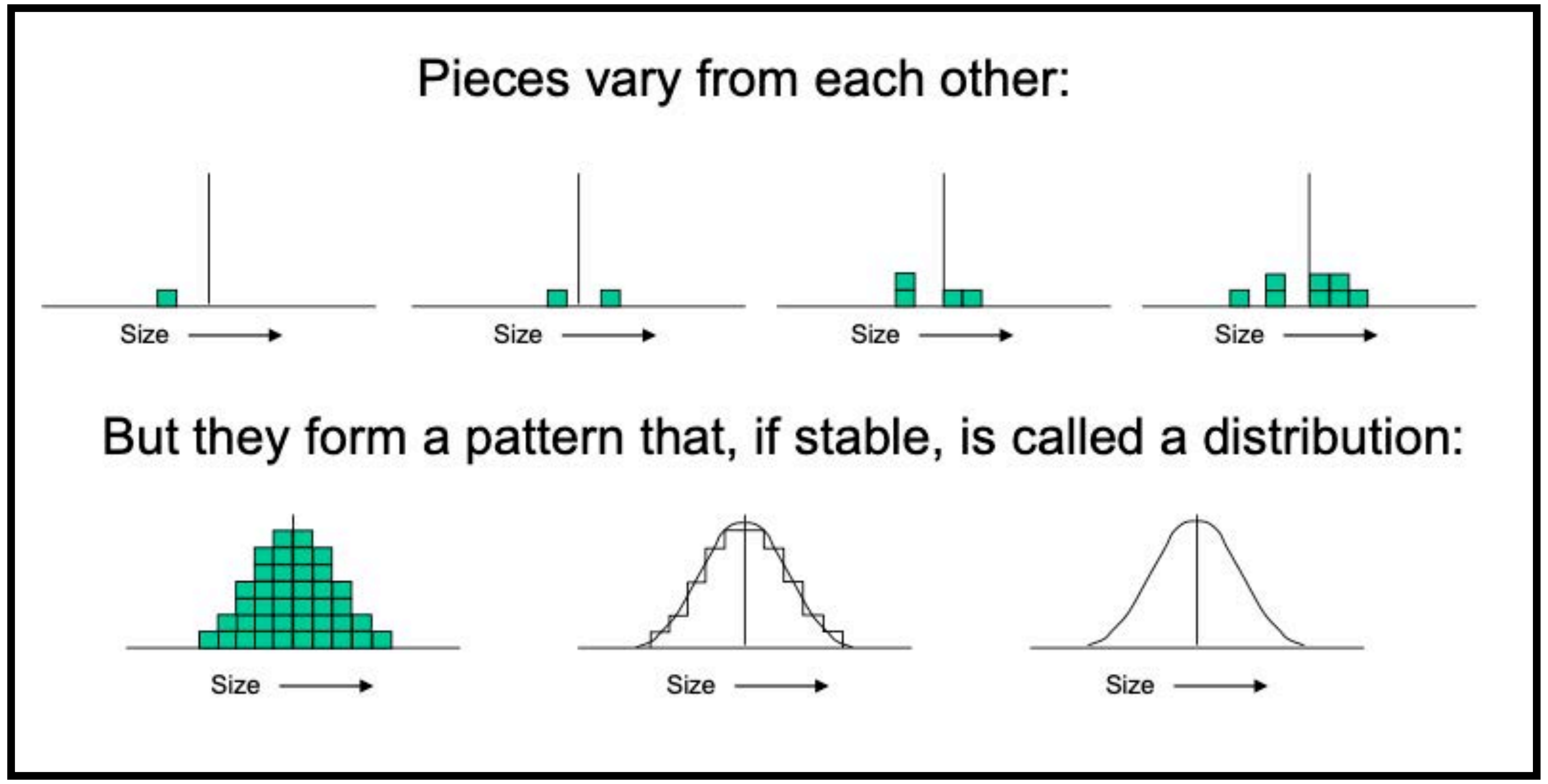
$$Total\ Yield \propto Cost$$

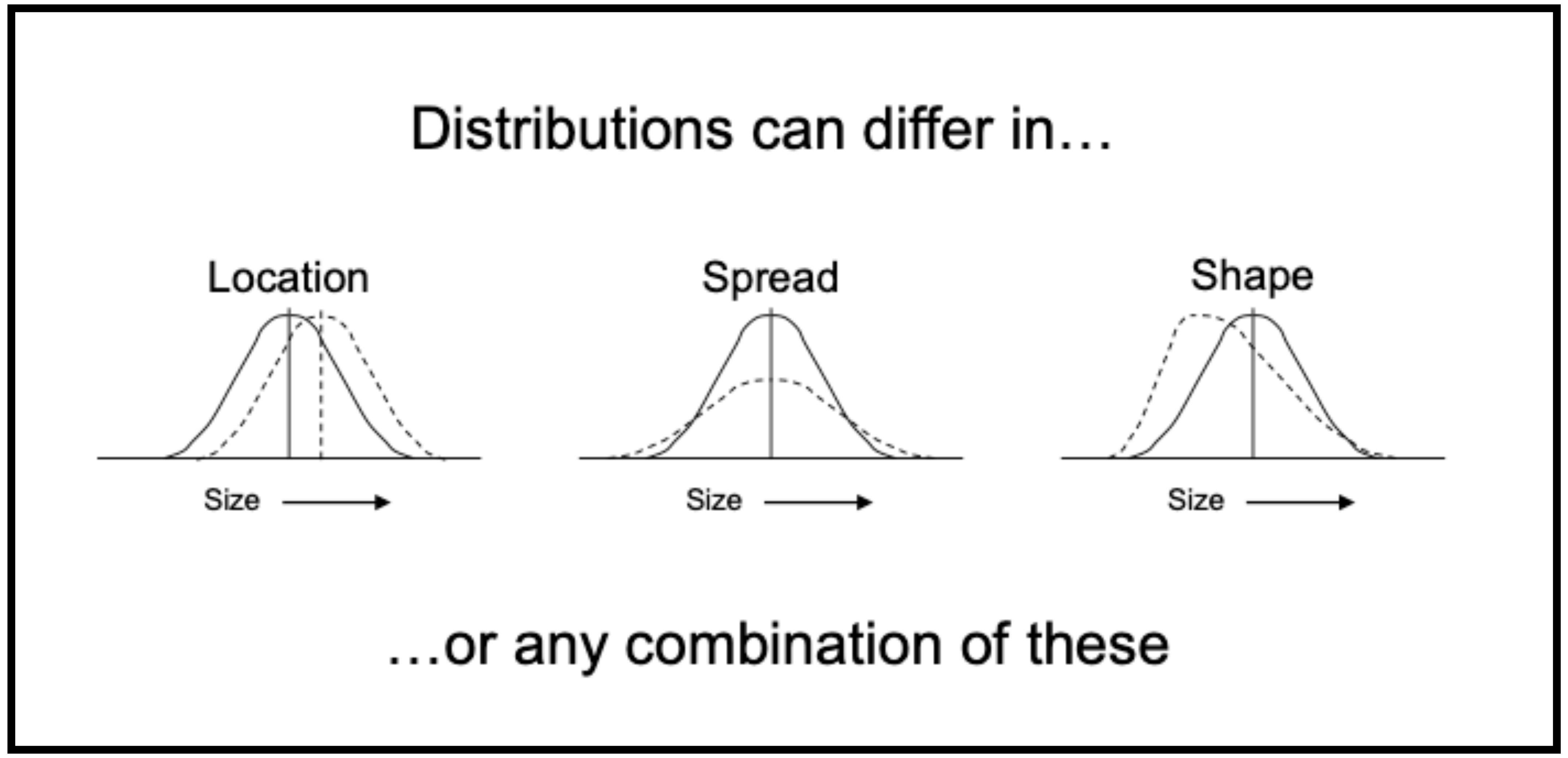
- processes with more stages will have lower yield
- total yield is highly sensitive to one low yield operation



Where do variations come from?







Where do variations come from?

Systematic / Assignable / Special Cause

detect, chase down, and eliminate or reduce

(you know or can figure out what is happening)

- tool is wearing out
- operator used the wrong depth of cut
- typically a “single direction” shift

Random / Un-assignable / Common Cause

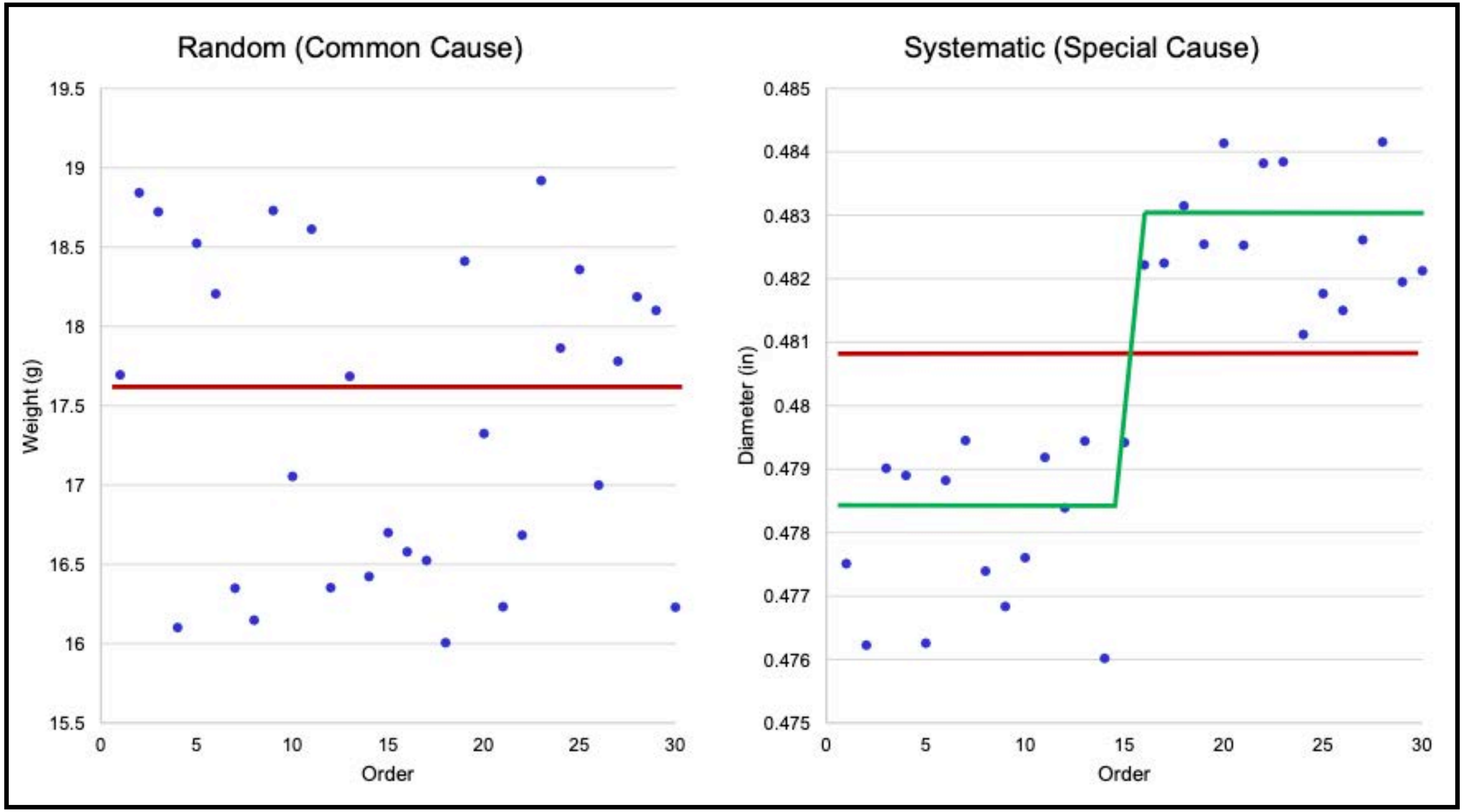
understand, minimize, and coordinate
(with designers, etc.)

