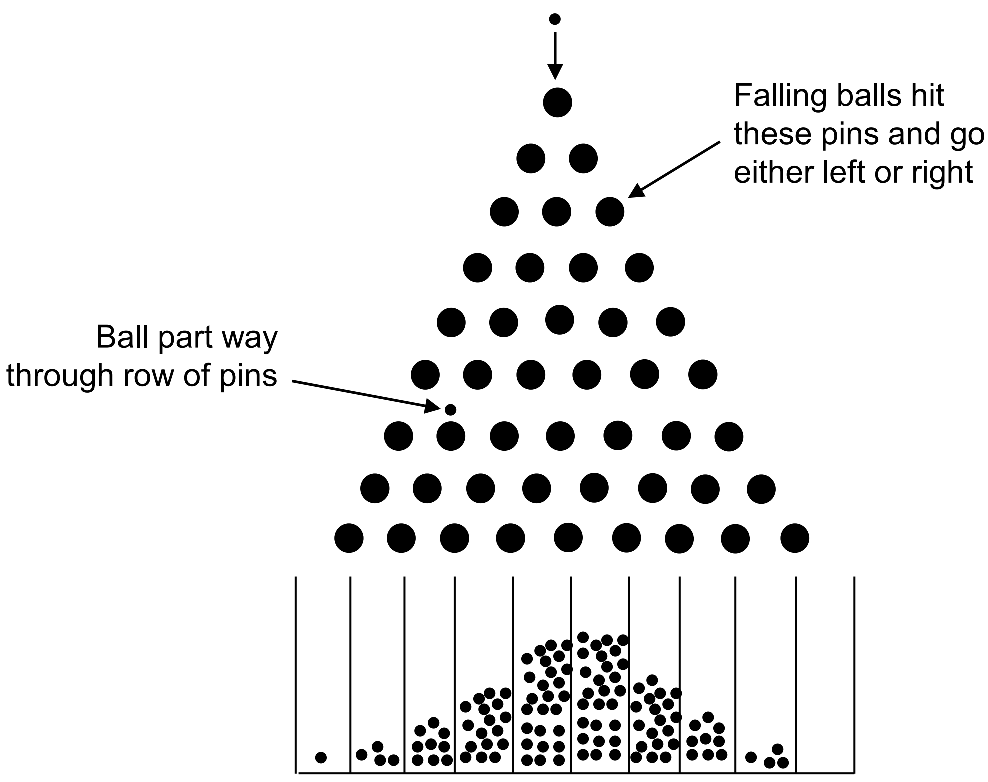
2.008

Process Control

Outline

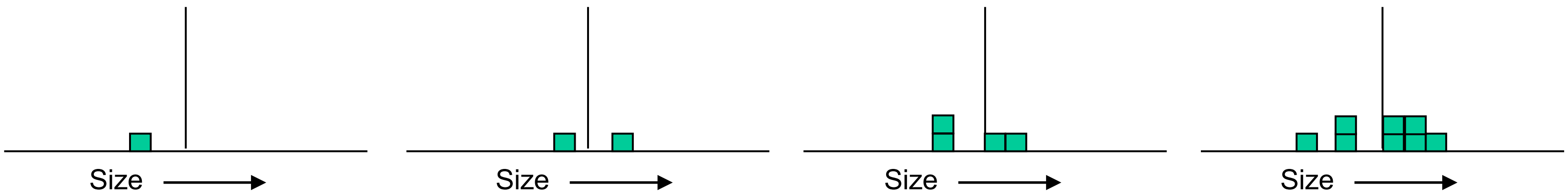
- Variations
- Optimization, In-process Control & Statistical Process Control
- Statistical Process Control

Manufacturing Outcome

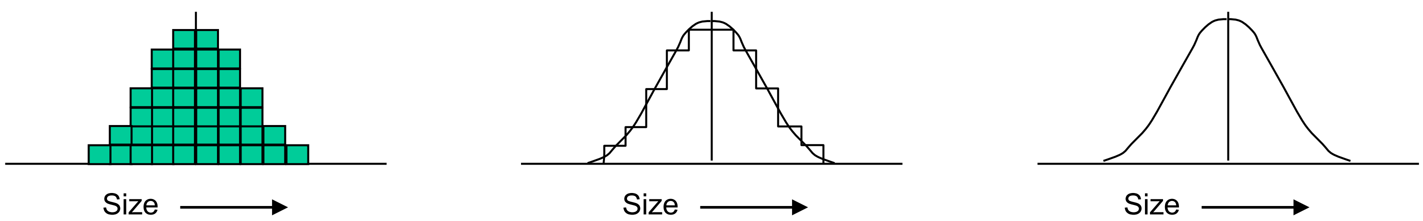


Variation: Common and Special Causes

Pieces vary from each other:

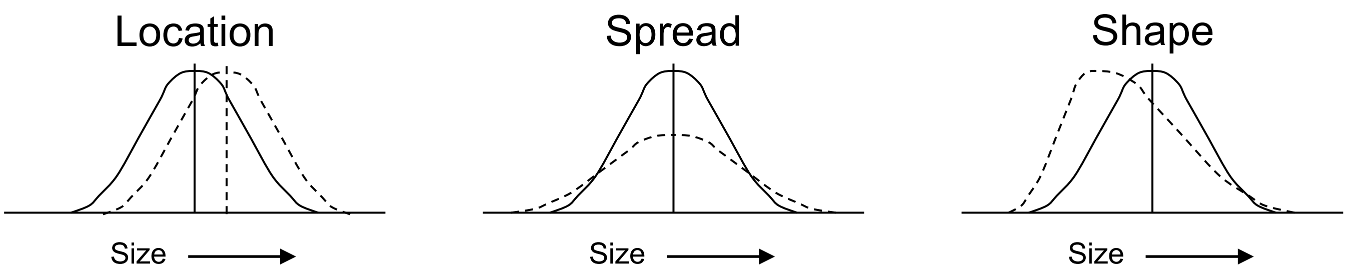


But they form a pattern that, if stable, is called a distribution:



Common and Special Causes (cont'd)