make sure these variations are **tolerated**

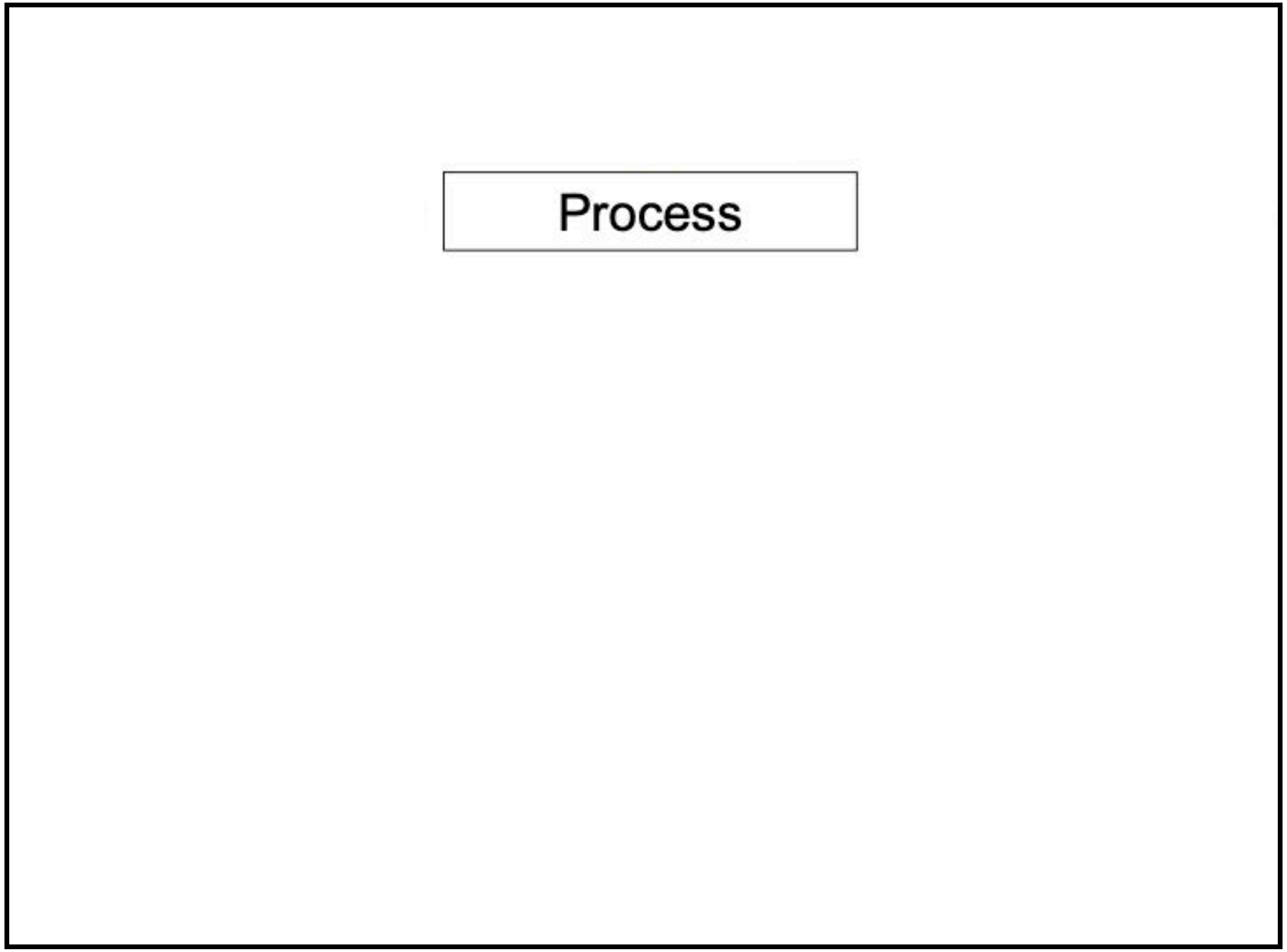
(there’s always some amount of randomness)

- there’s some natural variation due to vibrations, nonlinearities, etc.
- i.e. a truck passed by and caused extra vibration or there was a solar flare
- “positive and negative” shifts

Where do variations come from?

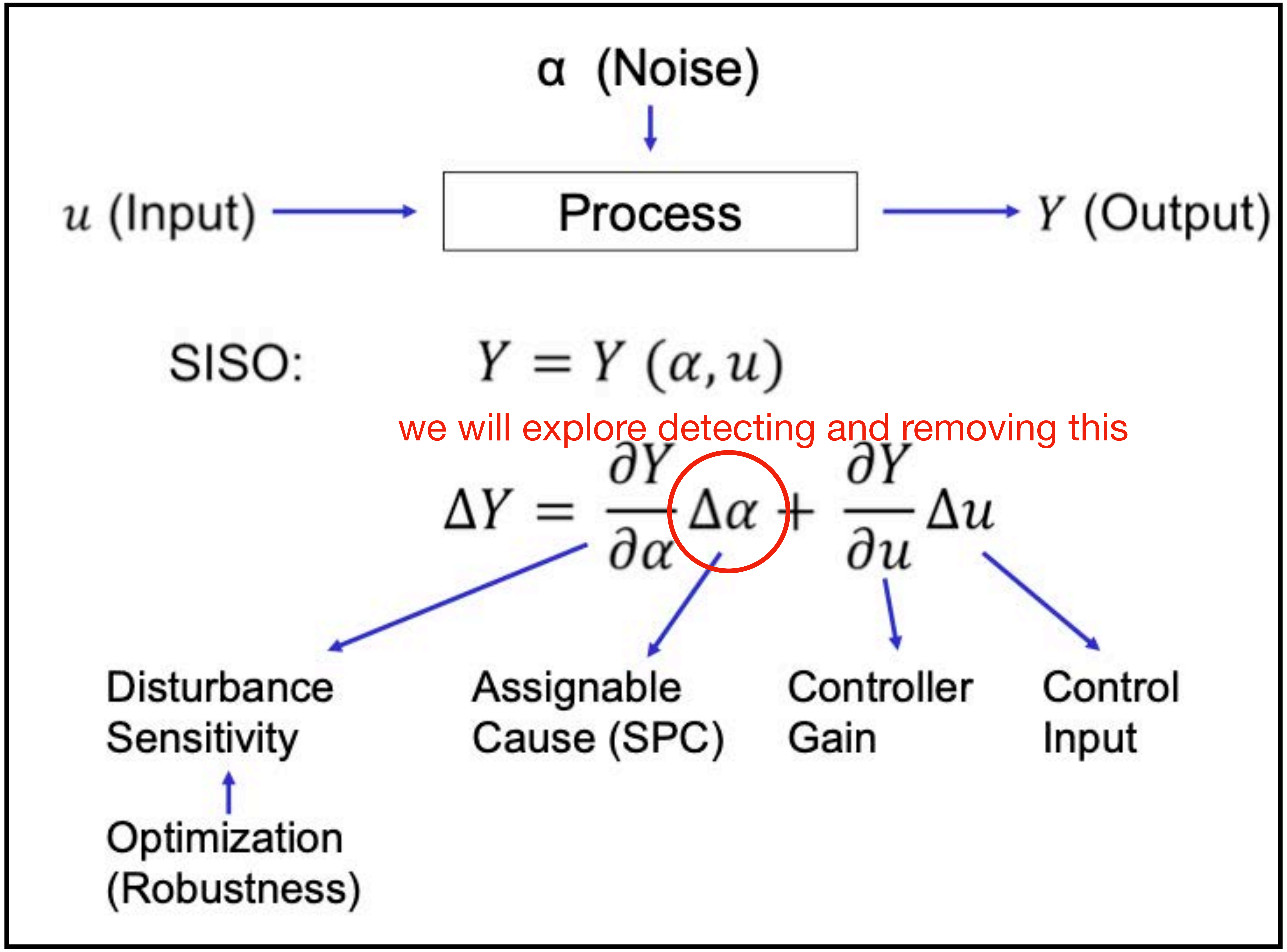


General Process Control Scheme



General Process Control Scheme

1. Detect disturbances (special causes)
2. Take corrective action



Variation and Quality

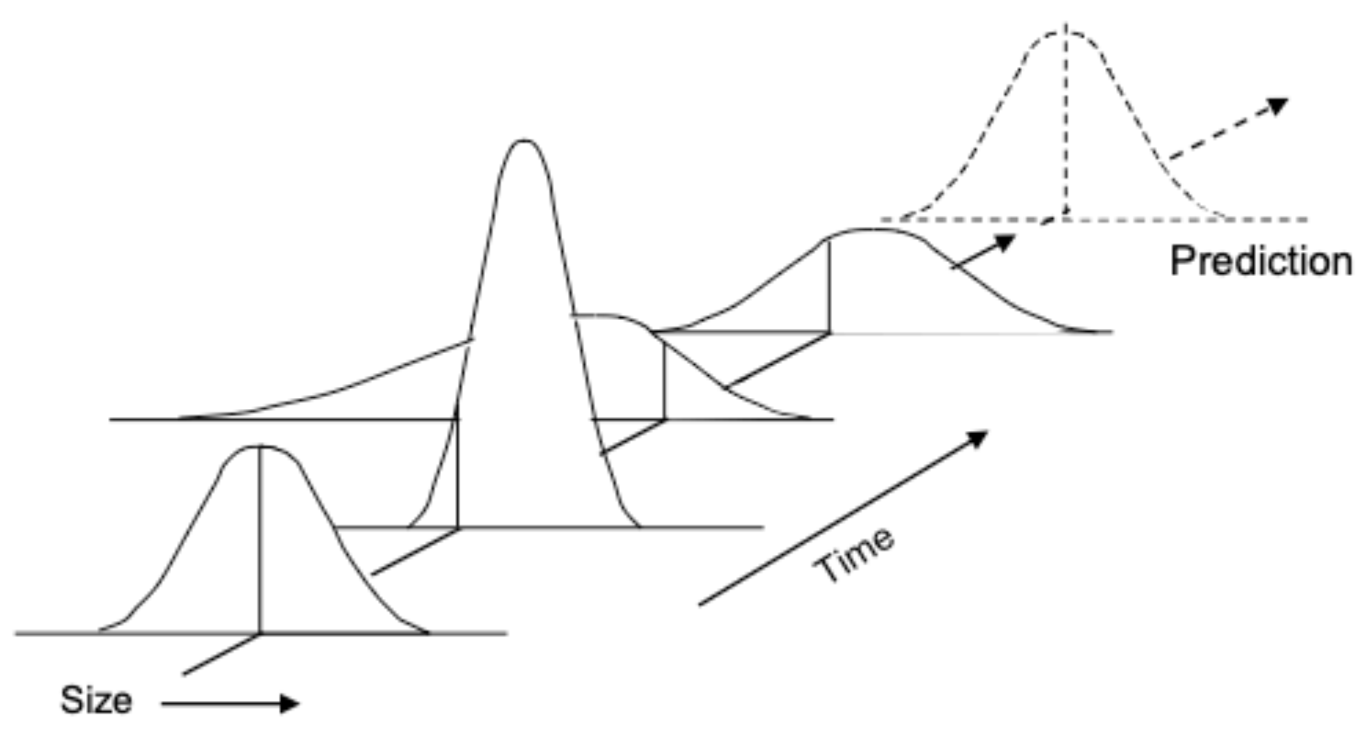
Defining, Measuring, and Controlling Quality in Manufacturing

Process Control

In Control is different from being **Capable**

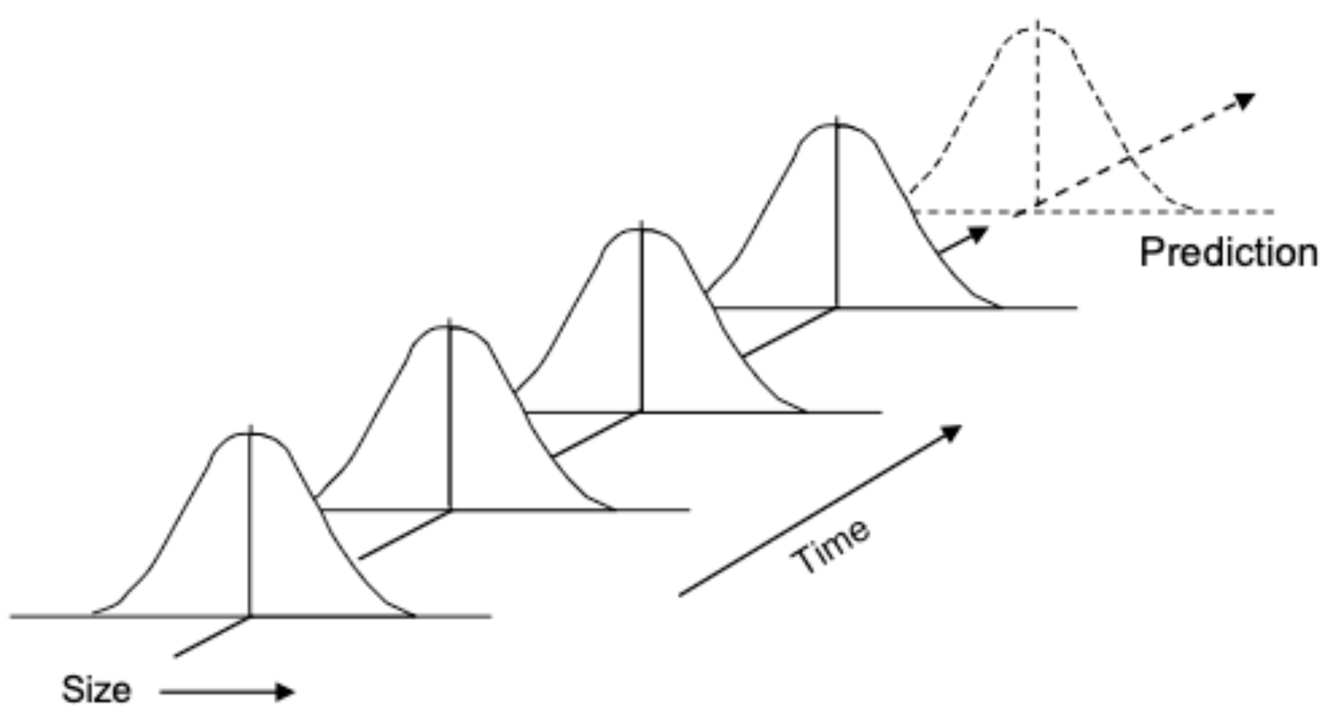
- a process is “in control” if it is producing **similar parts in a predictable way over time**
- this is in comparison to a **qualifying run** that provides a benchmark for production
- you need to be able to predict production to properly meet the demand of the customers

If special causes of variation are present, the process output is not stable over time and is not predictable:



out of control (special causes present)

If only common causes of variation are present, the output of a process forms a distribution that is stable over time and is predictable:

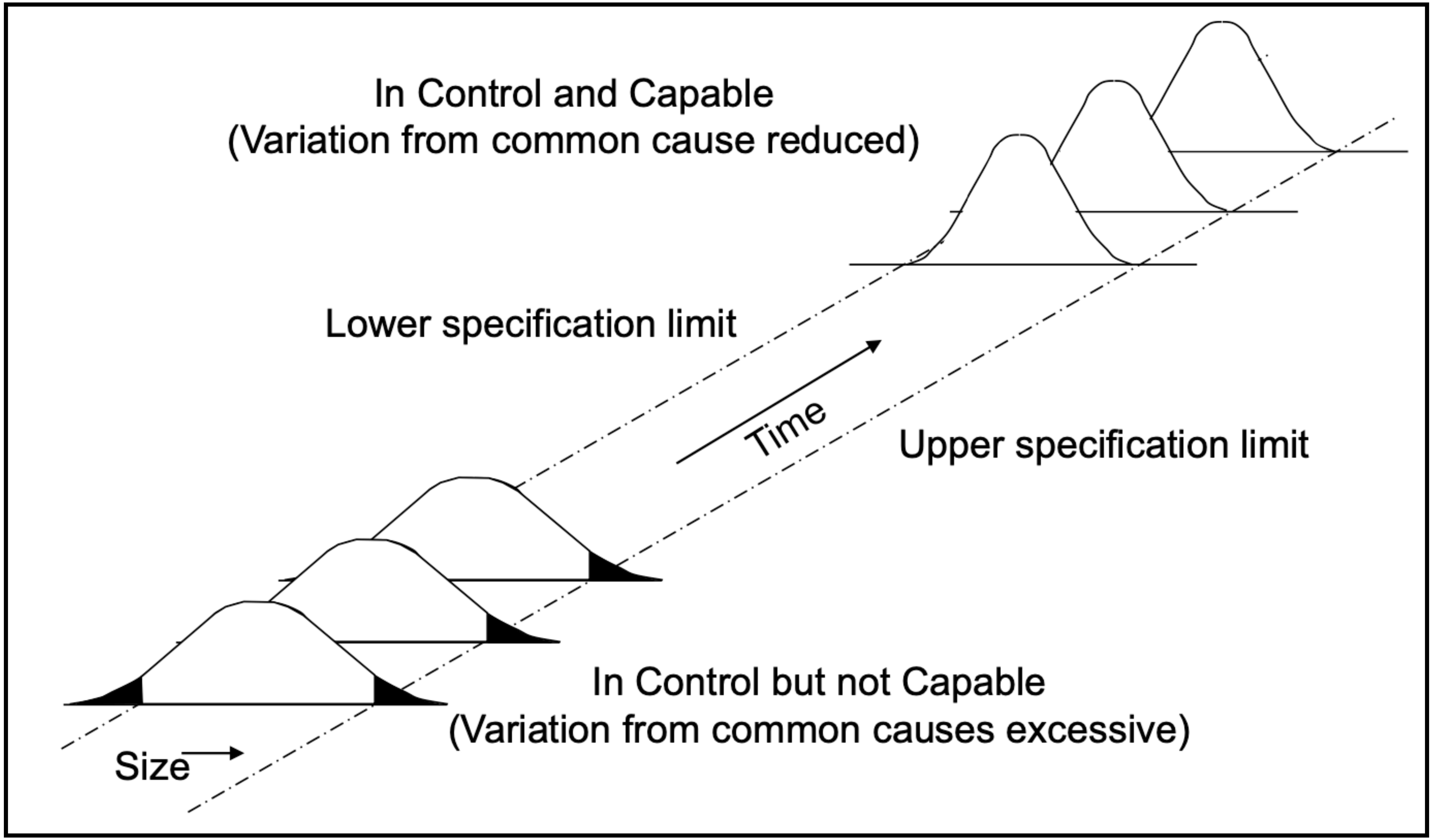


in control (special causes eliminated)

Process Control

Ideally a process would be both **in control** (stable over time) and **capable** (able to achieve the design specifications).

$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$



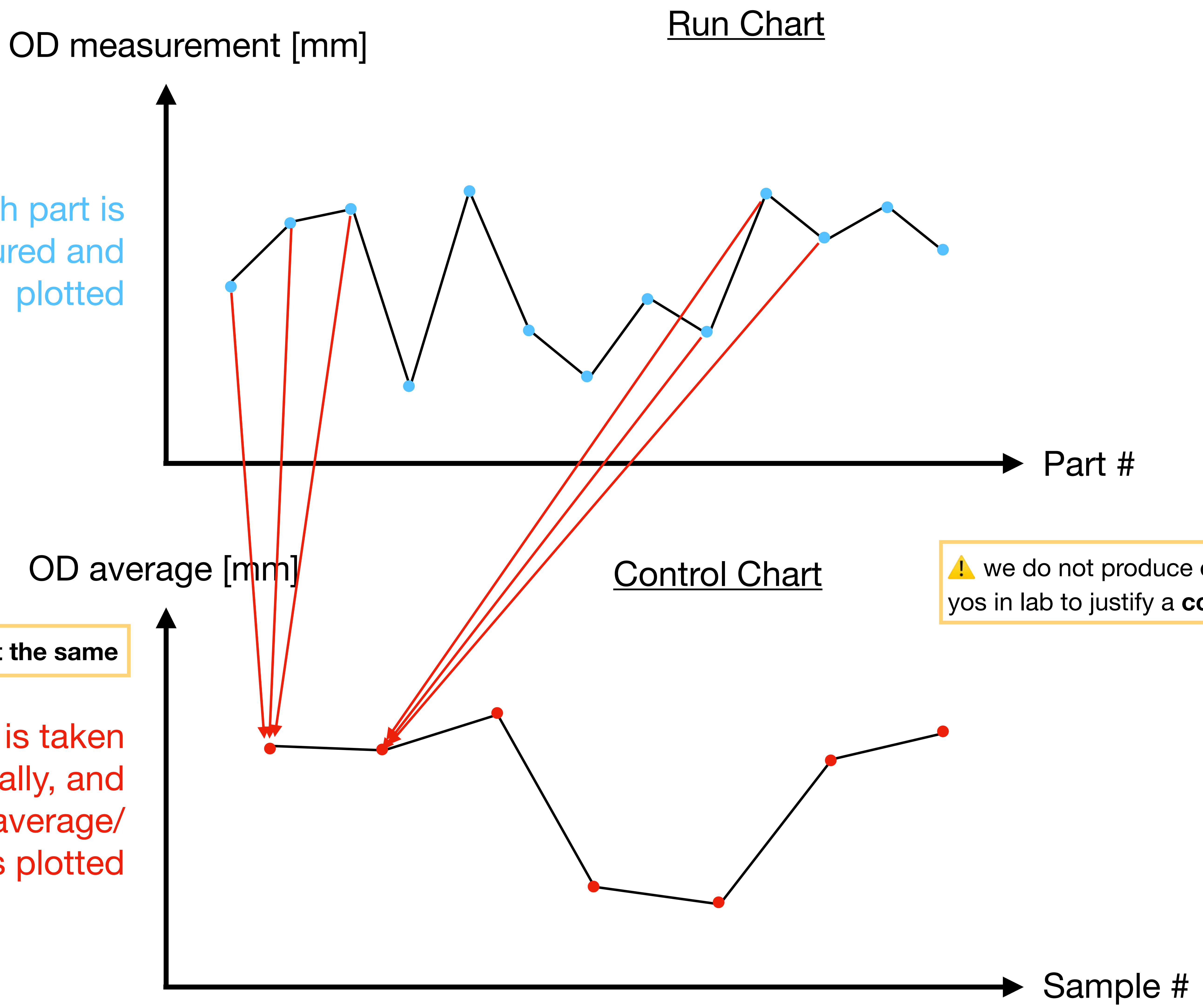
Process Control

So how do we determine if a process is **in control** (stable over time)?