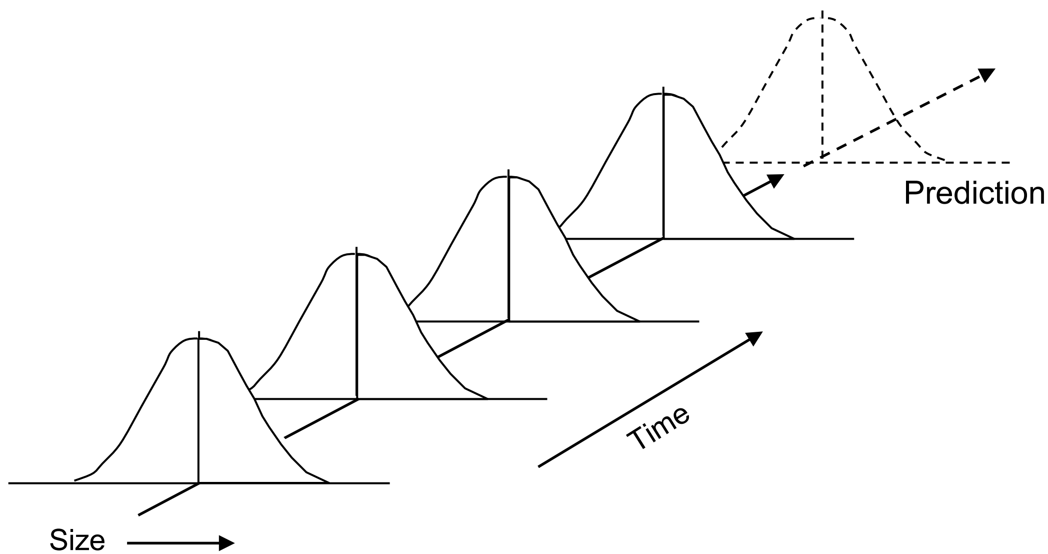
Distributions can differ in…



…or any combination of these

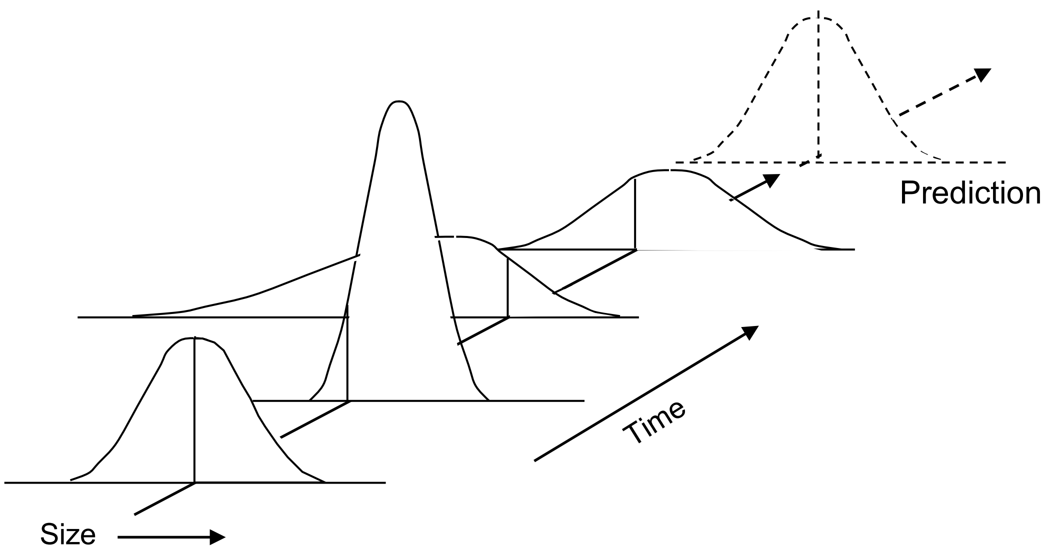
Common and Special Causes (cont'd)

If only common causes of variation are present, the output of a process forms a distribution that is stable over time and is predictable:

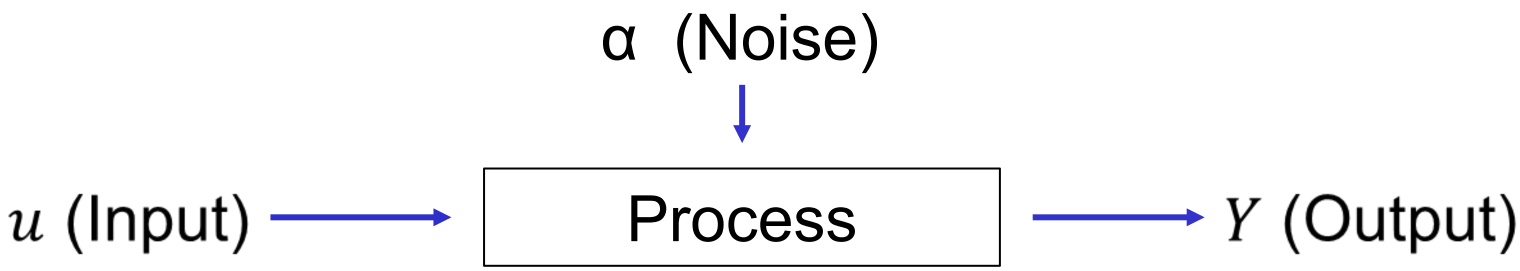


Common and Special Causes (cont'd)

If special causes of variation are present, the process output is not stable over time and is not predictable:



Process Control Schemes



SISO: $$Y = Y\,(\alpha, u)$$

$$\Delta Y = \frac{\partial Y}{\partial \alpha}\Delta\alpha + \frac{\partial Y}{\partial u}\Delta u$$

Disturbance Sensitivity

Assignable Cause (SPC)