We need to start **charting** the process.

Run Chart vs Control Chart

each part is measured and plotted



⚠️ vertical axis is similar, but **not the same**

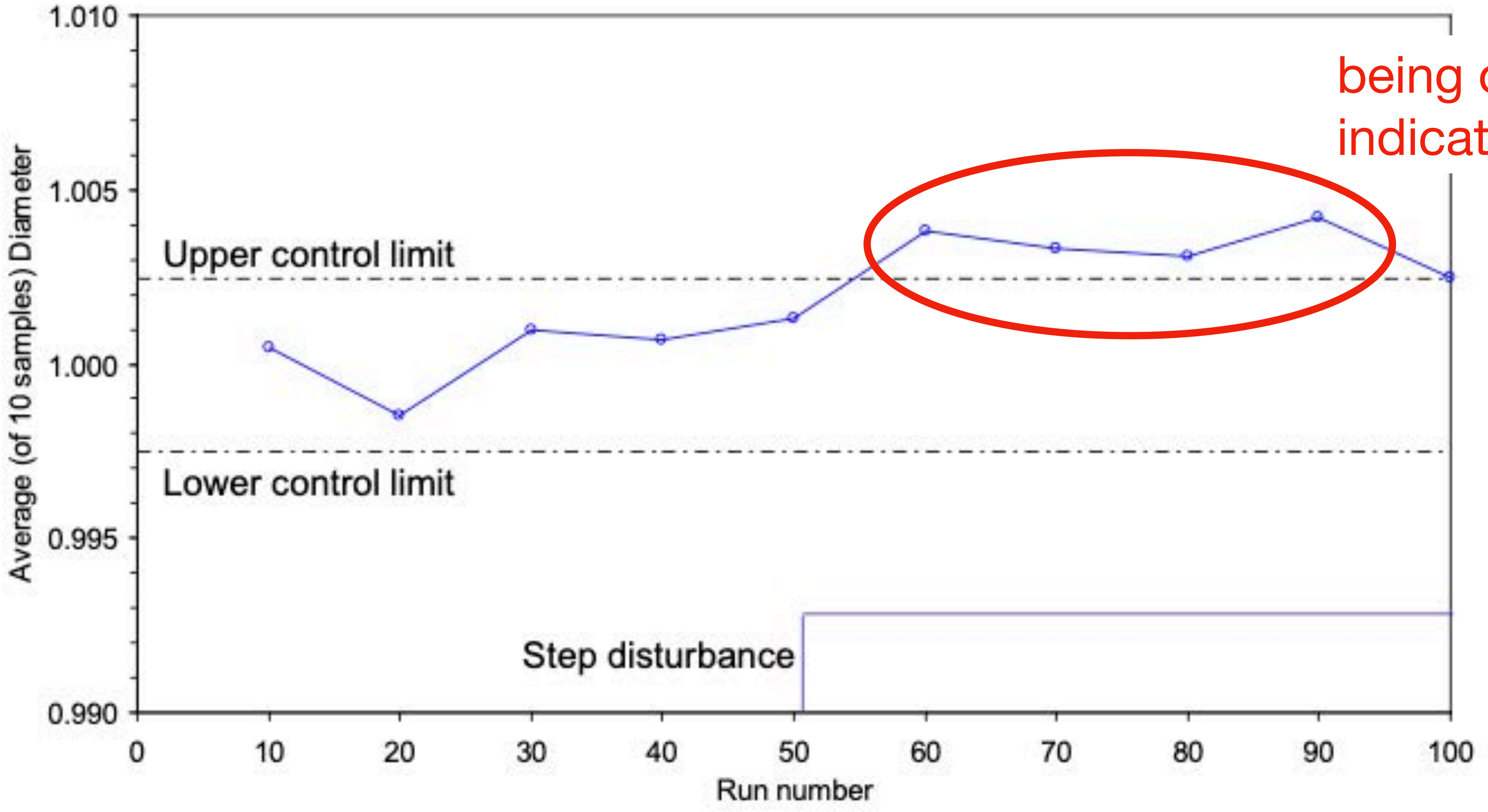
a sample is taken periodically, and the average/ range is plotted

⚠️ we do not produce enough yo-yos in lab to justify a **control chart**

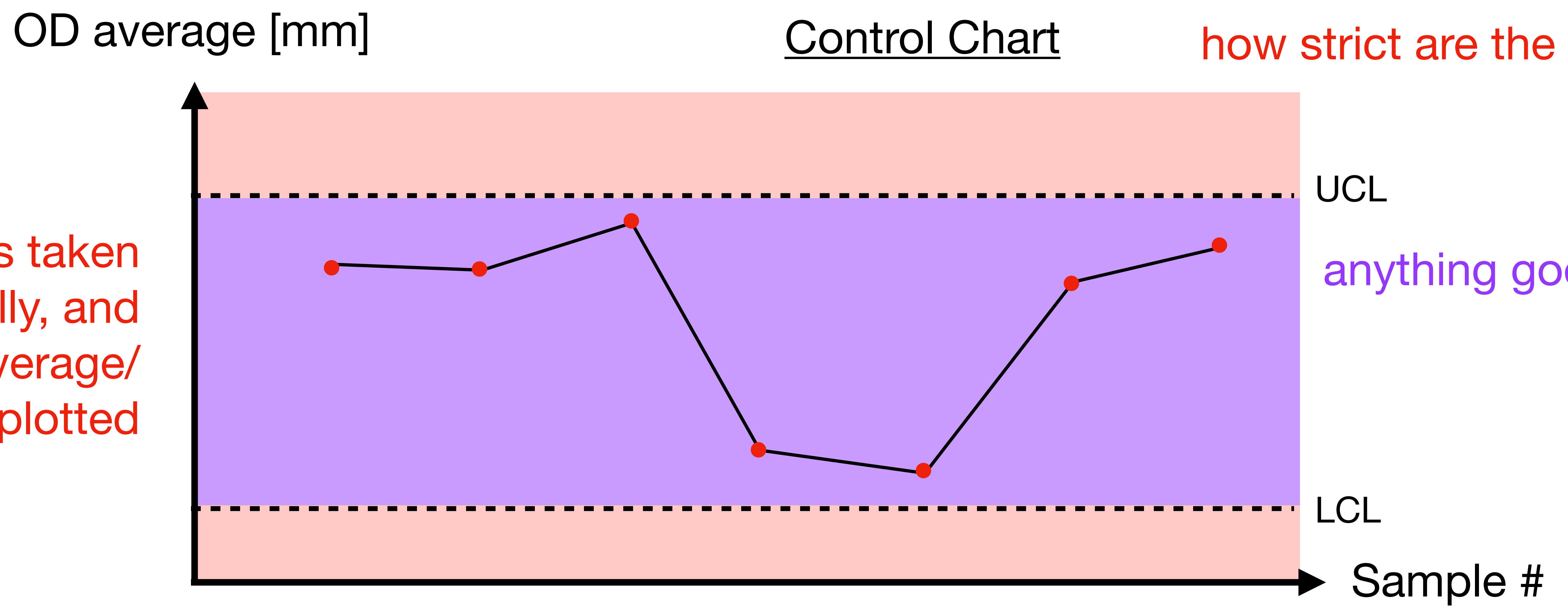
Control Charts ("Shewhart Charts")

Properly used, control charts can:

- Be used by operators for ongoing control of a process
- Help the process perform consistently, predictably for higher quality, lower cost and higher effective capacity
- Provide a common language for discussing process performance
- Distinguish special from common causes of variation; as a guide to local or management action



being outside the limits indicates a problem



Control Chart

how strict are the limits?

anything goes?

a sample is taken periodically, and the average/range is plotted

Control Limits

Points outside the limits will signal that something is wrong- an assignable cause.

$$UCL = \bar{x} + 3\sigma_{sg} = \bar{x} + \frac{\sigma_{process}}{\sqrt{n}}$$

$$\sigma_{sg} \neq \sigma_{process}$$

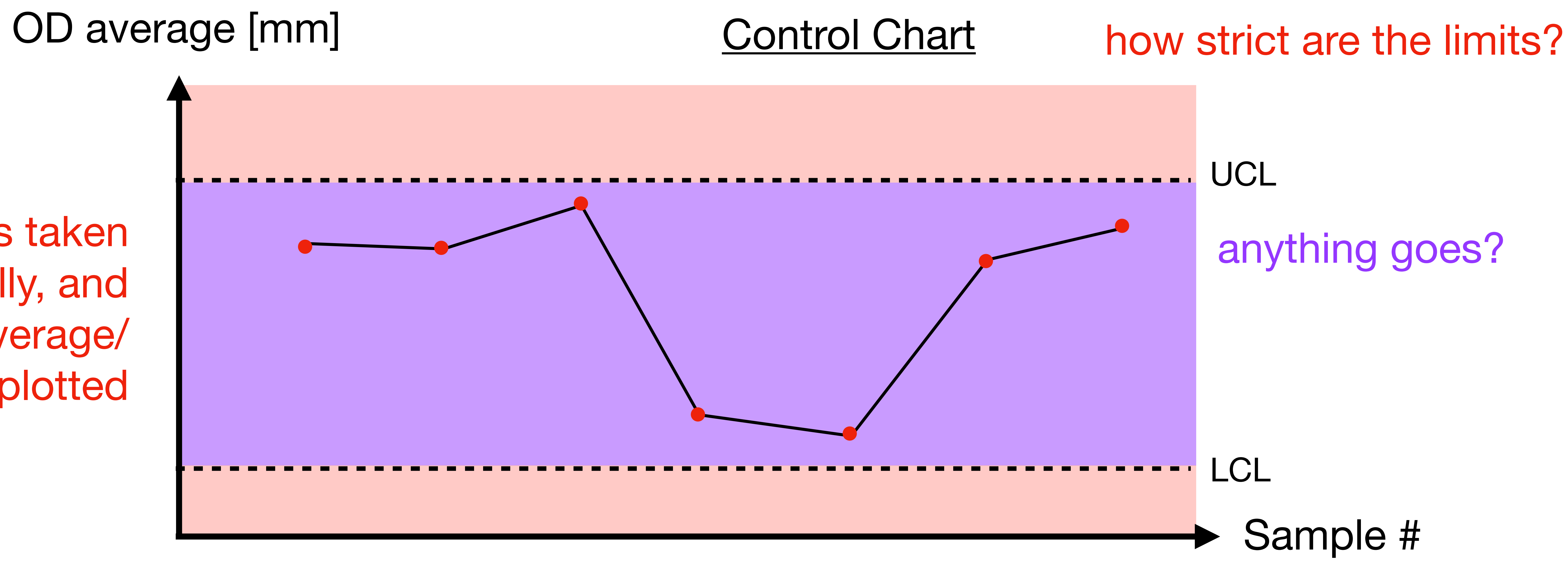
We want limits set so that assignable causes are highlighted, but few random causes are highlighted accidentally.

$$LCL = \bar{x} - 3\sigma_{sg} = \bar{x} - \frac{\sigma_{process}}{\sqrt{n}}$$

$$\sigma_{sg} = \frac{\sigma_{process}}{\sqrt{n}}$$

As n increases, the UCL and LCL move closer to the center line, making the control chart more sensitive to shifts in the mean.