Controller Gain

Control Input

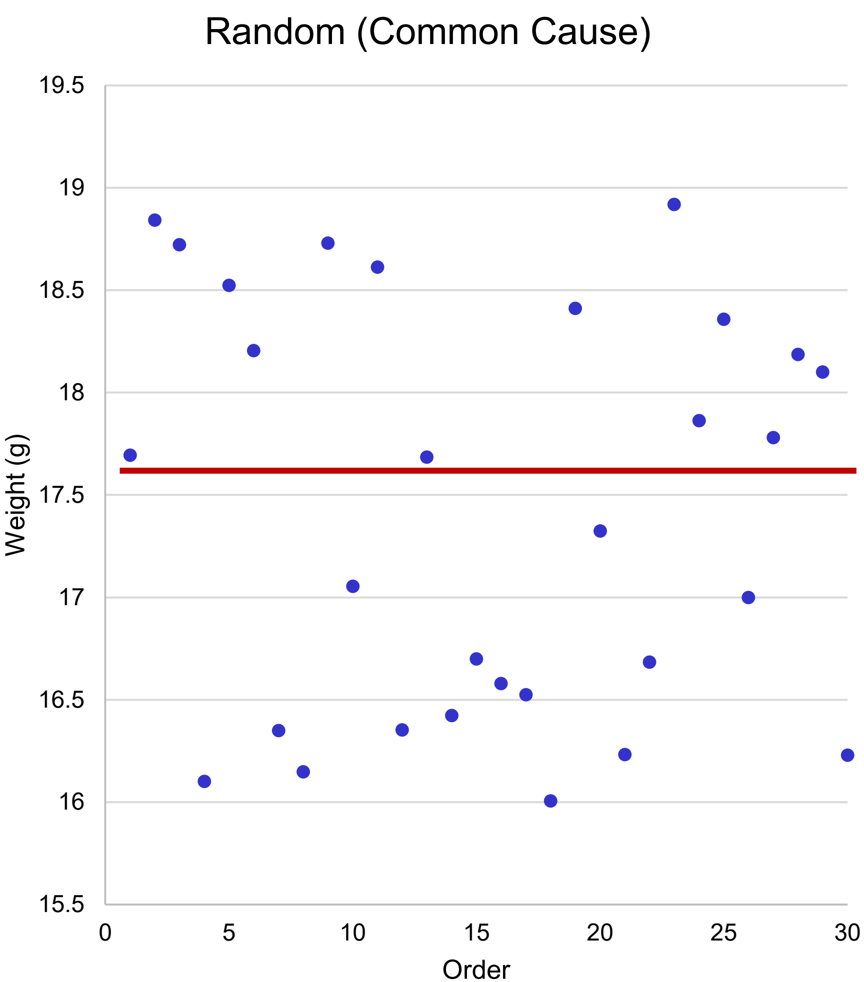
Optimization (Robustness)

Statistical Process Control

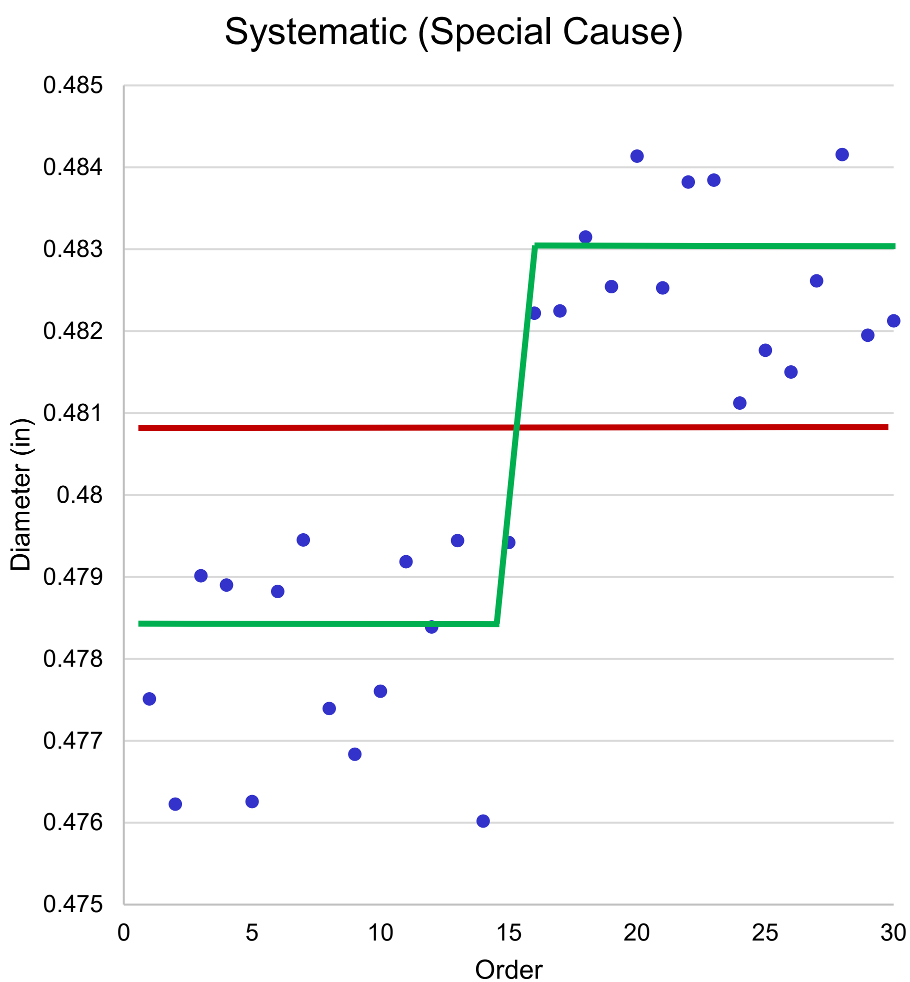
1. Detect disturbances
   (special causes)

2. Take corrective actions

Variations

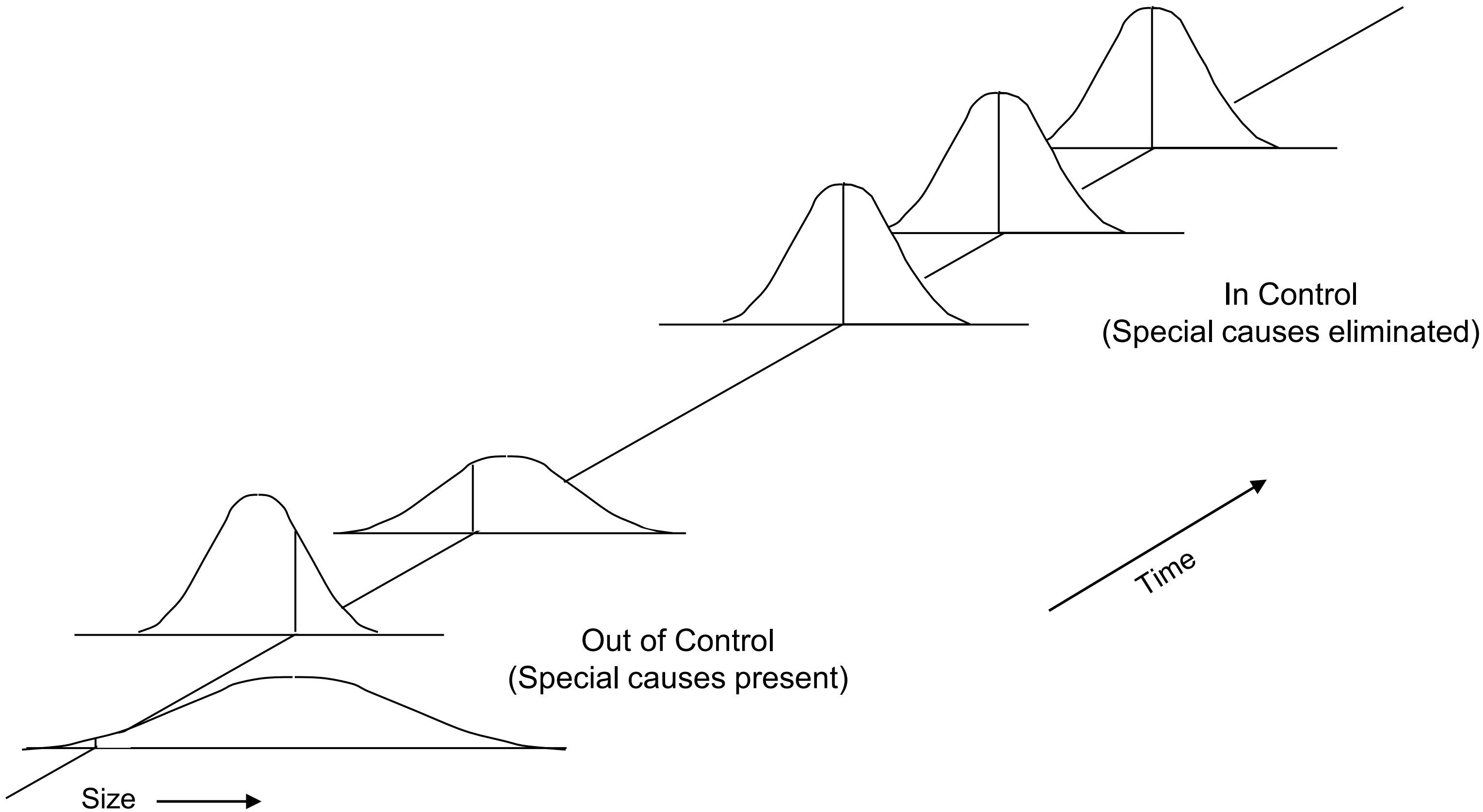


Random (Common Cause)
19.5
19
18.5
18
17.5
17
16.5
16
15.5
Weight (g)
0
5
10
15
20
25
30
Order



Systematic (Special Cause)
0.485
0.484
0.483
0.482
0.481
0.48
0.479
0.478
0.477
0.476
0.475
Diameter (in)
0
5
10
15
20
25
30
Order

Process Control

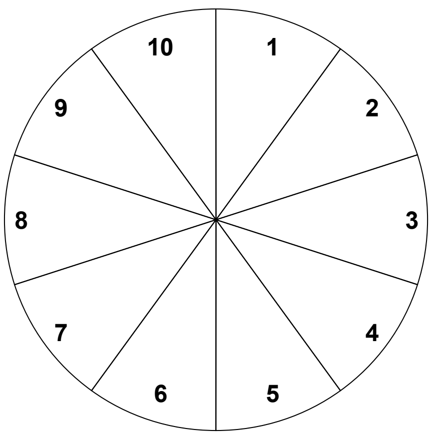


Central Limit Theorem

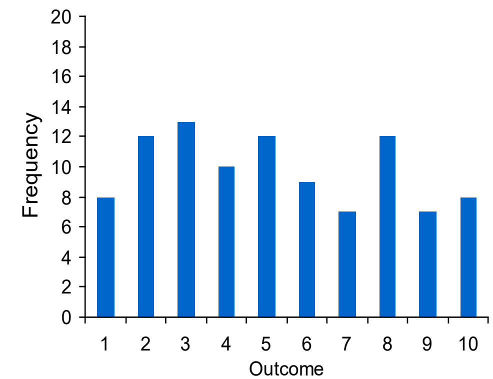
- A large number of independent events have a continuous probability density function that is normal in shape.
- Averaging more samples increases the precision of the estimate of the average.

Sampling and Histogram Creation

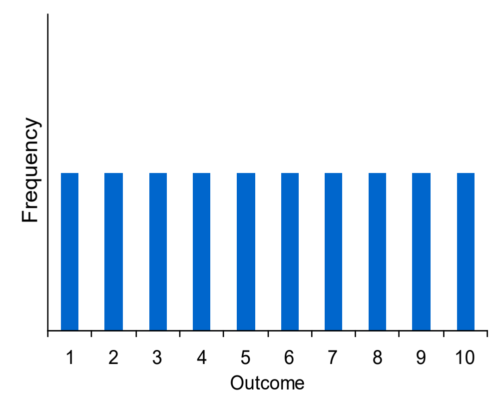
Wheel of Fortune: Equal probability of outcome 1-10, P=0.1



Taking 100 random samples, the resulting histogram would look like this

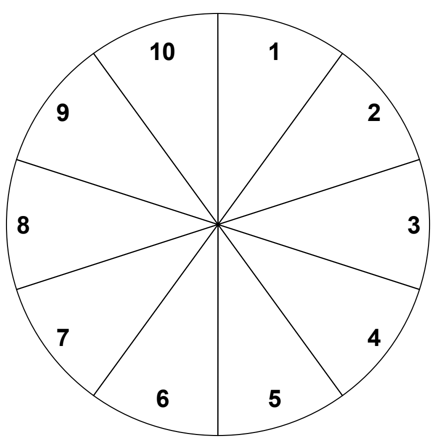


Taking ∞ random samples, the resulting histogram would look like this

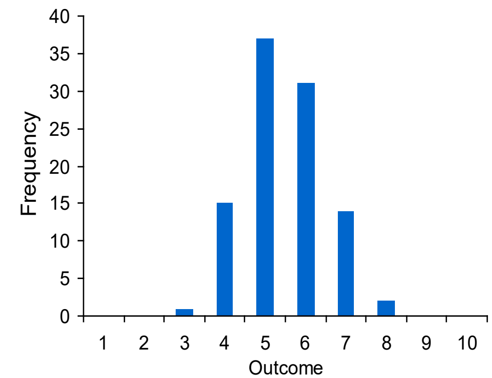


Sampling and Histogram Creation (cont'd)

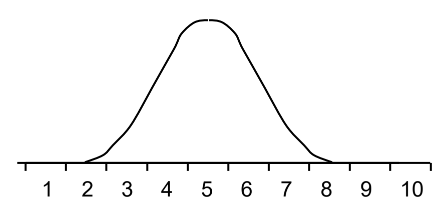
Wheel of Fortune: Equal probability of outcome 1-10, P=0.1



Take 10 random samples, calculate their average, and repeat 100 times, the resulting histogram would resemble