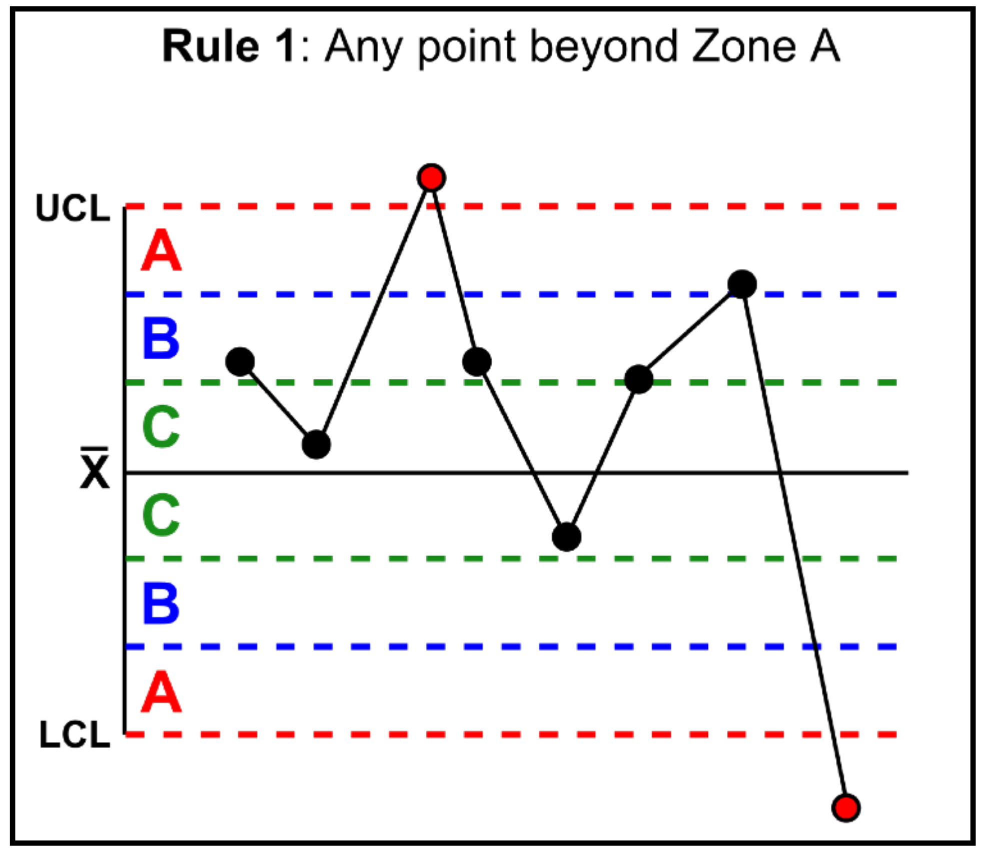
UCL: upper control limit
LCL: lower control limit
σ_{sg}: standard deviation of the sample
$\sigma_{process}$: standard deviation of the process



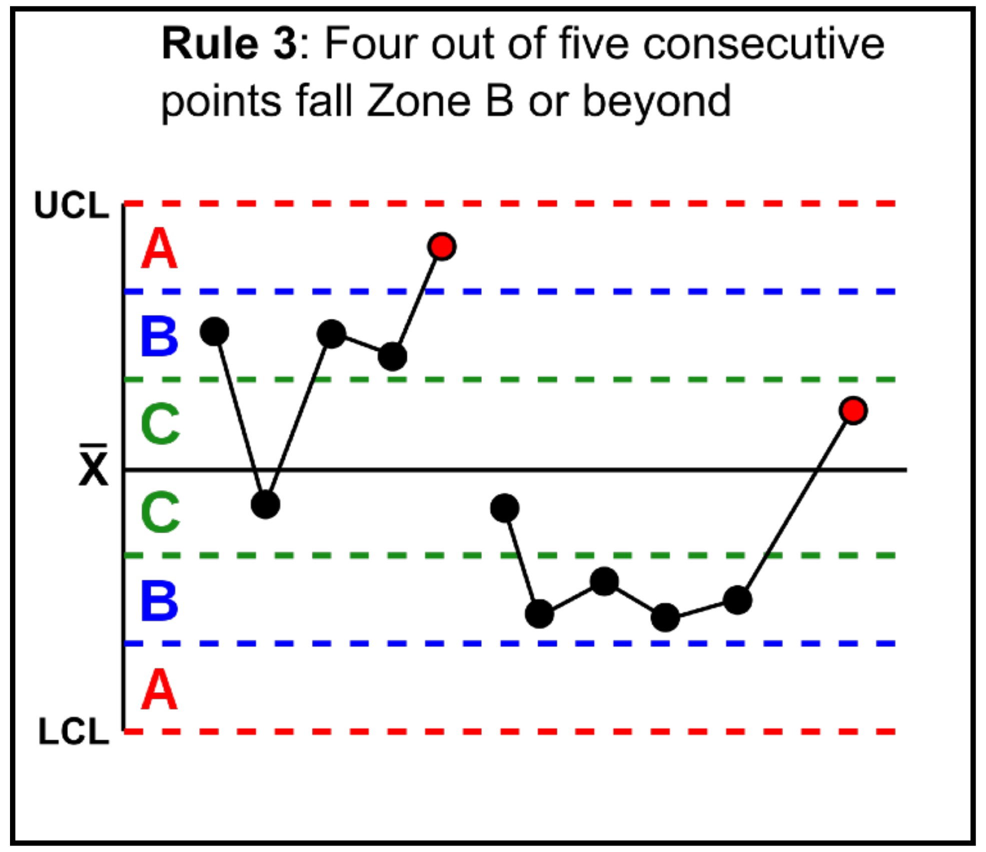
Western Electric Rules

The Western Electric Rules are chosen because each of these conditions is improbable:

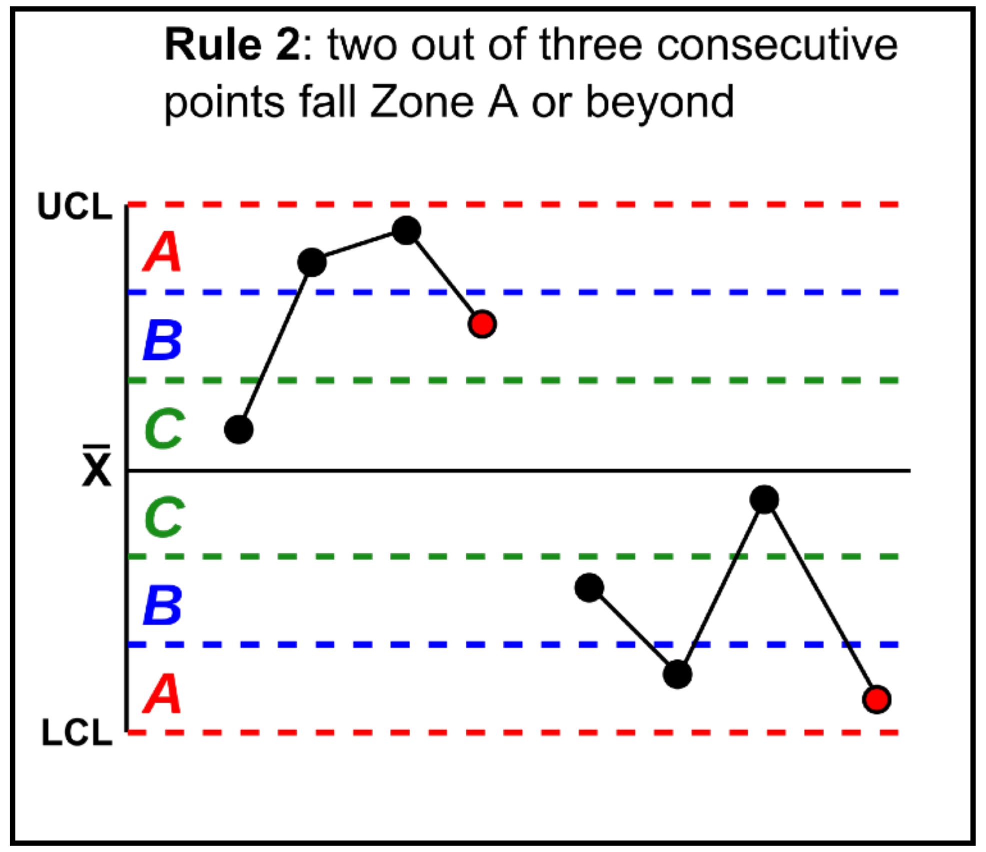
⚠️ we are looking at sample mean charts, but range charts can also be used



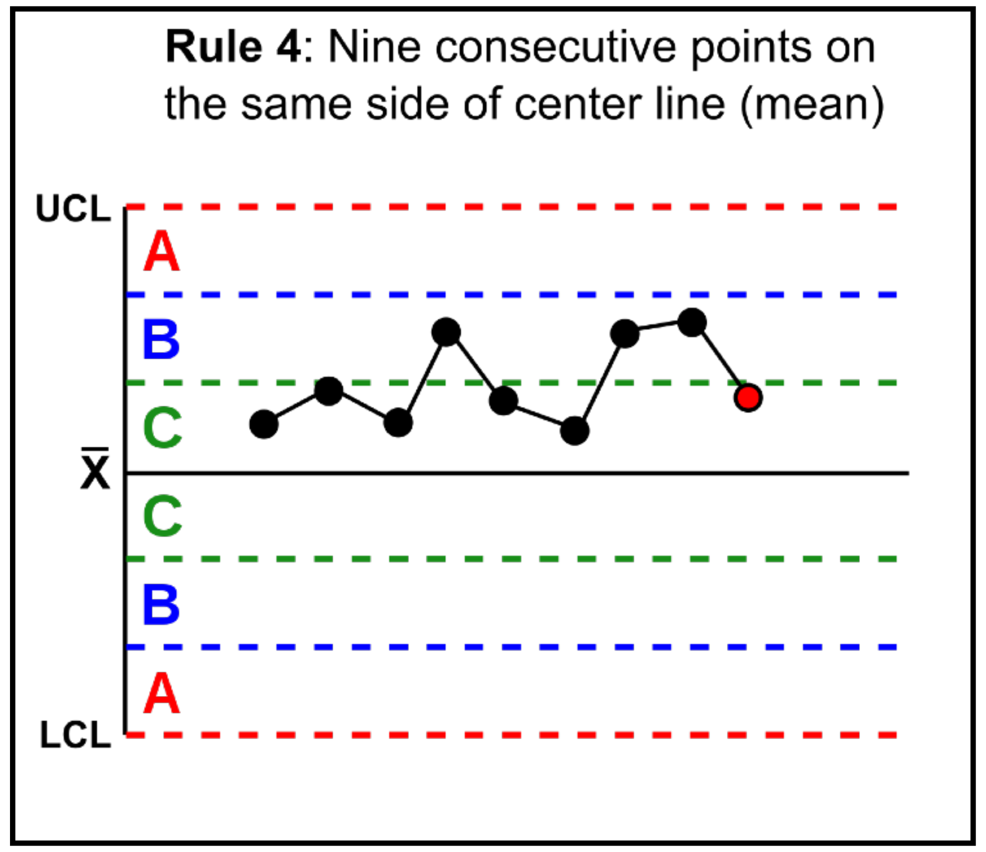
Rule 1: Any single data point falls outside of the 3σ-limit (UCL/LCL) from the centerline