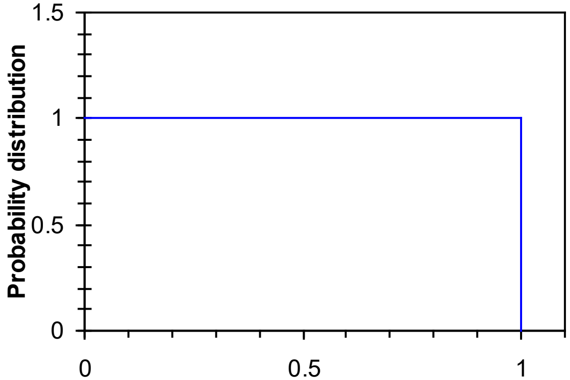


Take 10 random samples, calculate their average, and repeat ∞ times, the resulting histogram would approach the continuous distribution shown

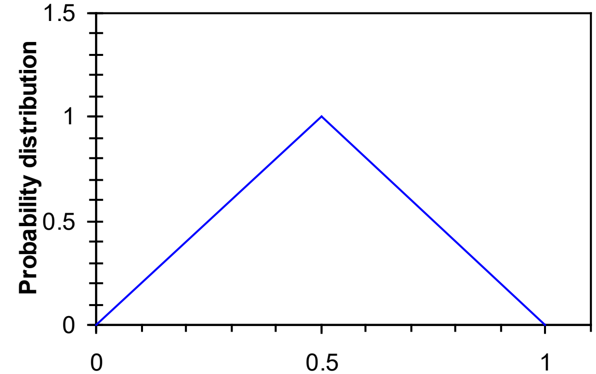


Uniform Distributions

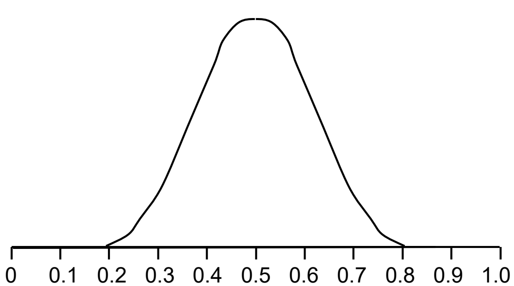
Underlying Distribution



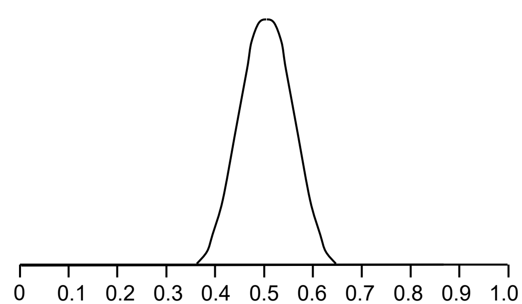
Distribution resulting from averaging of 2 random values



Distribution resulting from the average of 3 random values

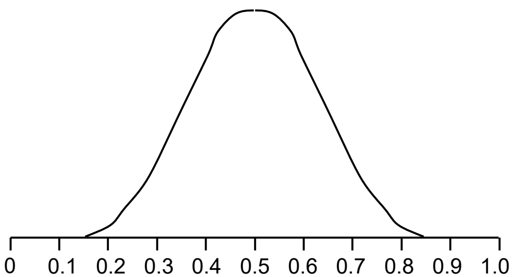


Distribution resulting from the average of 10 random values

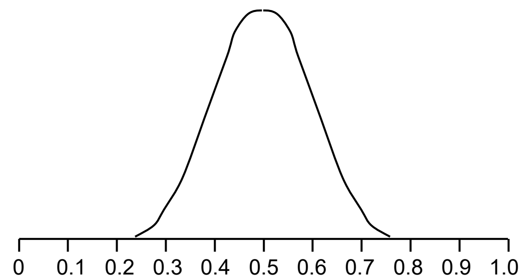


Normal Distribution

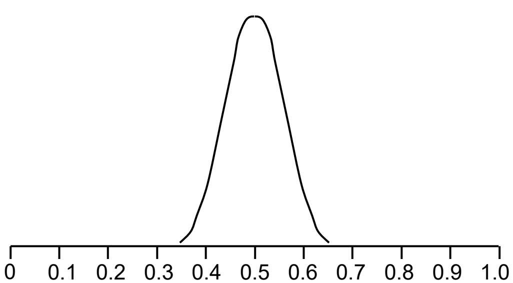
Underlying distribution



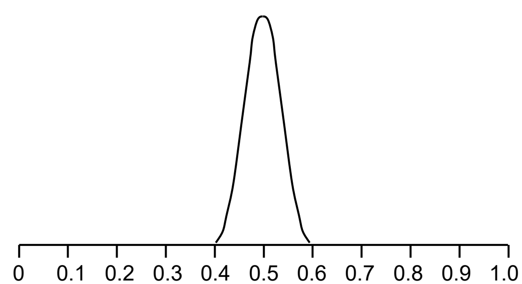
Distribution resulting from the average of 2 random values



Distribution resulting from the average of 3 random values



Distribution resulting from the average of 10 random values



Shewhart Control Chart

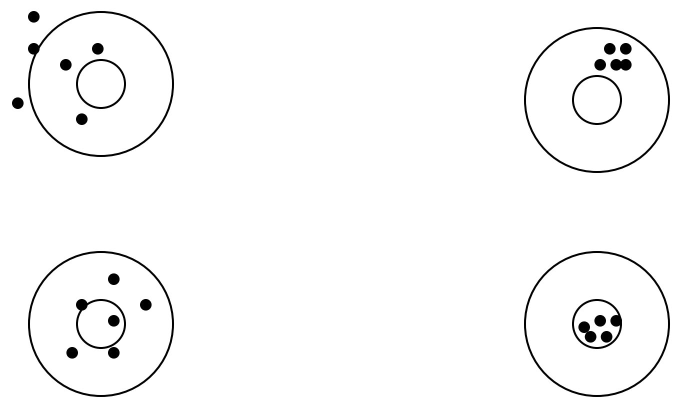
- Upper Control Limit (UCL), Lower Control Limit (LCL)
- Subgroup size ($5 < n < 20$)

Easier to Detect?