Rule 3: Four out of five consecutive points fall beyond the 1σ limit (Zone B or beyond), on the same side of the centerline



Rule 2: Two out of three consecutive points fall beyond the 2σ limit (Zone A or beyond) on the same side of the centerline



Rule 4: Nine consecutive points fall on the same side of the centerline (in Zone C or beyond)

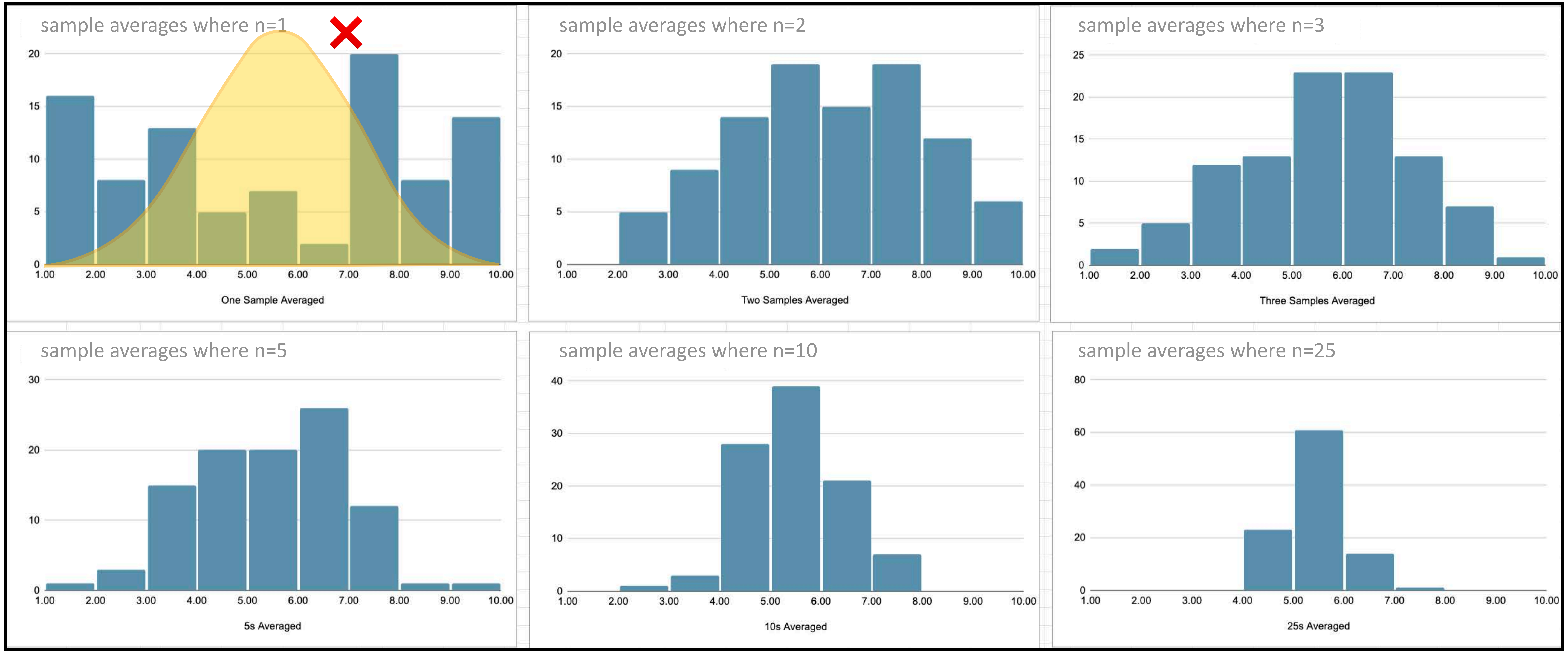
Process Control

What happens if your process is characterized by a **non-normal distribution?**

We need to rely on the **Central Limit Theorem**

Central Limit Theorem

a critical aspect of the sampling process is that **it shapes the data into a normal distribution**
you can start to see the effects even after a sample size of only 2
the bigger the sample size (n), the tighter and more normal the distribution



Central Limit Theorem

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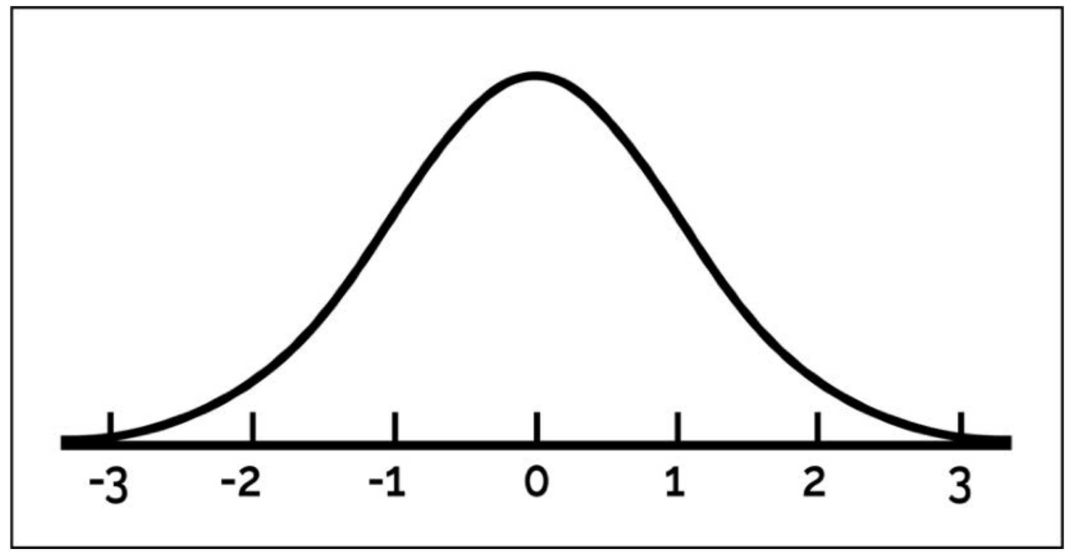
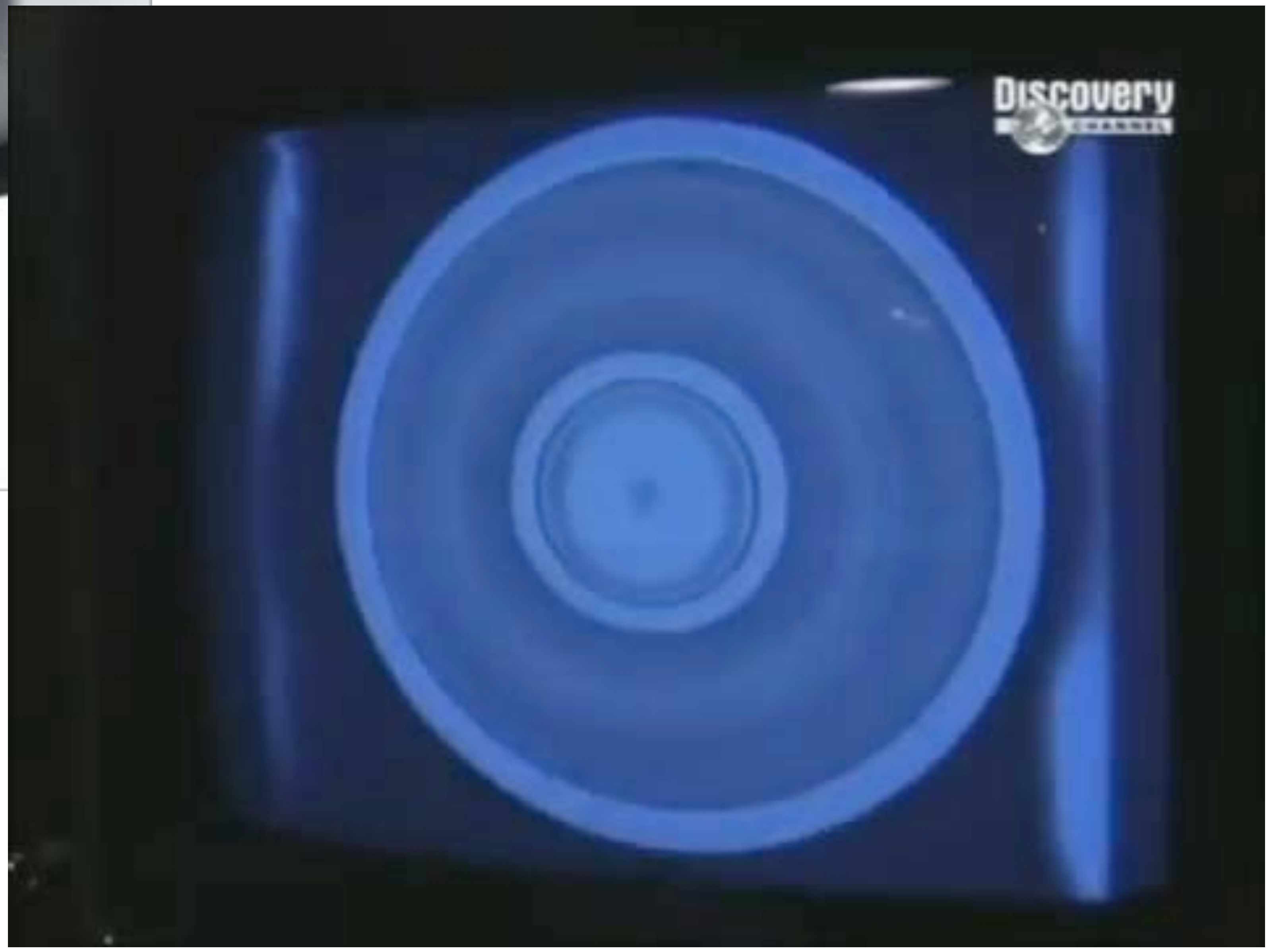
Control of Variations: Technological Development