Not precise　　Precise

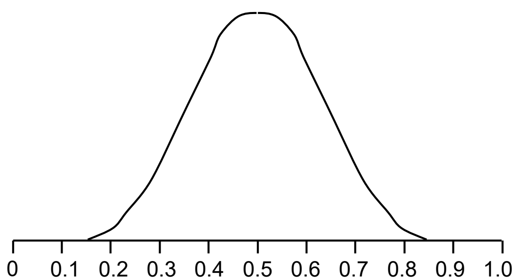
Not accurate

Accurate

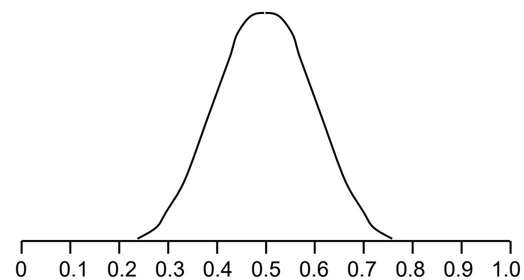


Normal Distribution

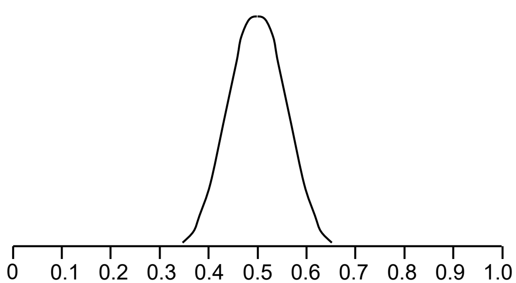
Underlying distribution



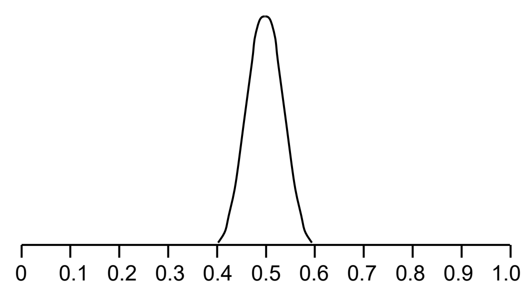
Distribution resulting from the average of 2 random values



Distribution resulting from the average of 3 random values



Distribution resulting from the average of 10 random values



Setting the Limits

Idea: Points outside the limits will signal that something is wrong- an assignable cause. We want limits set so that assignable causes are highlighted, but few random causes are highlighted accidentally.

Convention for Control Charts:

- Upper control limit (UCL) = $\bar{x} + 3\sigma_{sg}$
- Lower control limit (LCL) = $\bar{x} - 3\sigma_{sg}$

(Where σ_{sg} represents the standard deviation of a subgroup of samples)

Setting the Limits (cont'd)

Convention for Control Charts (cont'd):

$$\sigma_{\text{sub group}} = \sigma_{\text{sg}} \neq \sigma_{\text{process}}$$

$$\sigma_{\text{subgroup}} = \sigma_{\text{process}}/\sqrt{n}$$

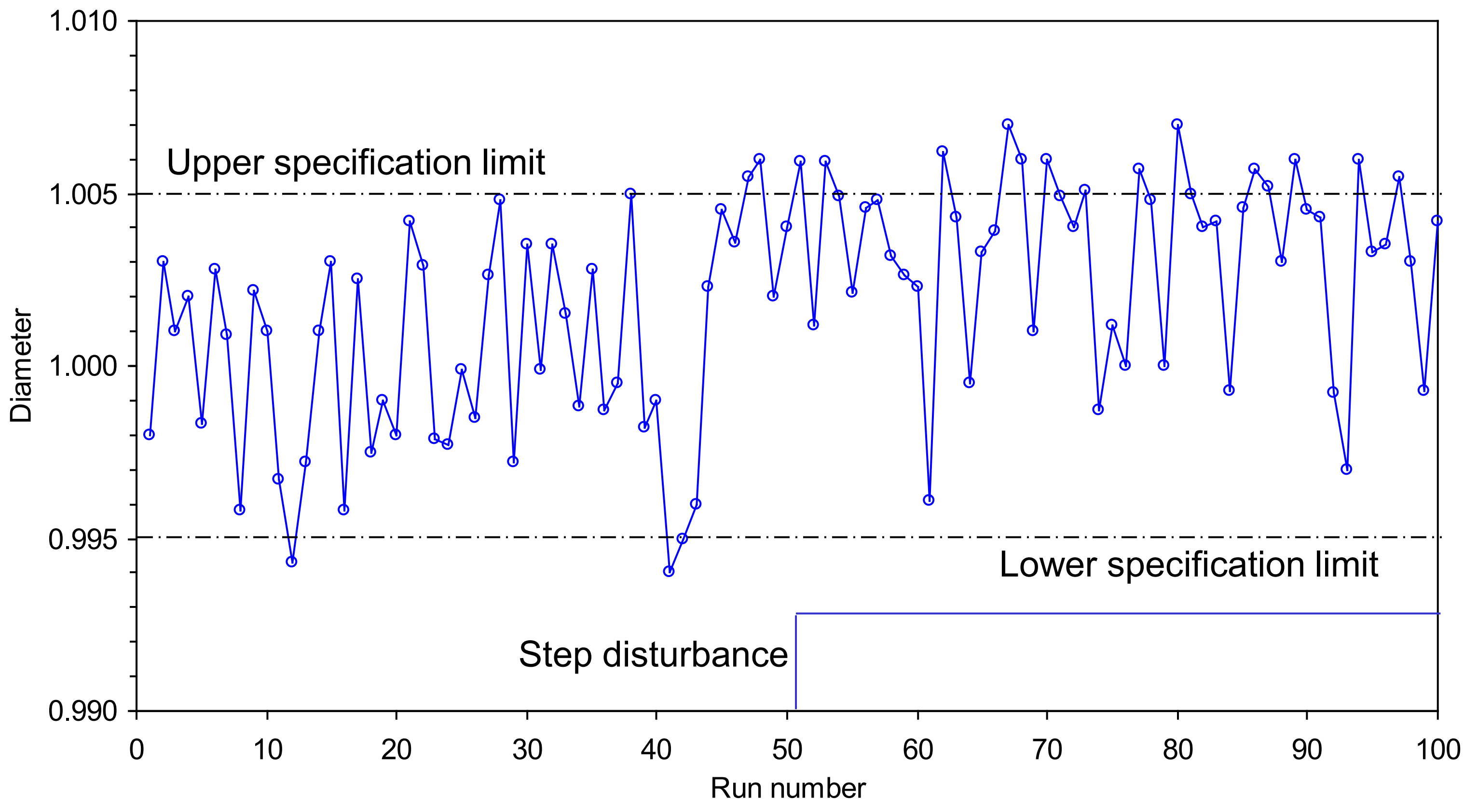
$$\text{UCL} = \bar{x} + 3\sigma_{\text{process}}/\sqrt{n}$$

$$\text{LCL} = \bar{x} - 3\sigma_{\text{process}}/\sqrt{n}$$

As n increases, the UCL and LCL move closer to the center line, making the control chart more sensitive to shifts in the mean.

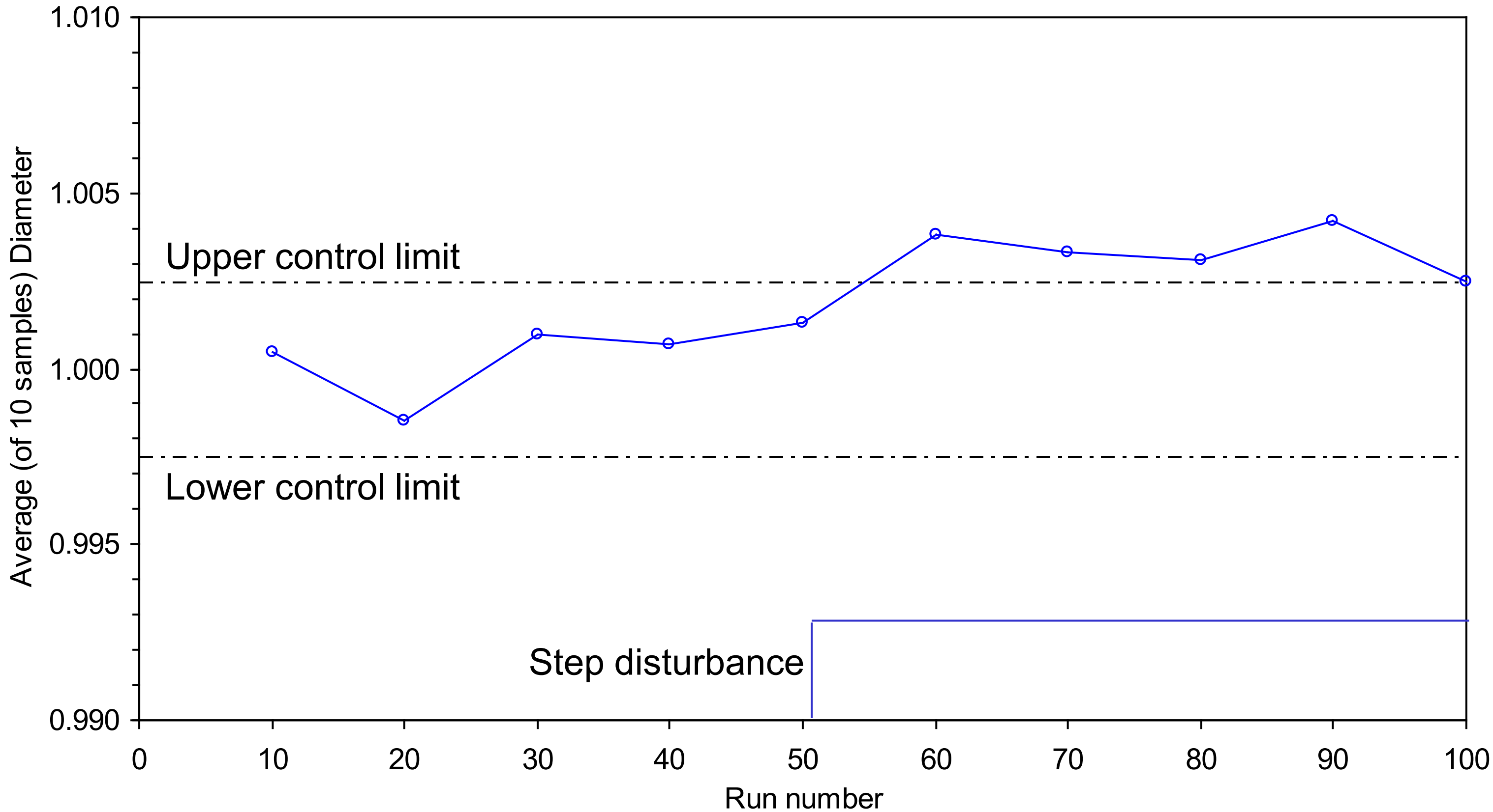
Run Chart

Run chart based on 100 observations

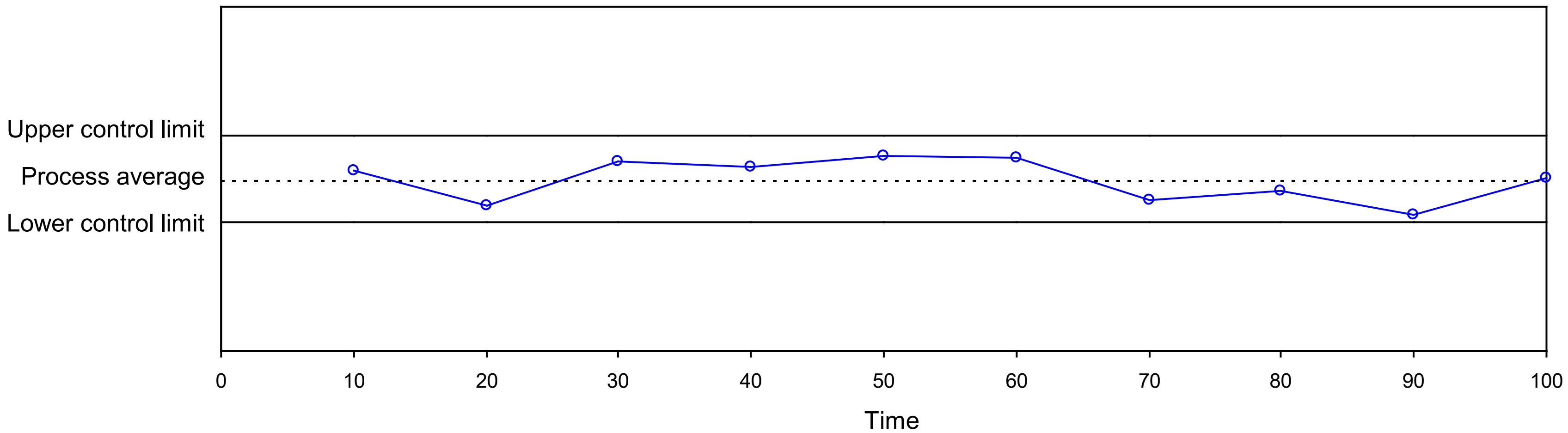


Shewhart Chart

Chart based on average of 10 samples, note the step change that occurs at run 51



Statistical Process Control



1. Collection:
   - Gather data and plot on chart.
2. Control:
   - Calculate control limits from process data, using simple formulae.
   - Identify special causes of variation; take local actions to correct.
3. Capability:
   - Quantify common cause variation; take action on the system.