statistical representation



continuous on-line measurement

no statistical
representation needed



Control Charts and Quality Loss

Variation & Quality

Control Chart Flow Chart

$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$

LSL: lower specification limit
USL: upper specification limit
LCL: lower control limit
UCL: upper control limit

design engineers design parts to meet performance/be part of an assembly → **design engineers** determined the **specifications** → capability can be determined and the process can be managed and improved by all

bad part < LSL ≤ good part ≤ USL < bad part

⚠️ specification limits and control limits are determined **independently**

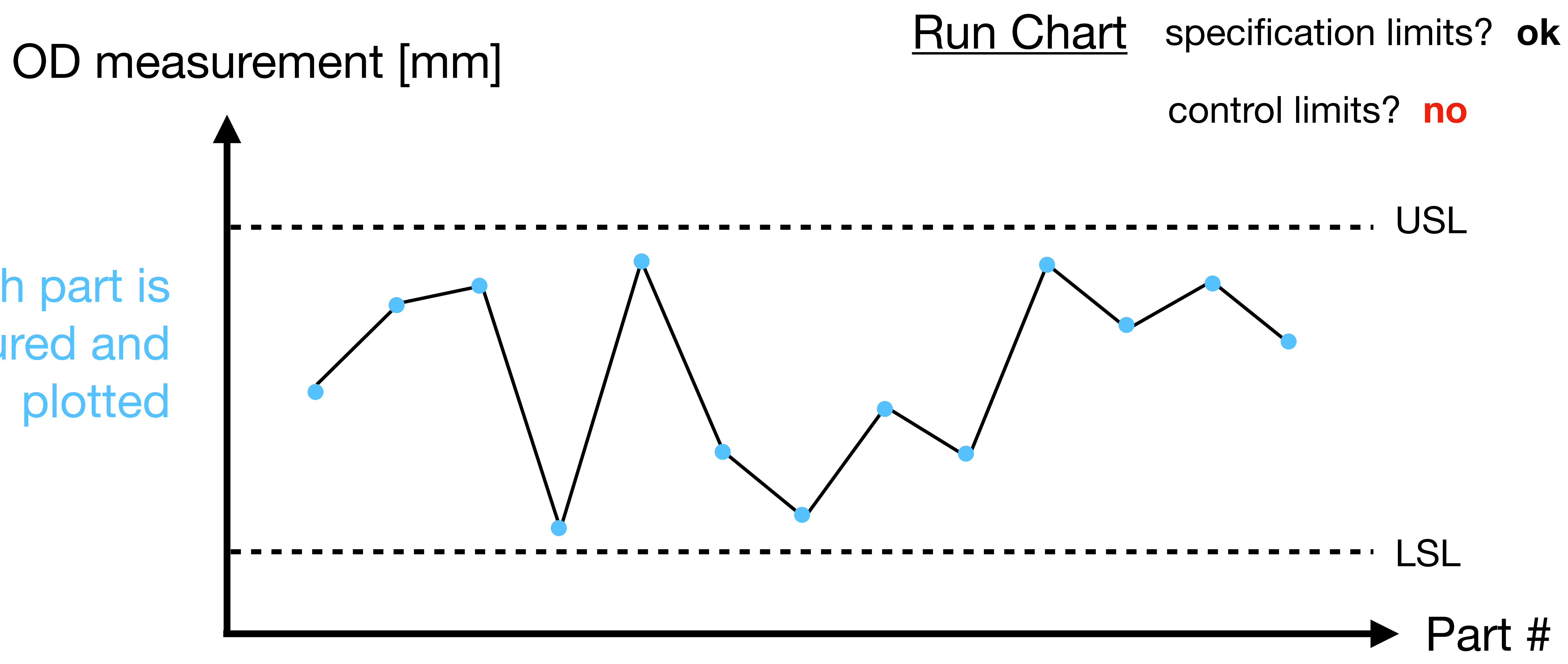
bad process < LCL ≤ good process≤ UCL < bad process

manufacturing engineers do a qualifying run of parts and figure out how to measure critical dimensions → a run chart is plotted to start gaining intuition → **manufacturing engineers** determine a reasonable sampling rate to start with, and sample means are plotted → statistics determines the control limits → **manufacturing engineers determine** the rules used to dictate when the process in <u>in control</u> or <u>out of control</u> → the control chart and rules are used to monitor control of a process → capability can be determined and the process can be managed and improved by all

e.g. Western Electric Rules

Run Chart vs Control Chart

each part is measured and plotted



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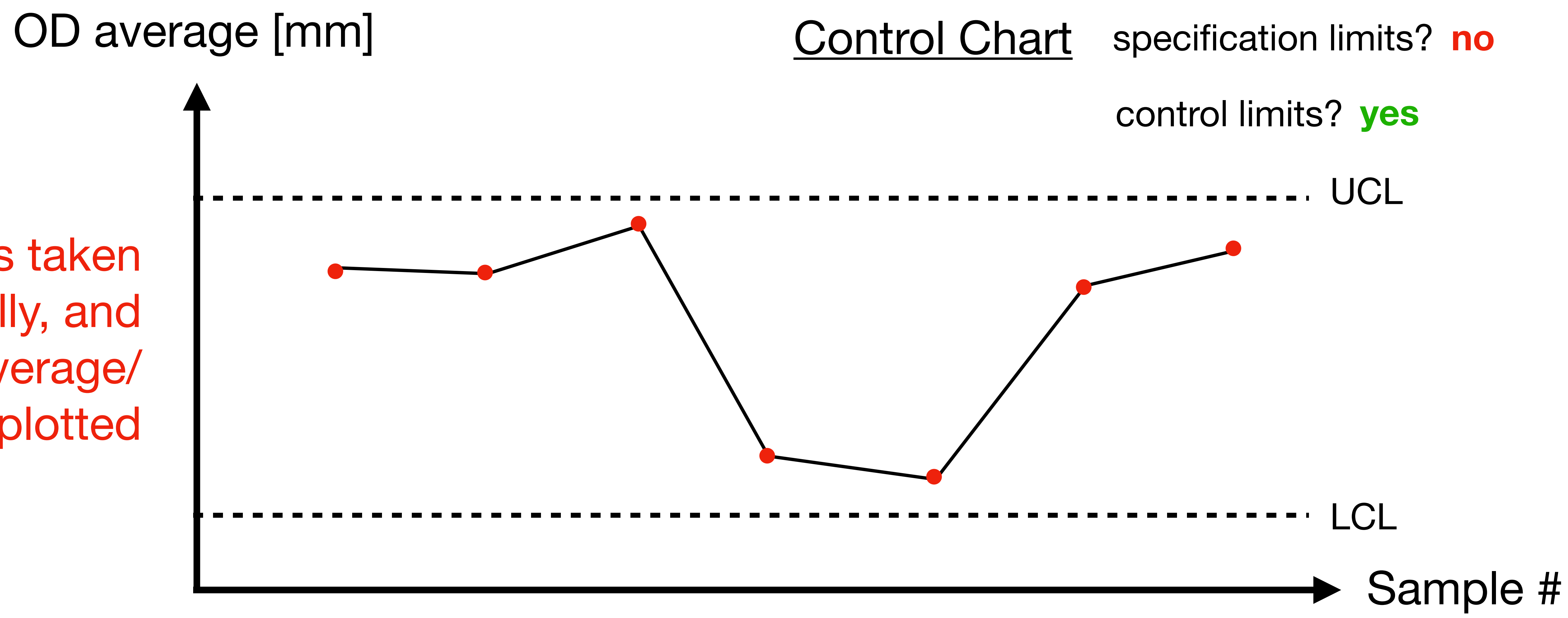
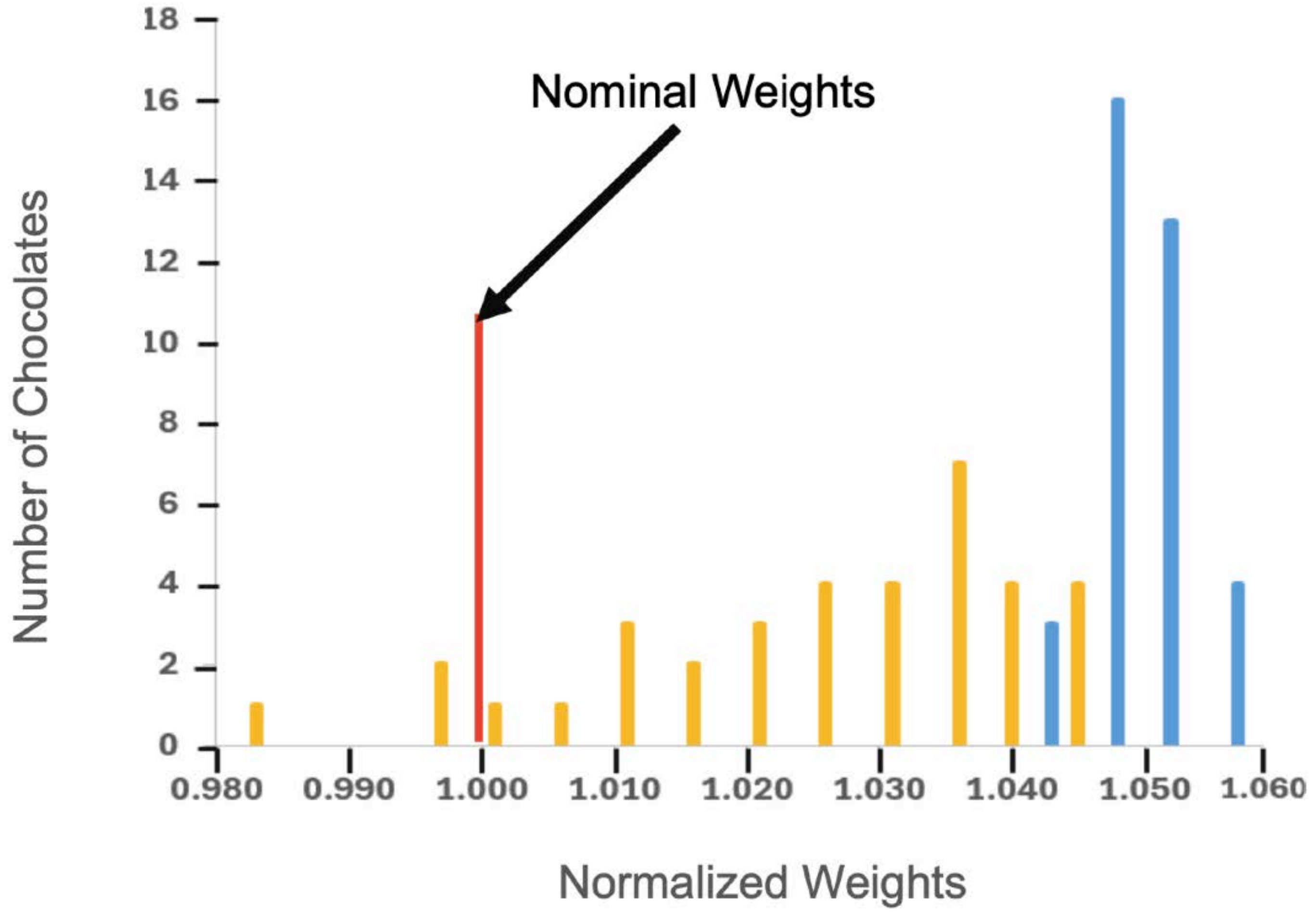
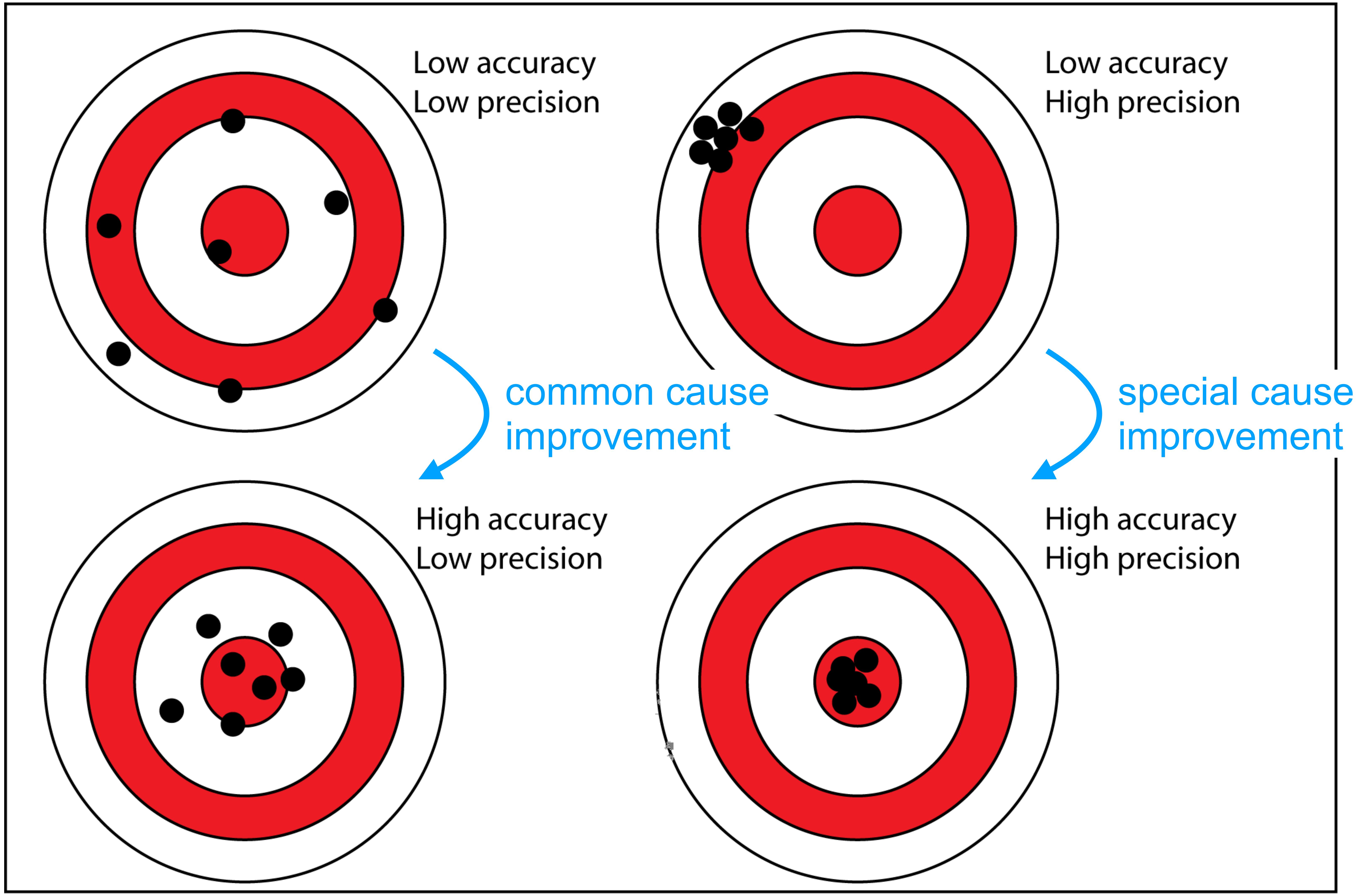
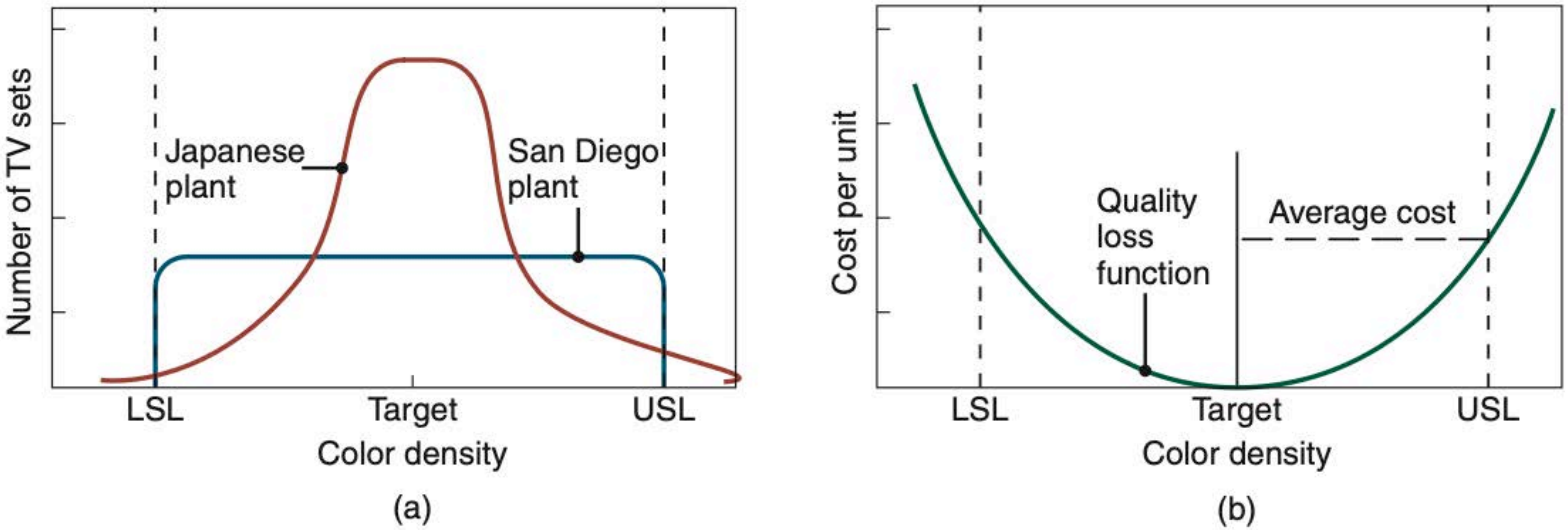