These three phases are repeated for continuing process improvement.

Benefits of Control Charts

Properly used, control charts can:

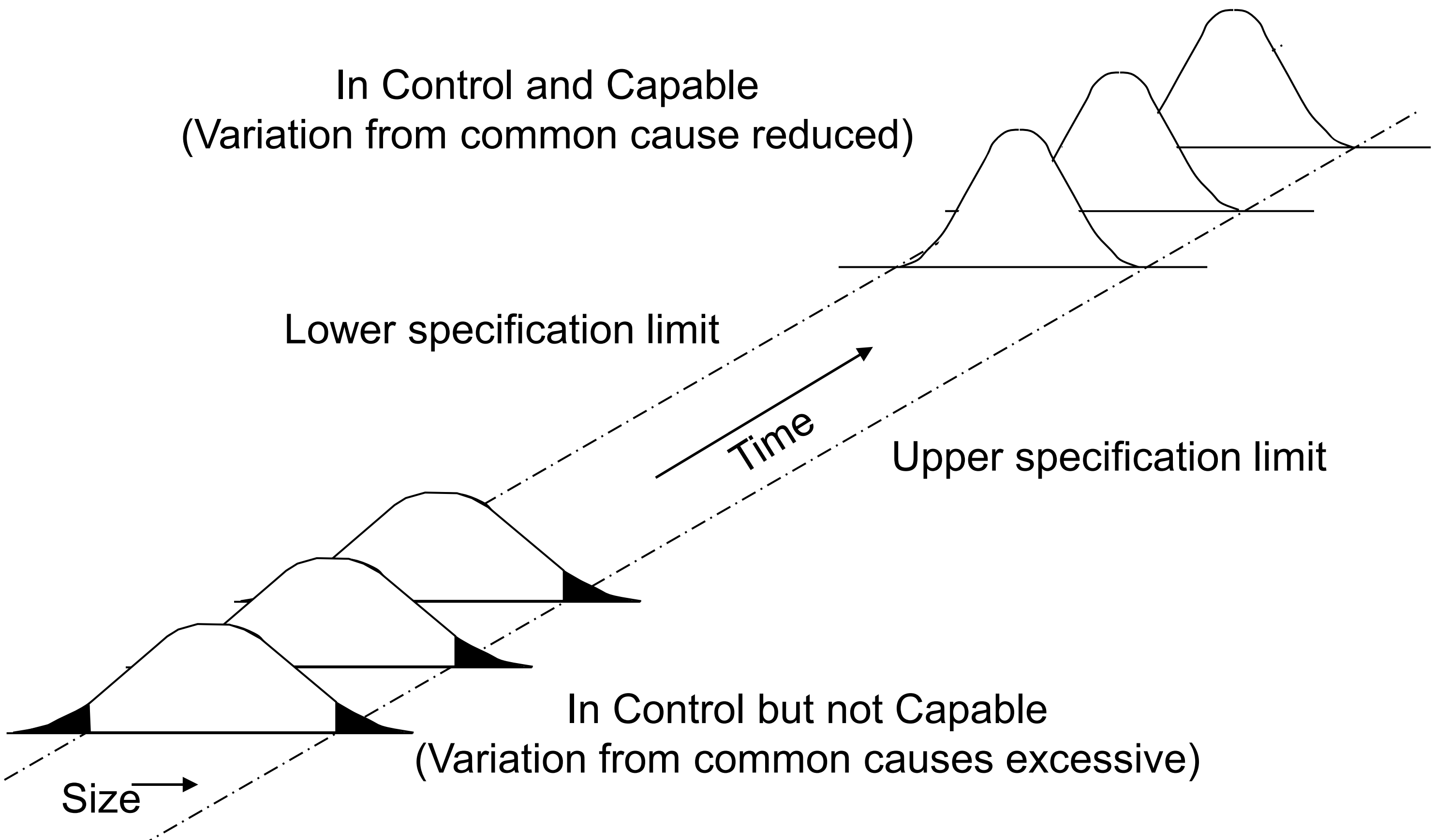
- Be used by operators for ongoing control of a process
- Help the process perform consistently, predictably for higher quality, lower cost and higher effective capacity
- Provide a common language for discussing process performance
- Distinguish special from common causes of variation; as a guide to local or management action

Improving Process Capability

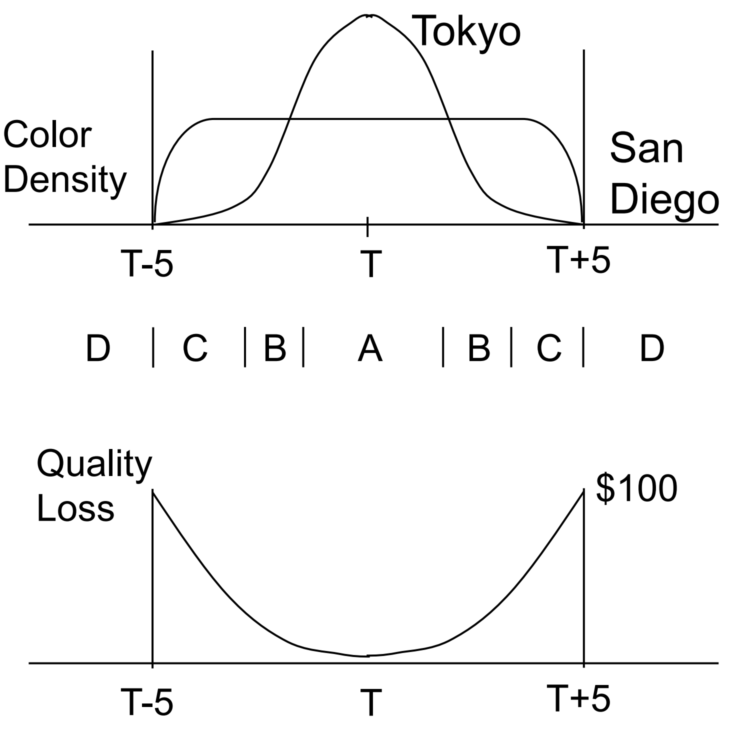
- To improve the chronic performance of the process, concentrate on the common causes that affect all periods. These will usually require management action on the system to correct.

- Chart and analyze the revised process:
  - Confirm the effectiveness of the system by continued monitoring of the Control Chart

Process Capability



A Tale of Two Factories



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