Accuracy vs Precision

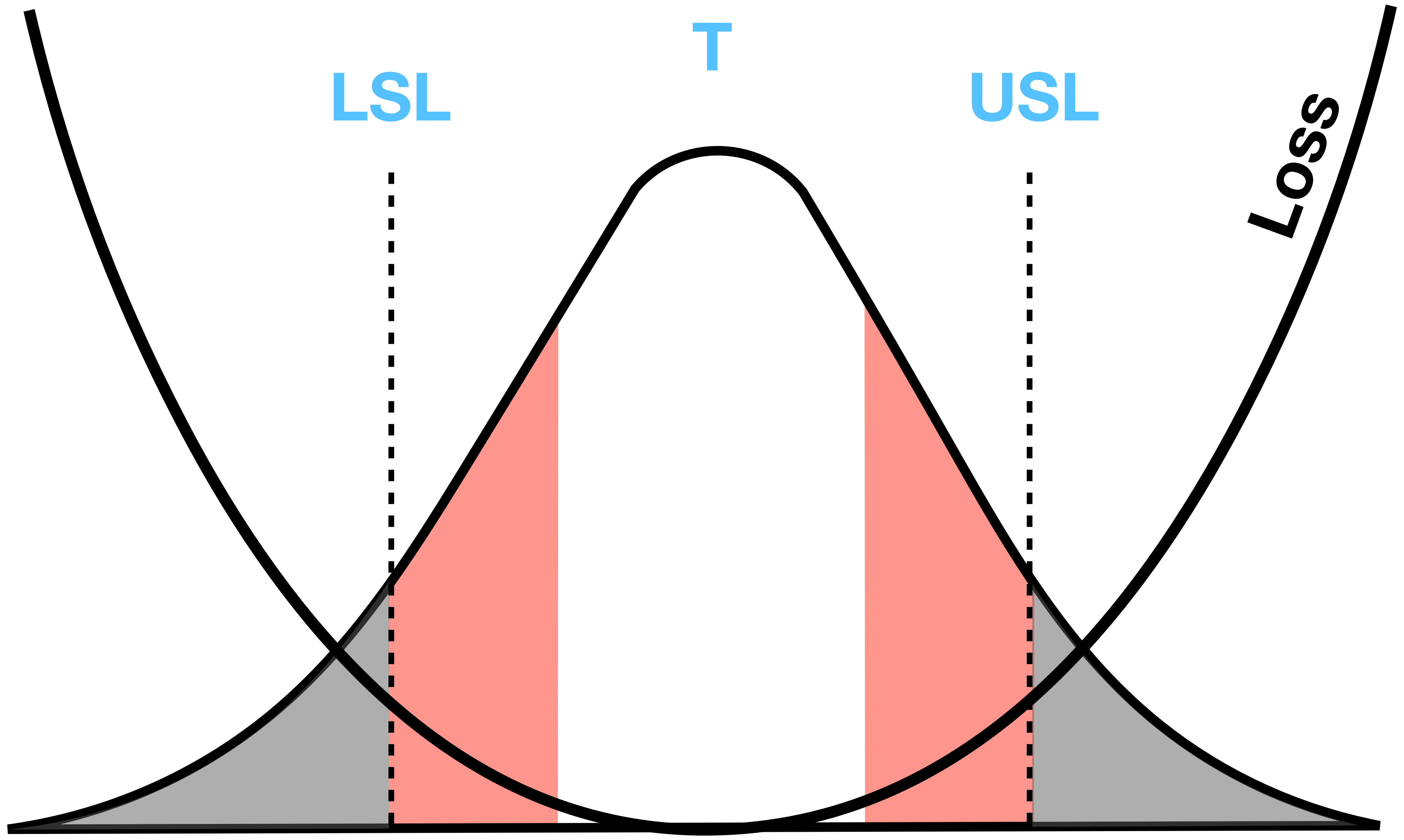




A Tale of Two Factories



Quality Loss Function



Often a company will start with standard USL/LSL and then move to QLF even if a product meets the specification, deviating from the nominal value can lead to a loss in perceived quality. The QLF attempts to quantify the loss to create more stringent requirements.

$$Loss = k(x - T)^2$$

x: mean value of sample observed
T: nominal or target value of the product
k: proportionality constant

$$k = \frac{C}{(USL - T)^2}$$

C: total cost for units produced with low quality or out of spec (e.g. at the USL)
USL: upper specification limit

k is empirically determined from a collection of data related to various costs associated with a product being perceived as poor quality and producing parts that are out of spec

these costs could include cost to produce, rework, dispose, refund, repair, recapture customer, brand damage, etc.

Image Credits

Slide 1:

Slide 21: