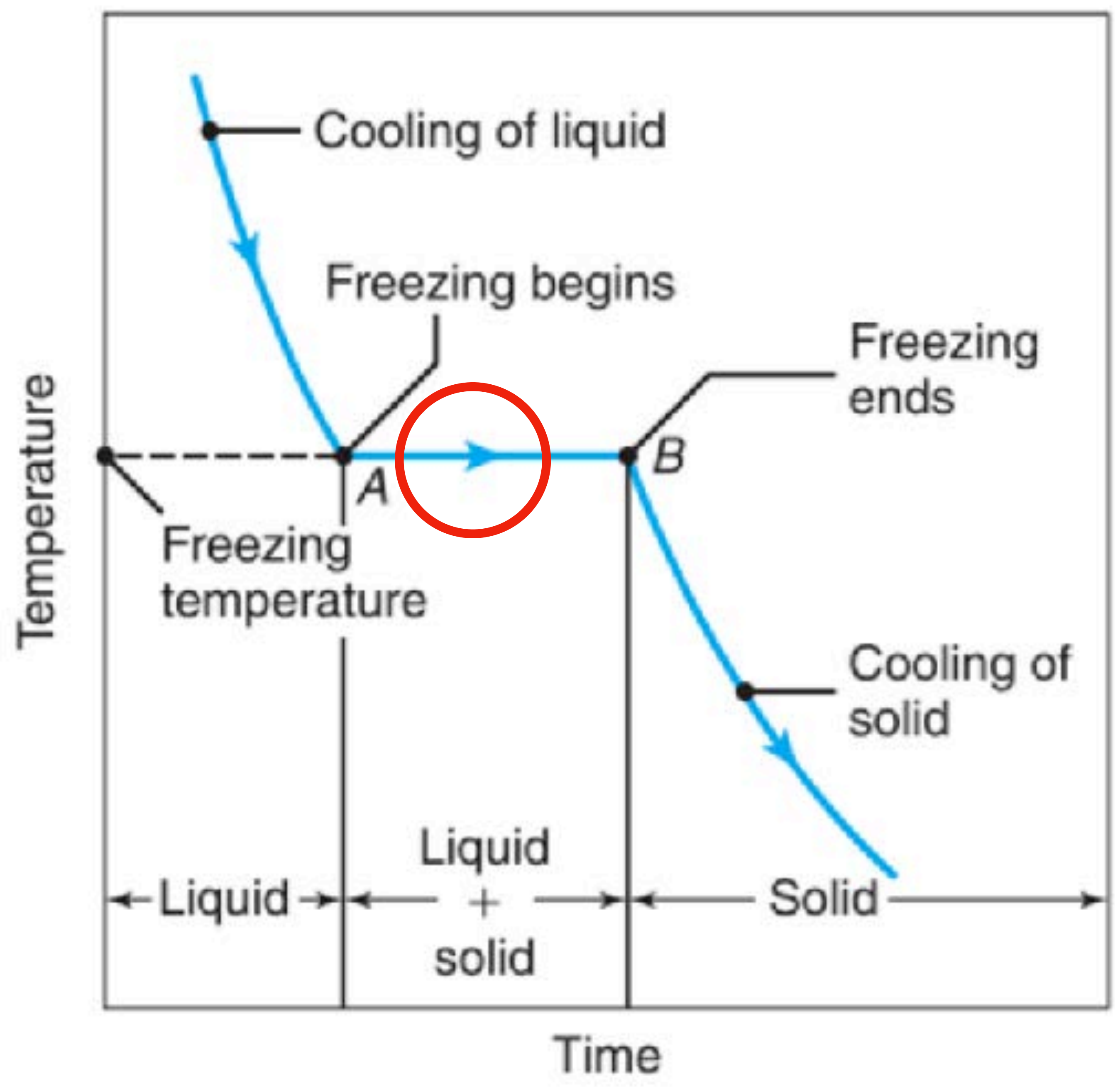
Solidification and Cooling

Casting

01

Temperature Distribution



Chvorinov's rule

$$t_{solidify} = C \cdot \left(\frac{V}{A}\right)^n$$

- V = volume of the casting
- A = surface area of the casting
- C = mold constant, depends on mold material and thermal properties of casting metal

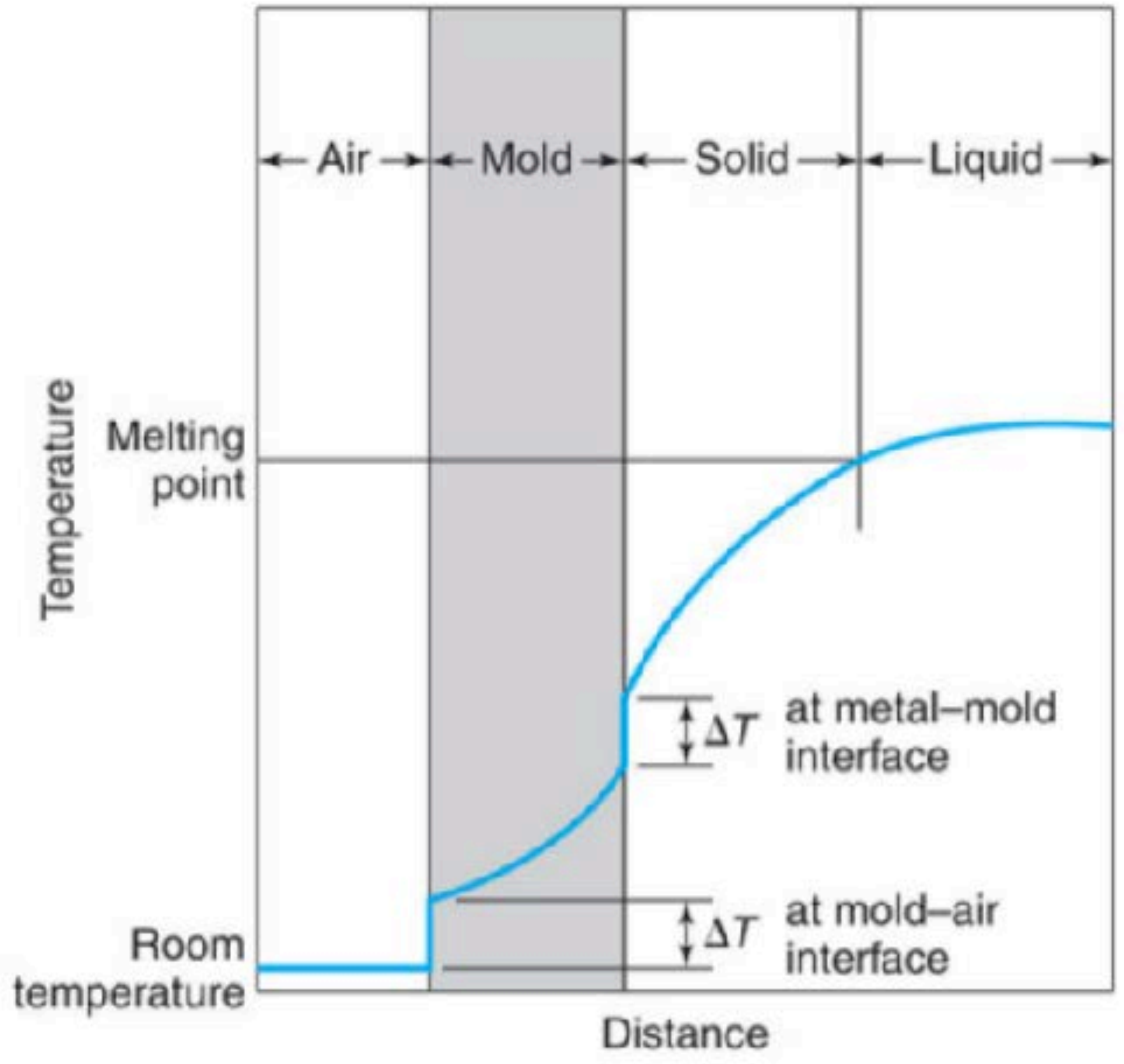


FIGURE 10.10 Temperature distribution at the interface of the mold wall and the liquid metal during the solidification of metals in casting.

metal is poured at T_{melt}

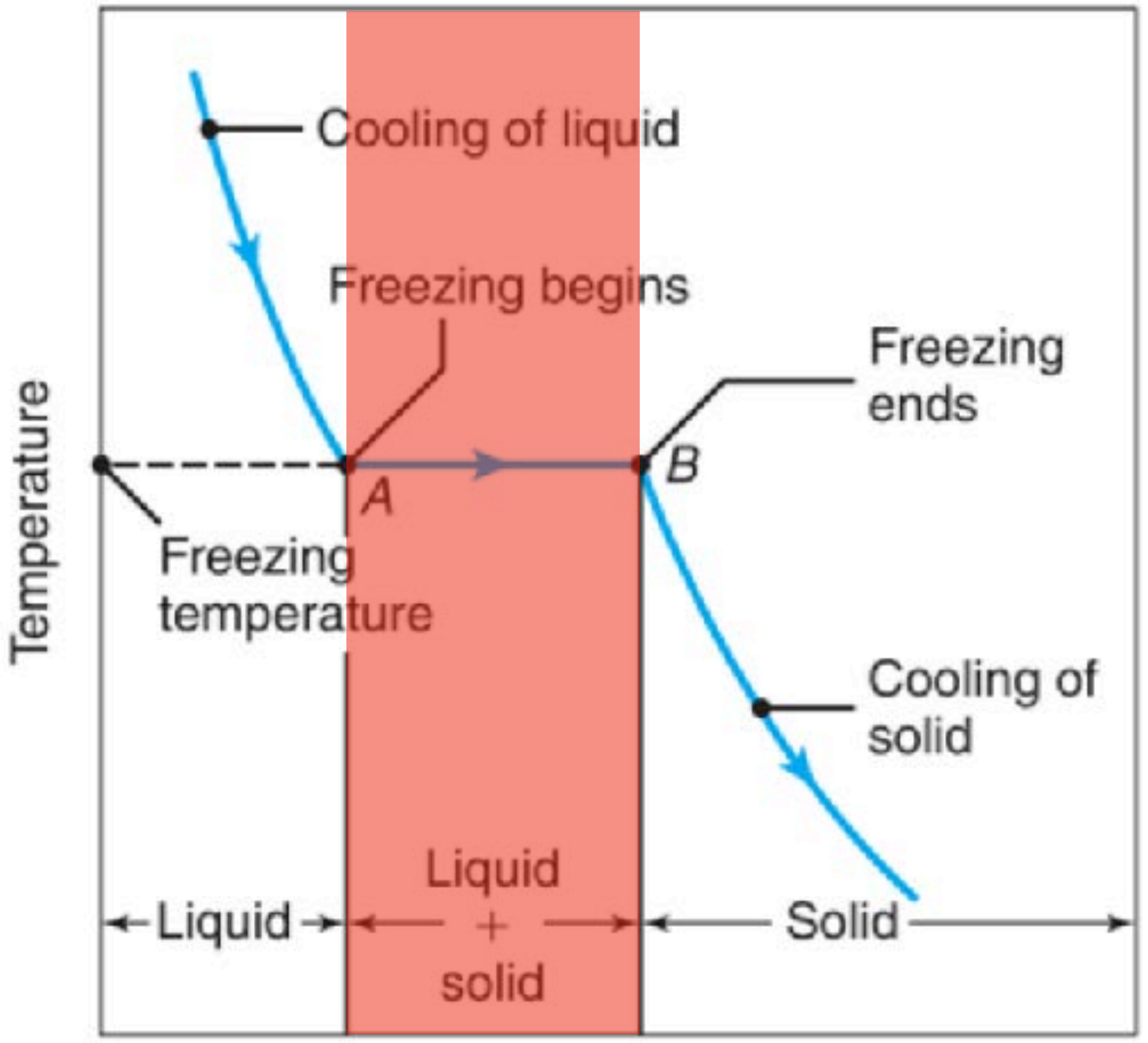
thermal diffusivity of the mold dictates 2 options:

1. temp distribution across the mold (sand)
2. mold temp stays constant and the temp drop is across the mold-part interface (die)

the part solidifies inwards at a constant temperature (T_{melt}), and then continues to cool

subscripts: s (solid), m (mold)

Cycle Time



$t_{liquid\ cool}$ $t_{solidify}$ $t_{solid\ cool}$

Chvorinov's Rule

$$t_{solidify} \neq t_{cool} \neq t_{cycle}$$

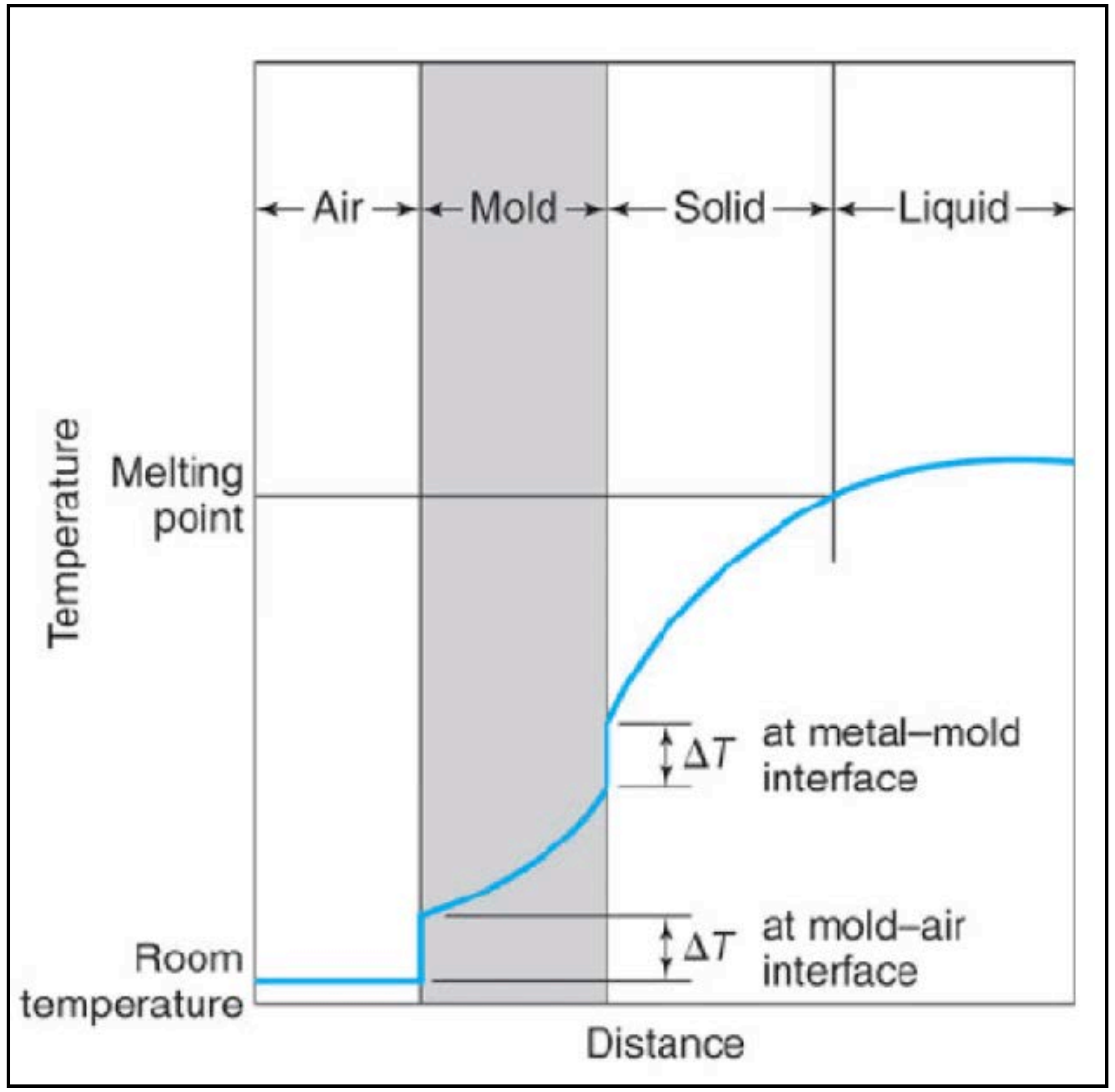
$$t_{cycle} \approx t_{solidify} + t_{cool}$$

liquid cooling might take place, then solidification, then solid cooling

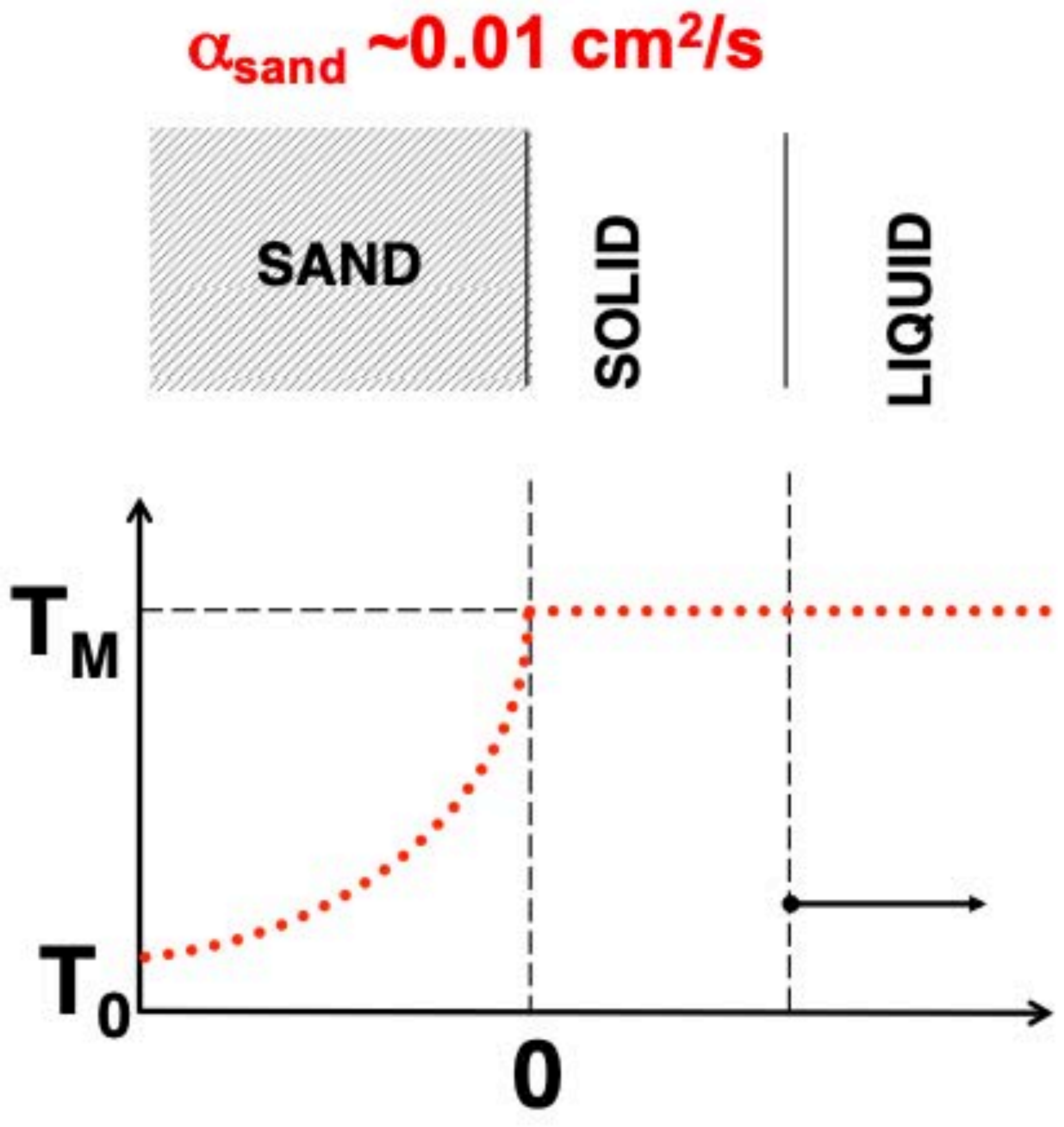
there are more steps besides the cooling ones, but only if solidification and cooling times are short, then the rest of the cycle becomes more significant

focus on solidification first with Chvorinov's rule, then explore solid cooling

Sand vs Die Casting



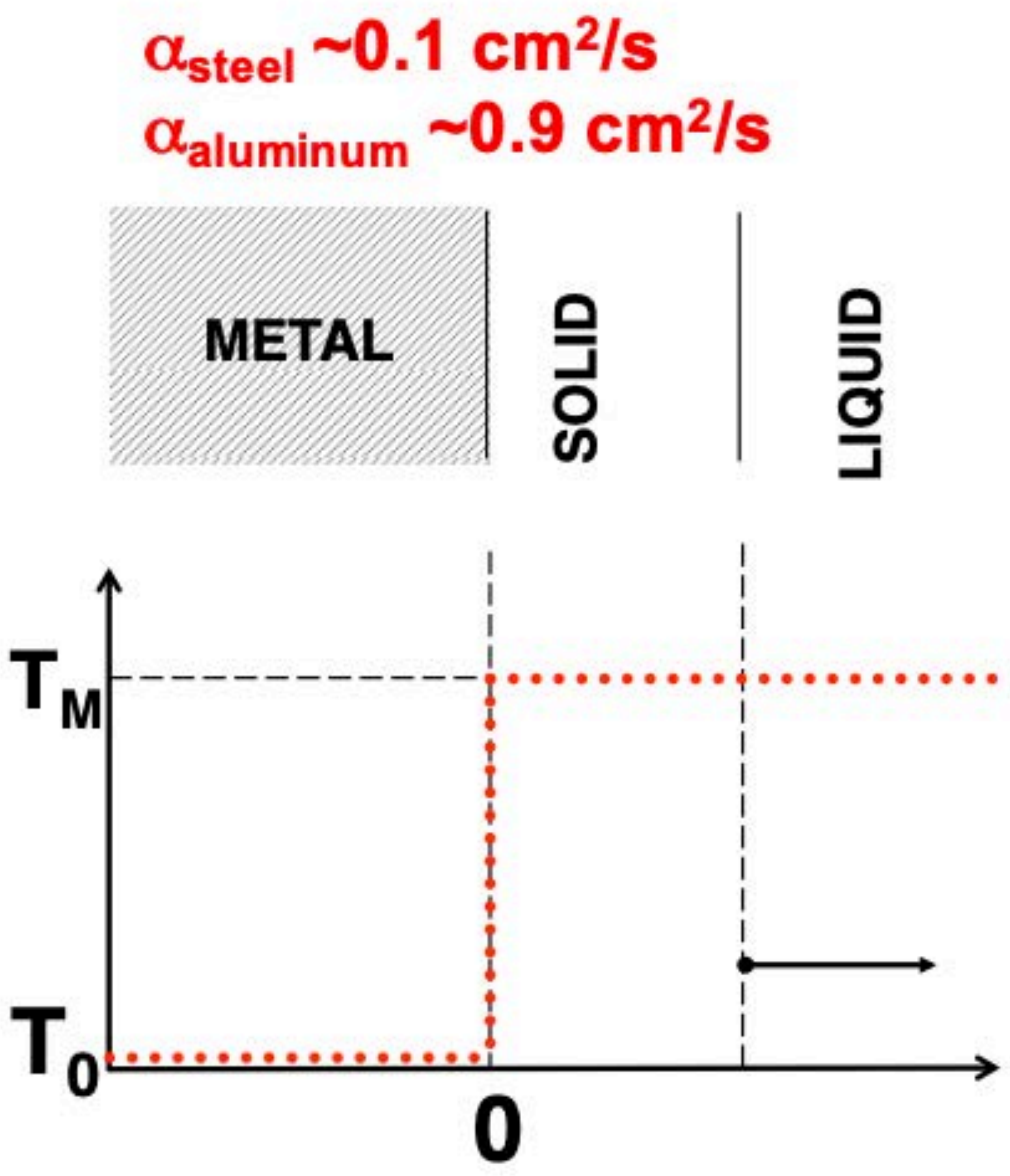
Sand Casting



$$t_{solidify} = C \cdot \left(\frac{V}{A}\right)^2$$

→ Like injection molding, but mold has low thermal conductivity

Die Casting



$$t_{solidify} = C \cdot \left(\frac{V}{A}\right)$$

→ Lower-bound (assumes constant mold temp)

Solidification and Cooling

Casting

Solidification in Sand Casting

Mold: Sand

heat flow: primary resistance dominated by conduction in sand

$$\frac{\partial T}{\partial t} = \alpha_m \frac{\partial^2 T}{\partial x^2}$$

initial condition: mold starts at T_0

boundary condition: constant surface temperature (phase change)

surface becomes T_{melt} @ $t = 0$

"semi-infinite" mold: outside does not change temp.

solve differential eqn

$$\frac{T - T_{melt}}{T_0 - T_{melt}} = erf\left(\frac{-x}{2\sqrt{\alpha_m t}}\right)$$

thermal gradient in the sand

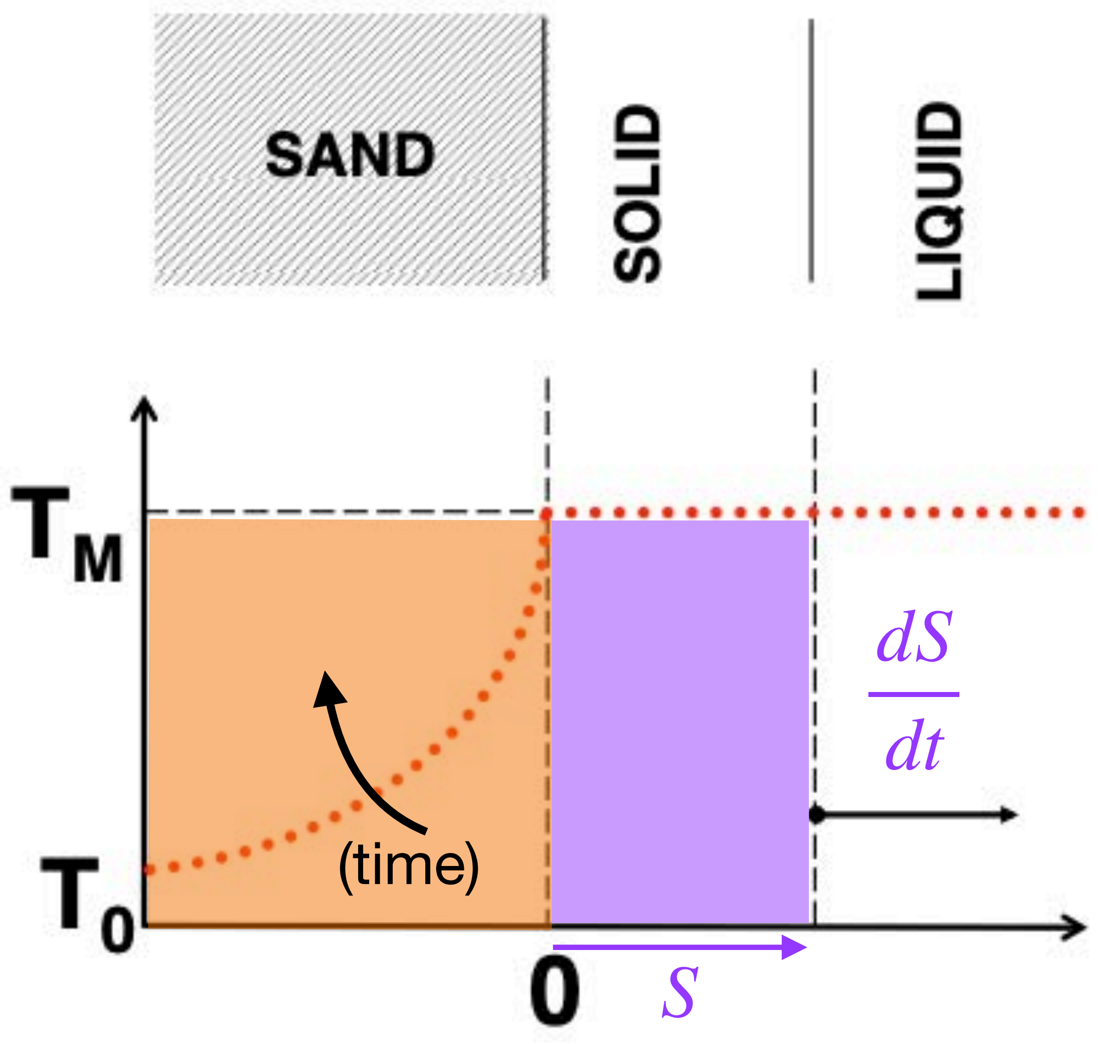
$$T - T_{melt} = erf\left(\frac{-x}{2\sqrt{\alpha_m t}}\right)(T_0 - T_{melt})$$

differentiate w.r.t. x

$$\left(q_{out}\right)_{x=0} = \left(-k_m \frac{\partial T}{\partial x}\right)_{x=0}$$

Fourier's Law $\rightarrow q_{out} = -\sqrt{\frac{k_m \rho_m c_m}{\pi t}}\left(T_{melt} - T_0\right)$

α_{sand} ~0.01 cm²/s



- T_0: ambient/mold starting temp
- T_{melt}: melt temperature of the metal
- T: temperature of the mold
- x: distance from the mold-metal interface
- α_m: thermal diffusivity of the mold
- t: time
- ρ_s: density of the solid metal part
- k_m: thermal conductivity of the mold
- ρ_m: density of the mold
- c_m: heat capacity of the mold
- S: distance to solidification front
- H: latent heat of fusion for metal part

$$at\ boundary\ (x = 0):\ q_{out} = q_{in}$$

$$q_{in} = -\rho_s H \frac{dS}{dt}$$ heat flux from solidification

$$q_{out} = -\sqrt{\frac{k_m \rho_m c_m}{\pi t}}\left(T_{melt} - T_0\right) = q_{in} = -\rho_s H \frac{dS}{dt}$$

$$\frac{dS}{dt} = \frac{(T_{melt} - T_0)}{\rho_s H}\sqrt{\frac{k_m \rho_m c_m}{\pi}}\sqrt{t}$$

(metal) (mold)

$$t_{solidify} = \frac{\frac{\pi}{4}(\rho_s H)^2}{k_m \rho_m c_m (T_{melt} - T_0)^2}\left(\frac{V}{A}\right)^2$$

integrate for S, solve for t, set $S \rightarrow \frac{V}{A}$

reference: see Flemings: Solidification Processing

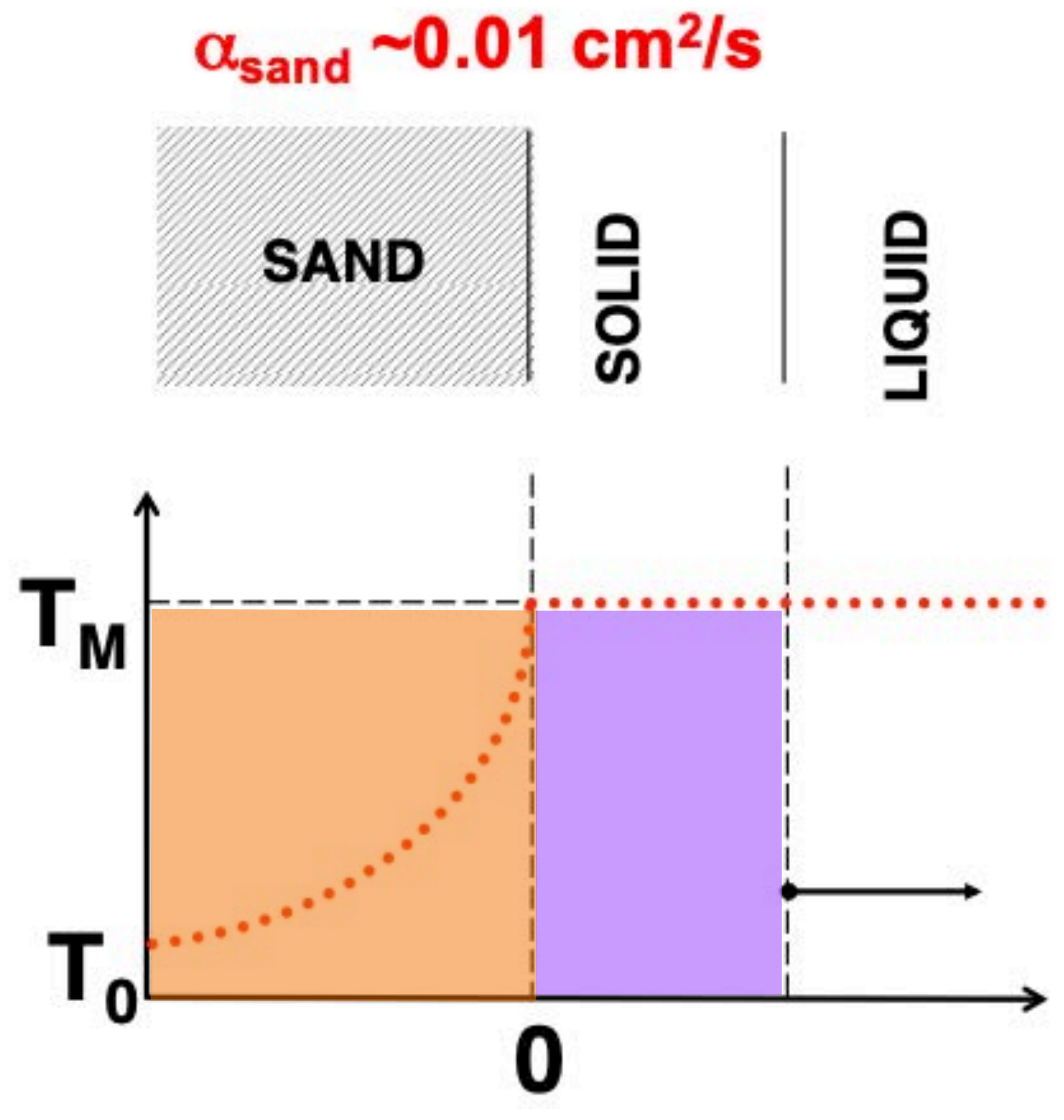
Solidification in Sand Casting

$$t_{solidify} = \frac{\frac{\pi}{4}(\rho_s H)^2}{k_m \rho_m c_m (T_{melt} - T_0)^2} \left(\frac{V}{A}\right)^2$$

2: from sand

constant C (also determined by experiment)

effective thickness



$$t_{solidify,sand} = C\left(\frac{V}{A}\right)^2$$

⚠️ similar form compared to injection molding, but not exactly the same

Key Takeaways

C: combines thermal properties of the metal and mold to influence freezing rate

for the metal: high melt temperature and low latent heat of fusion leads to faster solidification

for the mold: 3 parameters influence how well the mold absorbs heat

solidification is primarily determined by the mold, the conductivity of the metal has little influence

solidify time is parabolic: solidification happens quickly at first, then slows at the mold heats up

this analysis is for a wall, what about convex/concave surfaces?

Material Properties

metal	p (g/cm^3)	p (g/m^3)	H (J/g)	c (J/gK)	k (W/mK)	W/cmK	Tm	deltaT	Material contribution	Mold contribution	C (with sand as mold)	pH	pc	k/pc
zinc	7.1	7100000	112	0.388	122	1.22	419	394	4.073	0.30	120.45	795200000	2754800	0.44
nickel	8.9	8900000	293	0.461	94	0.94	1453	1428	3.335	0.26	98.61	2607700000	4102900	0.23
copper	8.9	8900000	206	0.385	401	4.01	1085	1060	2.992	0.07	88.46	1833400000	3426500	1.17
aluminum	2.7	2700000	398	0.897	236	2.36	660	635	2.864	0.17	84.69	1074600000	2421900	0.97
brass	8.5	8500000	168	0.375	109	1.09	930	905	2.490	0.29	73.62	1428000000	3187500	0.34
iron	7.9	7900000	247	0.449	84	0.84	1538	1513	1.663	0.34	49.18	1951300000	3547100	0.24
steel	7.8	7800000	245	0.49	43	0.43	1510	1485	1.656	0.61	48.97	1911000000	3822000	0.11
silver	10.5	10500000	105	0.235	428	4.28	961	936	1.387	0.09	41.03	1102500000	2467500	1.73
gold	19.3	19300000	63	0.129	318	3.18	1063	1038	1.372	0.13	40.58	1215900000	2489700	1.28
tungsten	19.3	19300000	190	0.132	182	1.82	3400	3375	1.181	0.22	34.91	3667000000	2547600	0.71
titanium	4.5	4500000	390	0.523	22	0.22	1670	1645	1.138	1.93	33.66	1755000000	2353500	0.09
magnesium	1.7	1700000	358	0.105	157	1.57	650	625	0.948	3.57	28.04	608600000	178500	8.80
sand	1.6	1600000	156	0.83	2	0.02	1700	1675	0.022	37.65	0.66	249600000	1328000	0.02

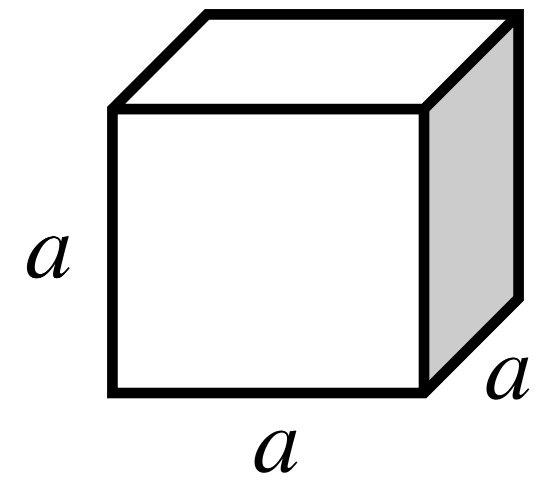
$\rho_s H$ energy released during solidification (and energy needed to melt)

$\rho_s c$ energy needed to change temperature (how much energy you get cooling down)

Volume / Surface Area

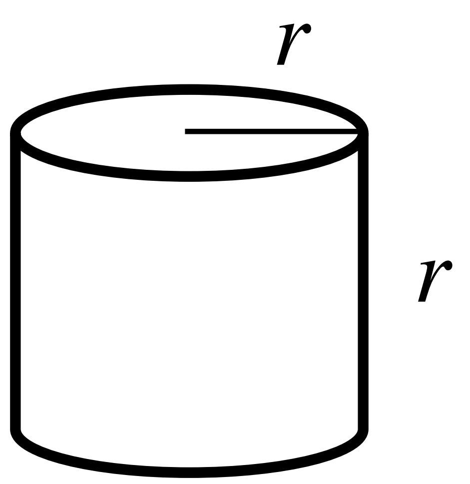
$$t_{solidify,sand} \propto \left(\frac{1}{surface\ area}\right)^2 \qquad t_{solidify,sand} = C\left(\frac{V}{A}\right)^2$$

compare different shapes with V = 1



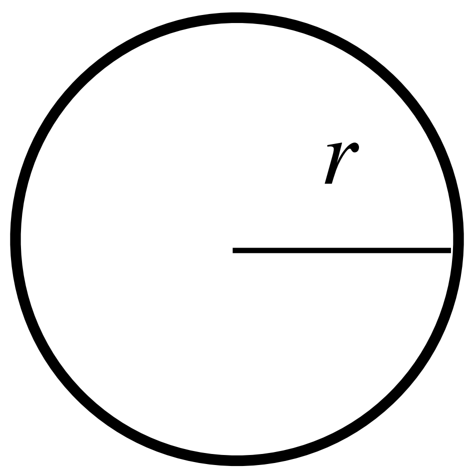
$$\left.\begin{array}{l} V = a^3 \rightarrow a = 1 \\ A = 6a^2 \end{array}\right\} \frac{V}{A} = \frac{a}{6} = 0.167$$

$$t_{cube} = 0.028C$$



$$\left.\begin{array}{l} V = \pi r^2 h = \pi r^3 \rightarrow r = \left(\frac{1}{\pi}\right)^{\frac{1}{3}} \\ A = 2\pi r^2 + 2\pi r h = 4\pi r^2 \rightarrow 4\pi\left(\frac{1}{\pi}\right)^{\frac{2}{3}} \end{array}\right\} \frac{V}{A} = \frac{r}{4} = 0.171$$

$$t_{cylinder} = 0.029C$$



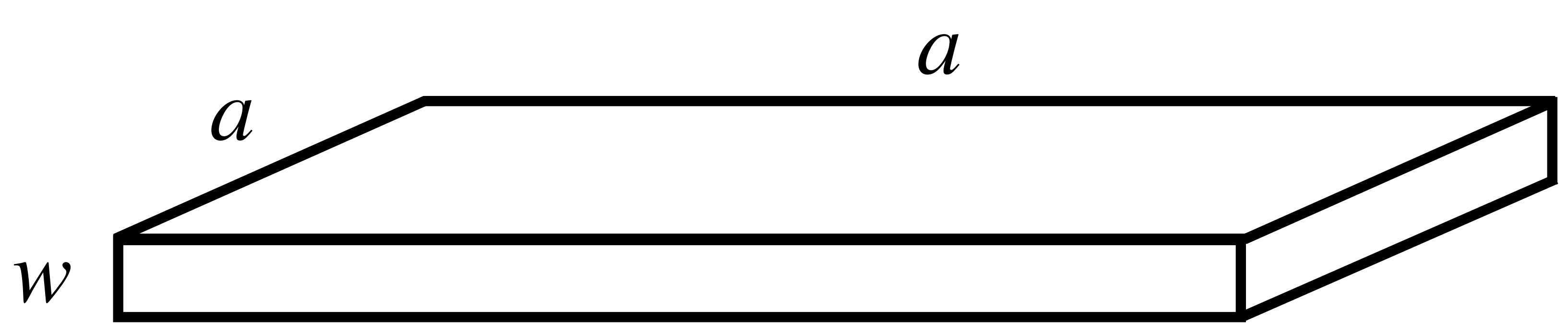
$$\left.\begin{array}{l} V = \frac{4}{3}\pi r^3 \rightarrow r = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \\ A = 4\pi r^2 \rightarrow 4\pi\left(\frac{3}{4\pi}\right)^{\frac{2}{3}} \end{array}\right\} \frac{V}{A} = \frac{r}{3} = 0.207$$

$$t_{sphere} = 0.043C$$

$$t_{sphere} > t_{cylinder} \approx t_{square}$$

Especially important as it comes to riser design because the riser must solidify after the part, but you also don't want to waste volume and energy!

Volume / Surface Area



$$\left.\begin{aligned} V &= a^2 w \\ A &\approx 2a^2 \end{aligned}\right\} \quad \frac{V}{A} = \frac{w}{2}$$

for thin sheets where 2 sides dominate the surface area

Solidification in Die Casting

⚠️ the mold is metal too

Mold: Metal heat flow: primary resistance dominated by the mold-metal interface

$$q_{x=0} = -h(T_{melt} - T_0)$$

note: heat flux is constant, whereas w/sand it varied over time

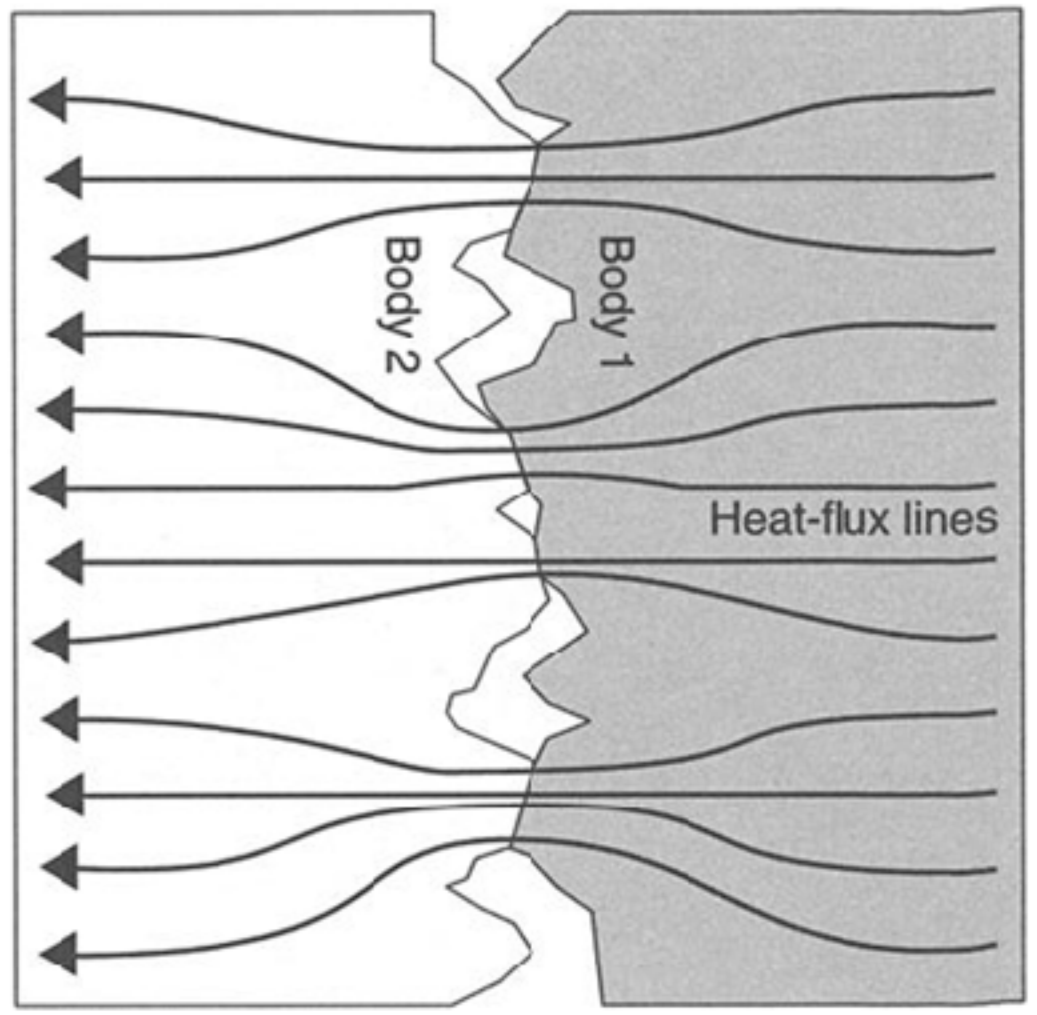
initial condition: mold starts at T_0

boundary condition: dictated by coefficient h ("convection-like")

"semi-infinite" mold: outside does not change temp.

heat transfer coefficient (film thickness)

all surfaces have imperfections!



reference: see Flemings: Solidification Processing

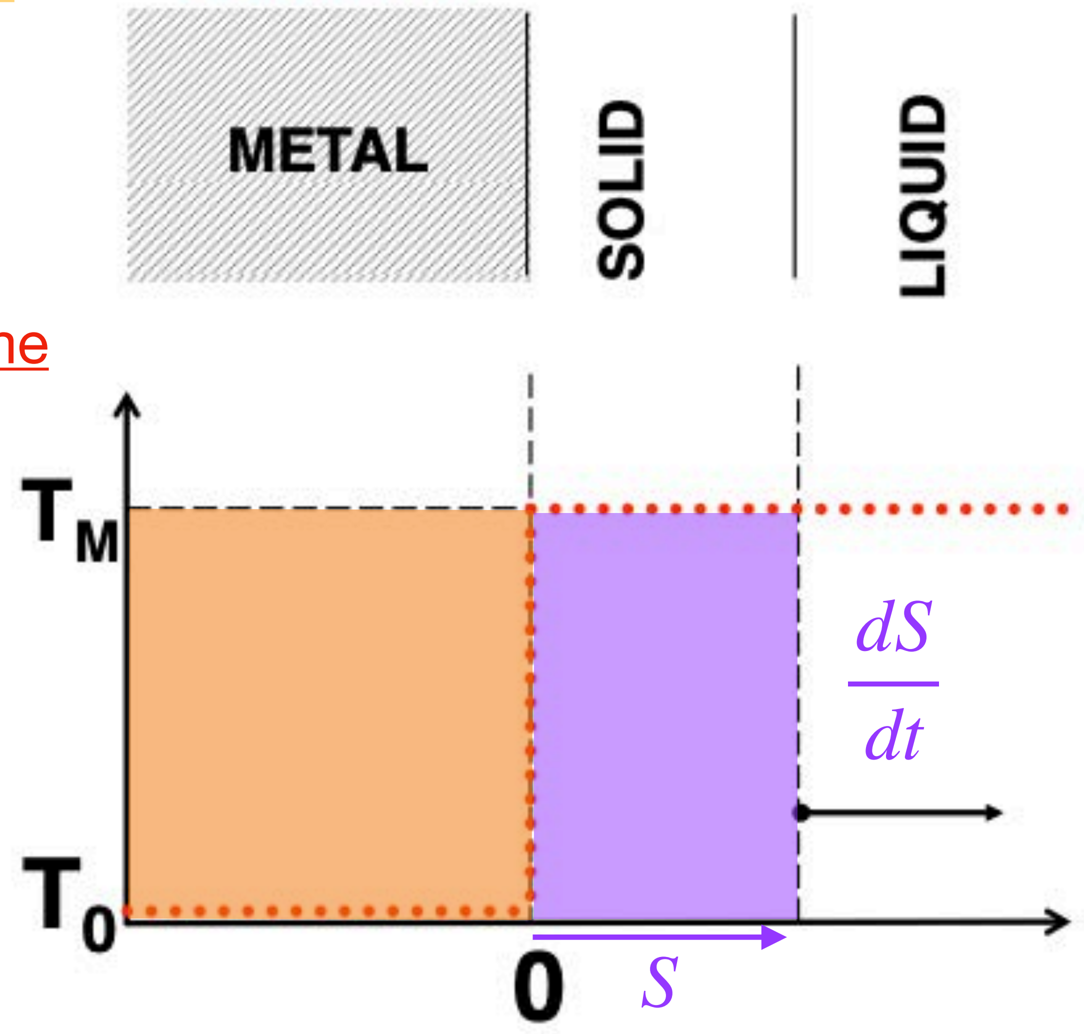
$$q_{out} = -h(T_{melt} - T_0)$$

influenced by roughness, gaps, material properties (conductivities), hardness, etc. at the operating temperatures

Type	*h* (W/m²K)
Natural convection	1
Flowing air	50
Carbon coating	1,000
High pressure	5,000
Polished die	10,000

α_{steel} ~0.1 cm²/s

$\alpha_{aluminum}$ ~0.9 cm²/s



T_0: ambient/mold starting temp
T_{melt}: melt temperature of the metal part
t: time
S: distance to solidification front
H: latent heat of fusion for metal part
ρ_s: density of the solid metal part
h: heat transfer coefficient between the part and the mold (film thickness)

$$at\ boundary\ (x=0): q_{out} = q_{in}$$

$$q_{in} = -\rho_s H \frac{dS}{dt}$$ heat flux from solidification

$$q_{out} = -h(T_{melt} - T_0) = q_{in} = -\rho_s H \frac{dS}{dt}$$

$$\frac{dS}{dt} = \frac{-h(T_{melt} - T_0)}{\rho_s H}$$

(metal + interface only)

$$t_{solidify} = \frac{\rho_s H}{h(T_{melt} - T_0)} \left(\frac{V}{A}\right)$$

integrate for S and t, solve for t, set $S \rightarrow \frac{V}{A}$

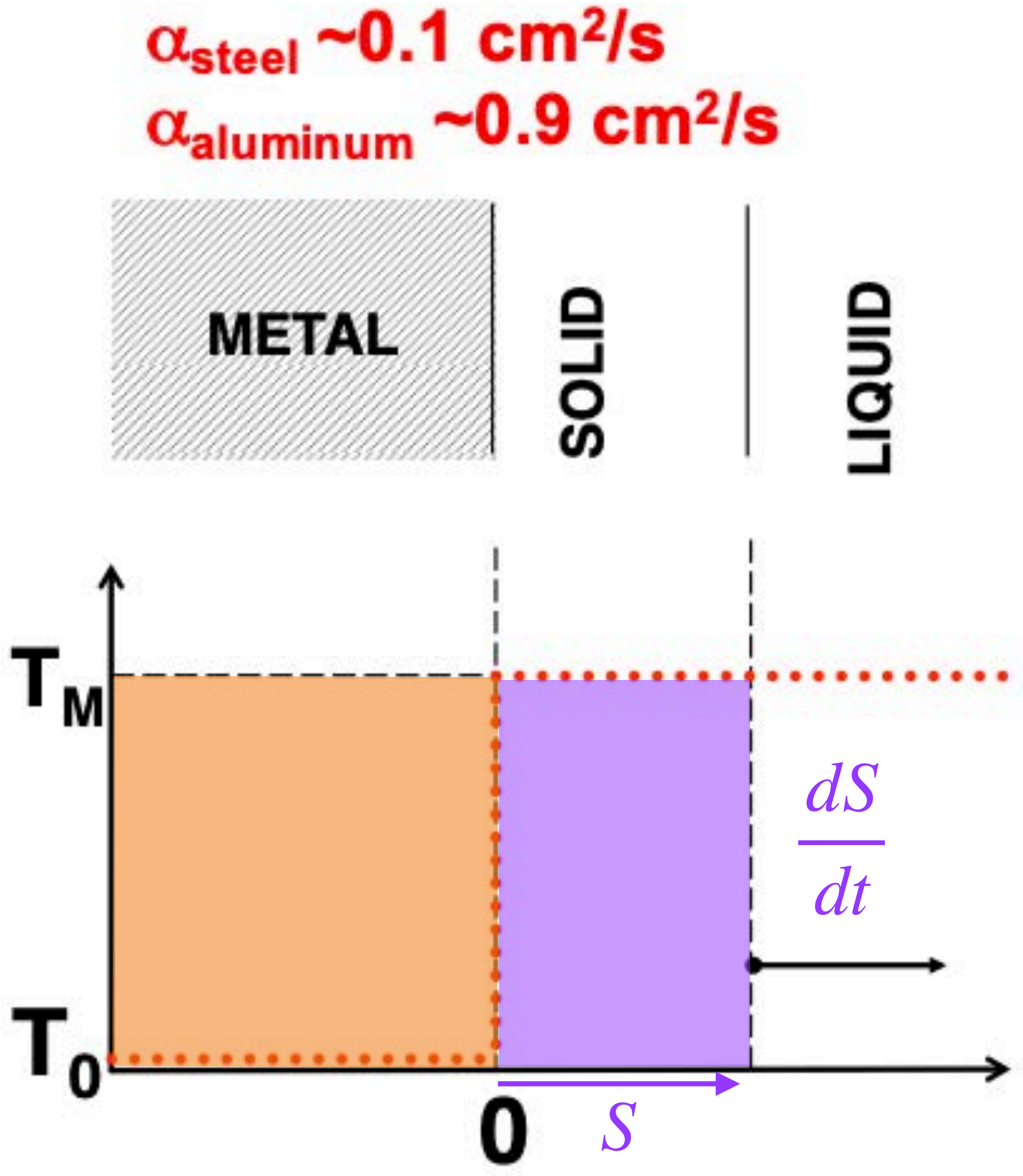
Solidification in Die Casting

$$t_{solidify} = \frac{\rho_s H}{h(T_{melt} - T_0)} \left(\frac{V}{A}\right)^1$$

1 vs 2 (vs sand casting)

constant C (also determined by experiment)

effective thickness



$$t_{solidify,sand} = C\left(\frac{V}{A}\right)^2$$

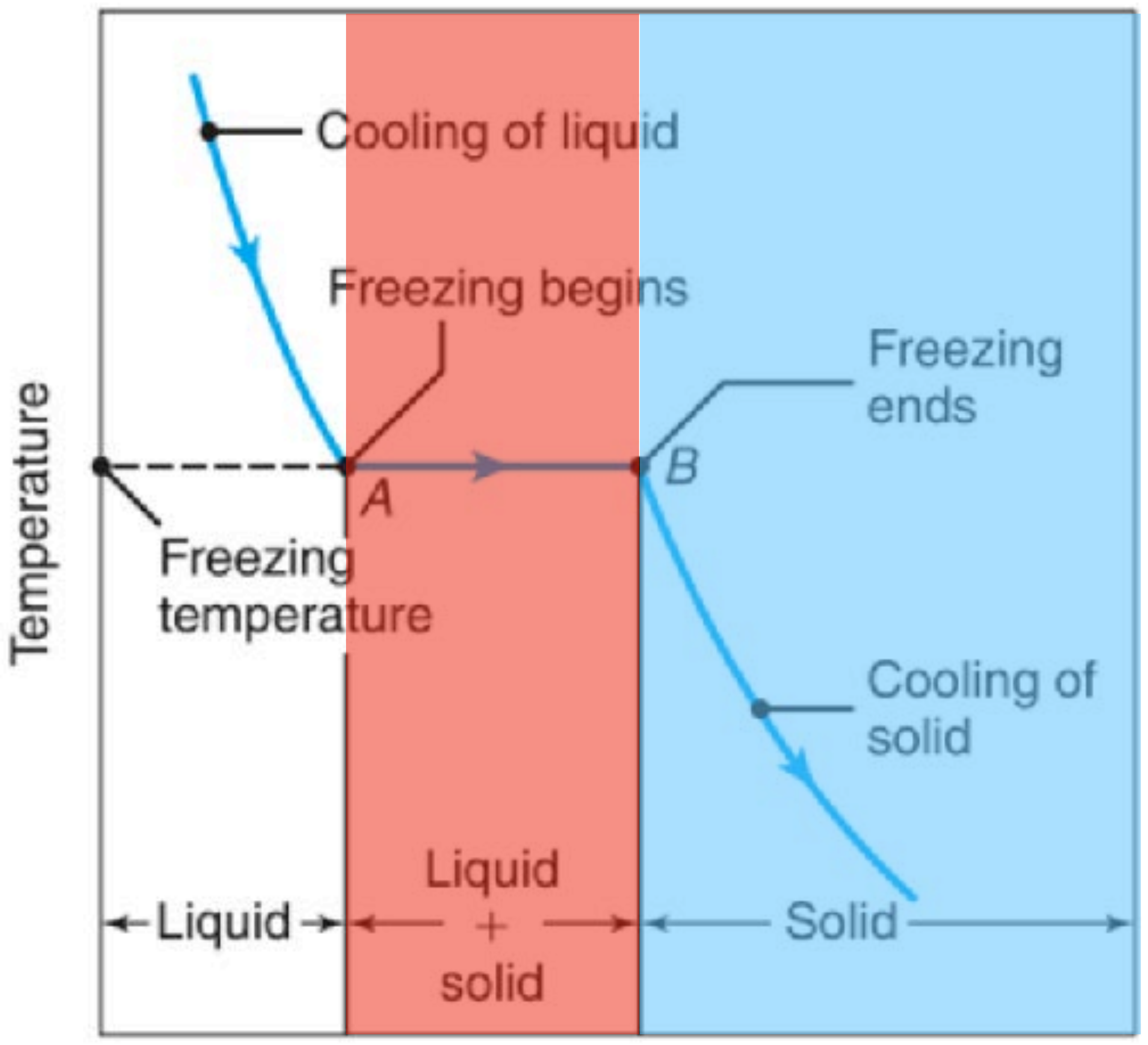
Key Takeaways

solidify time is linear: happens at a constant rate

this analysis is for a wall, what about convex/concave surfaces? doesn't matter

solidification is determined by the interface

Solid Cooling Time in Die Casting



$t_{liquid\ cool}$ $t_{solidify}$ $t_{solid\ cool}$

Chvorinov's Rule

$$t_{solidify} \neq t_{cool} \neq t_{cycle}$$

$$t_{cycle} \approx t_{solidify} + t_{cool}$$

Solid Cooling $at\ boundary\ (x = 0) : q_{out} = q_{in}$

$$-Ah(T - T_0) = m_s c_s \frac{dT}{dt}$$

$\theta = (T - T_0)$

$$-Ah\theta = m_s c_s \frac{d\theta}{dt}$$

separation of variables

$$\frac{1}{\theta} d\theta = -\frac{Ah}{m_s c_s} dt$$

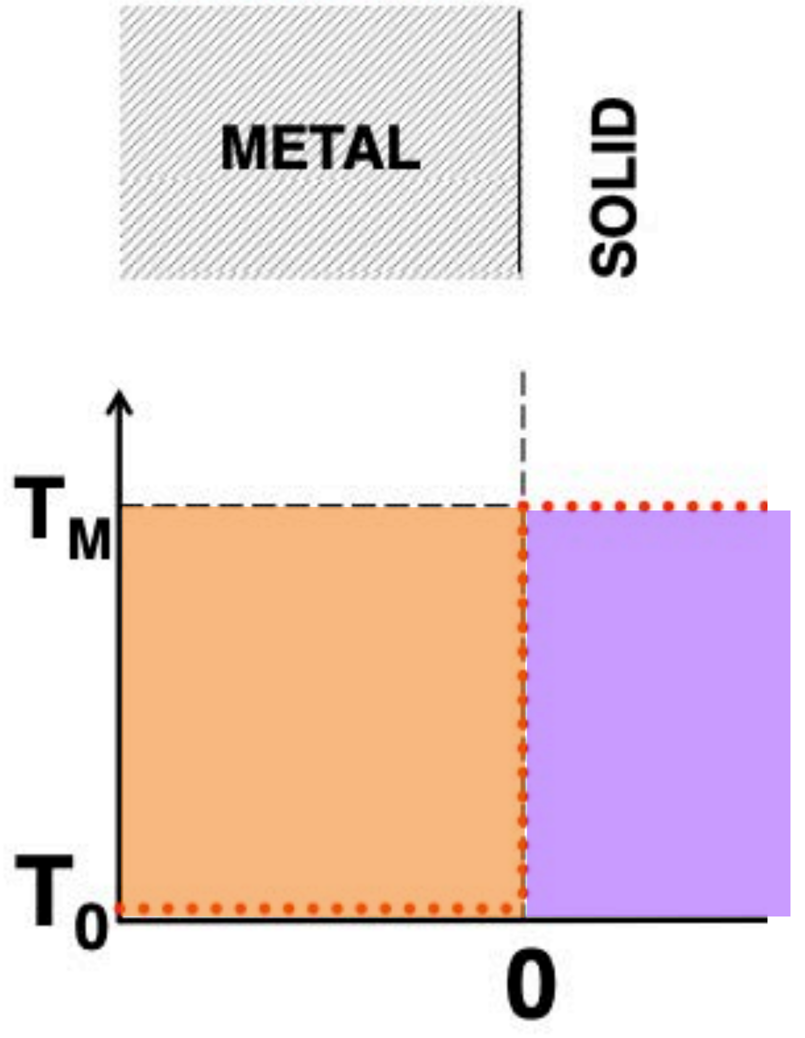
integrate

$$ln\left(\frac{\theta_f}{\theta_i}\right) = -\frac{Ah}{m_s c_s} t$$

$$t = \frac{\rho_s V c_s}{Ah} ln\left(\frac{T_{melt} - T_0}{T_{eject} - T_0}\right)$$

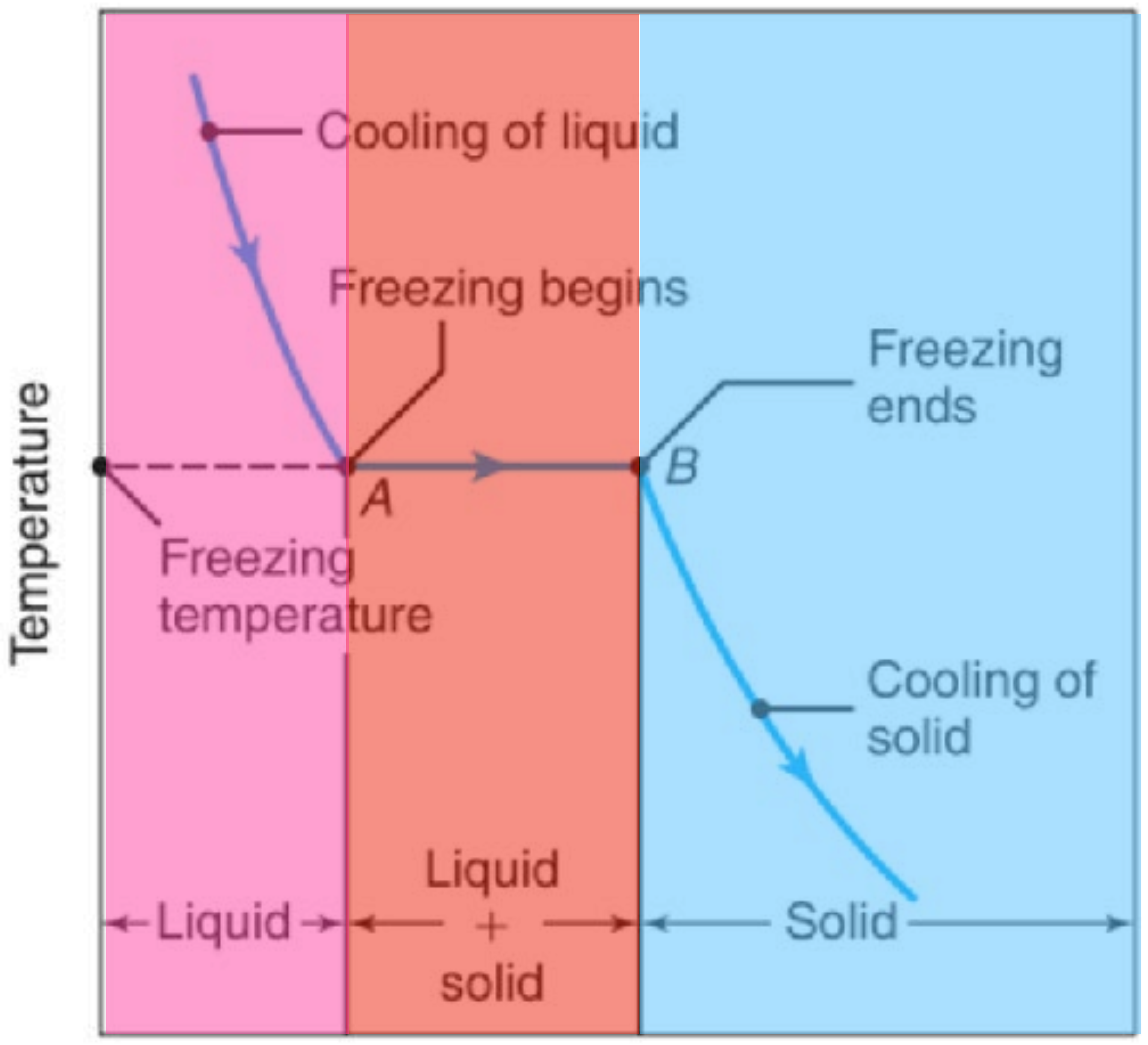
if thin sheet cooled on both sides: $\frac{V}{A} = \frac{w}{2}$

$$t_{solid\ cool} = \frac{\rho_s c_s w}{2h} ln\left(\frac{T_{melt} - T_0}{T_{eject} - T_0}\right)$$



T_0: ambient/mold starting temp
T_{melt}: melt temperature of the metal
T: temperature of the mold
t: time
m_s: mass of the solid metal part
c_s: heat capacity of the solid metal part
A: surface area of the interface
V: volume of the solid metal part
w: thickness of the solid metal part
h: heat transfer coefficient

Solid Cooling Time in Die Casting



$t_{liquid\ cool}$ $t_{solidify}$ $t_{solid\ cool}$

Chvorinov's Rule

$$t_{solidify} \neq t_{cool} \neq t_{cycle}$$

$$t_{cycle} \approx t_{solidify} + t_{cool}$$

$$t_{solid\ cool} = \frac{\rho_s c_s w}{2h} ln\left(\frac{T_{melt} - T_0}{T_{eject} - T_0}\right)$$

improves fluidity

$$t_{solid\ +\ liquid\ cool} = \frac{\rho_s c_s w}{2h} ln\left(\frac{T_{melt} + \Delta T_{superheat} - T_0}{T_{eject} - T_0}\right)$$

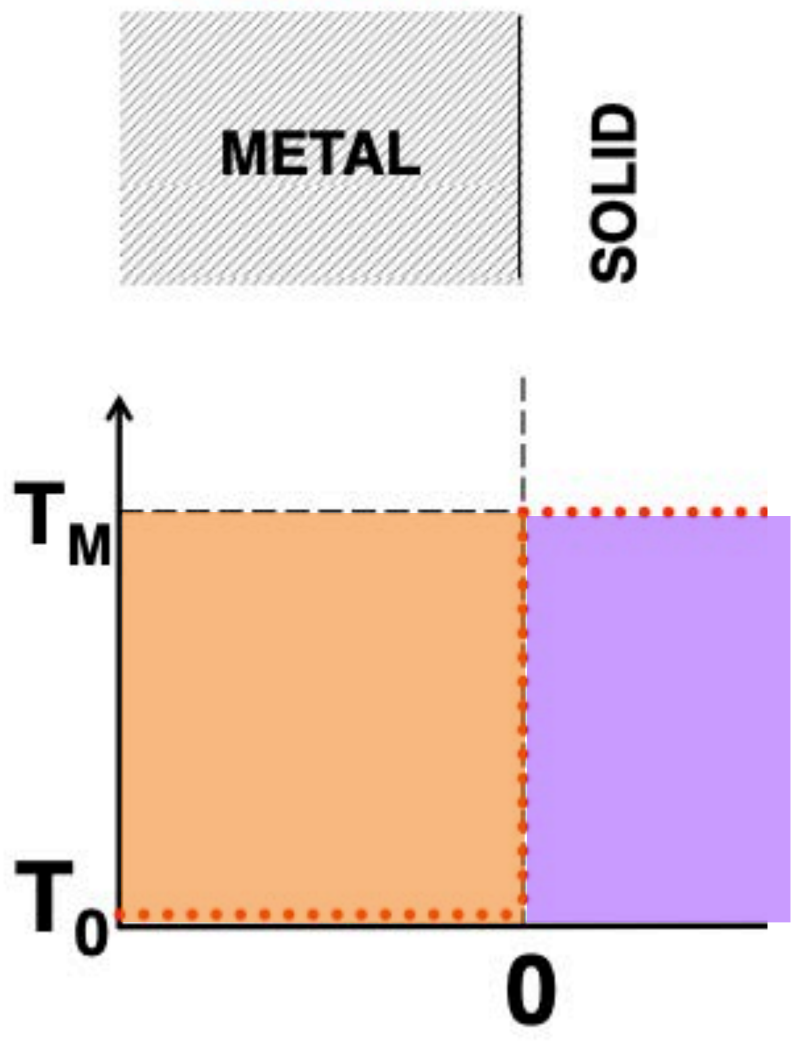
⚠️ sometimes the "superheat" adjustment is used to estimate solidification

Total Cycle Time Approximation

$$t_{solid\ +\ solidification} = \frac{\rho_s c_s w}{2h} ln\left(\frac{T_{melt} + \Delta T_{superheat} - T_0}{T_{eject} - T_0}\right)$$

$$where\ \ \Delta T_{superheat} = \frac{H}{c_s}$$ (temp. change equivalent of phase change)

⚠️ estimating the solidification time this way includes an assumption that the heat flux is changing as the temperature is changing, which is slightly different from when we derived Chvorinov's rule and the heat flux was constant due to the phase change



- T_0: ambient/mold starting temp
- T_{melt}: melt temperature of the metal
- T: temperature of the mold
- t: time
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- c_s: heat capacity of the solid metal part
- A: surface area of the interface
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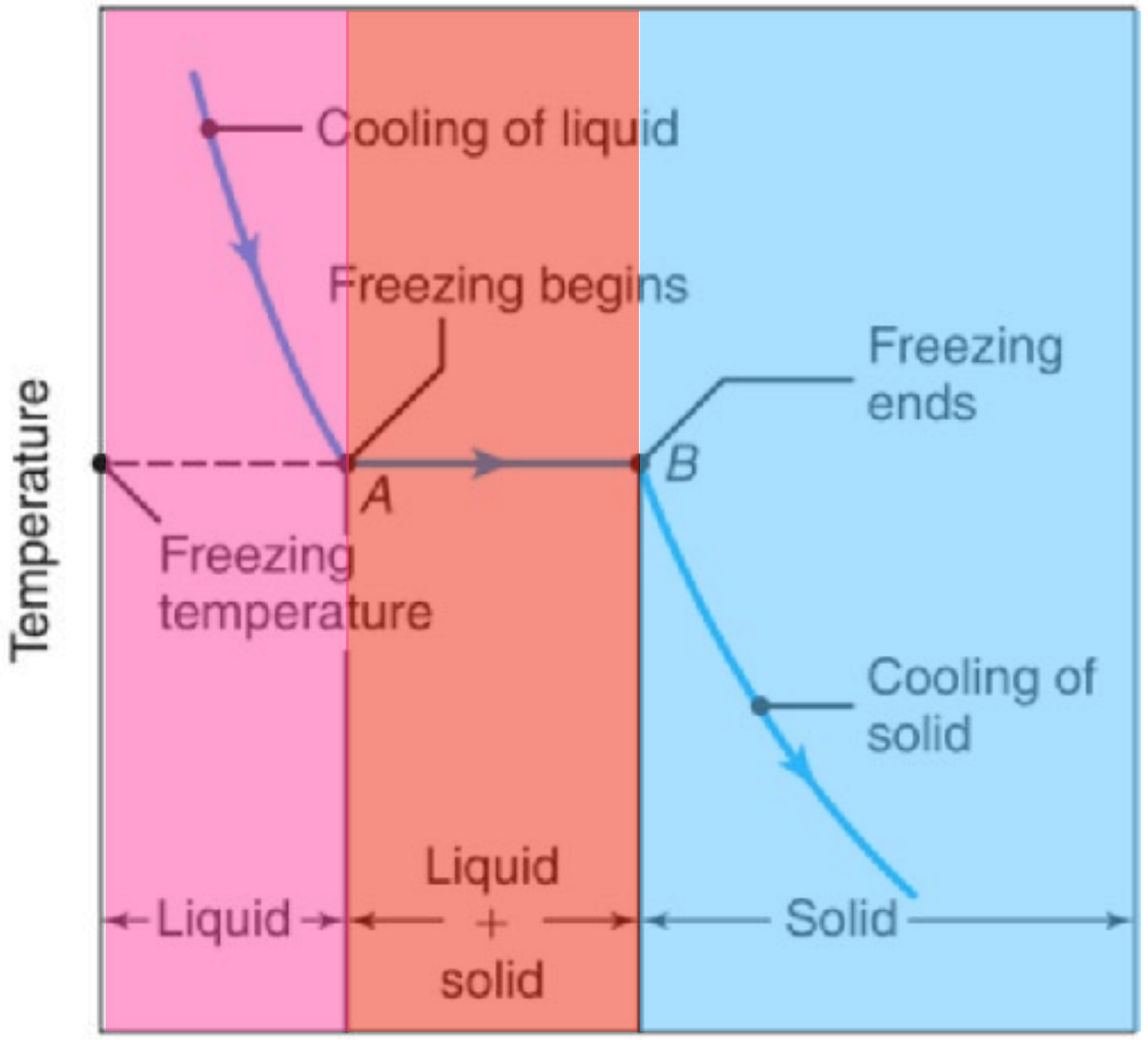
- w: thickness of the solid metal part
- h: heat transfer coefficient

Solid Cooling Time in Die Casting



$t_{liquid\ cool}$ $t_{solidify}$ $t_{solid\ cool}$

Chvorinov's Rule

$$t_{solidify} \neq t_{cool} \neq t_{cycle}$$

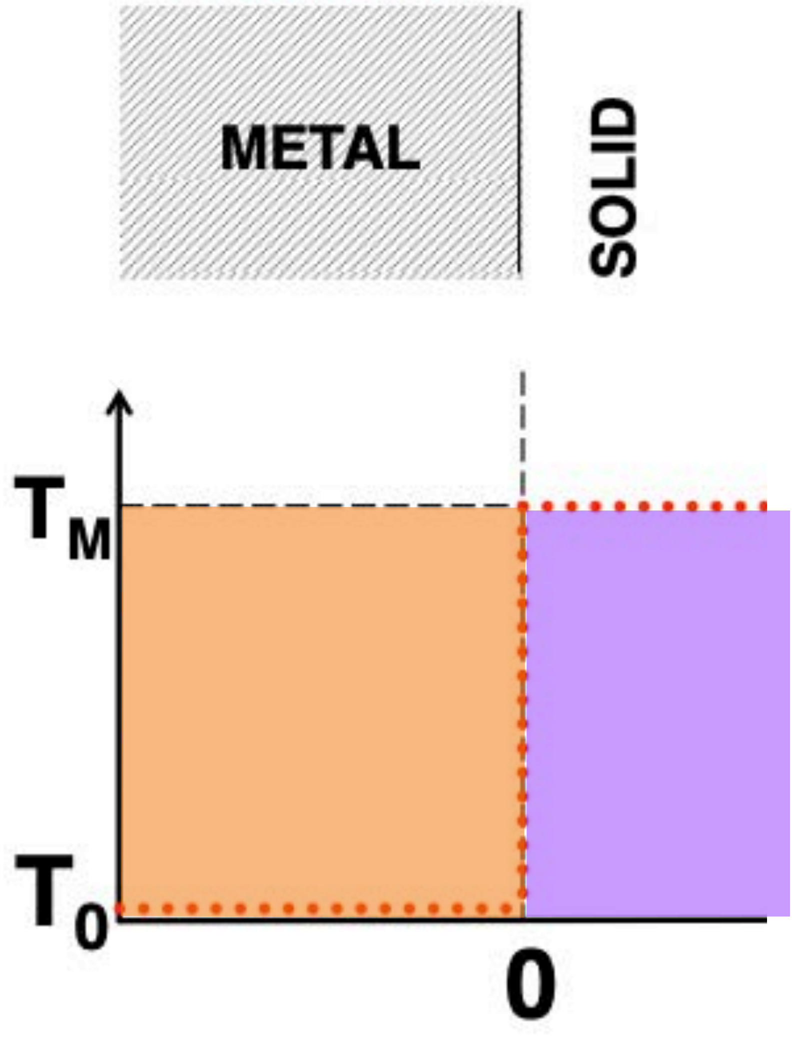
$$t_{cycle} \approx t_{solidify} + t_{cool}$$

cycle time for given thickness	
slow	$Copper: t = 0.63 \frac{sec}{mm} * w$
	$Aluminum: t = 0.47 \frac{sec}{mm} * w$
	$Zinc: t = 0.42 \frac{sec}{mm} * w$
fast	$Magnesium: t = 0.31 \frac{sec}{mm} * w$

<u>Total Cycle Time Approximation</u>

$$t_{solid\ +\ solidification} = \frac{\rho_s c_s w}{2h} ln\left(\frac{T_{melt} + \Delta T_{superheat} - T_0}{T_{eject} - T_0}\right)$$

$$where\ \Delta T_{superheat} = \frac{H}{c_s}$$ (temp. change equivalent of phase change)



- T_0: ambient/mold starting temp
- T_{melt}: melt temperature of the metal
- T: temperature of the mold
- t: time
- m_s: mass of the solid metal part
- c_s: heat capacity of the solid metal part
- A: surface area of the interface
- V: volume of the solid metal part
- w: thickness of the solid metal part
- h: heat transfer coefficient

02

Example Casting from 2.810

chassis top

chassis bottom

vs Injection Molding

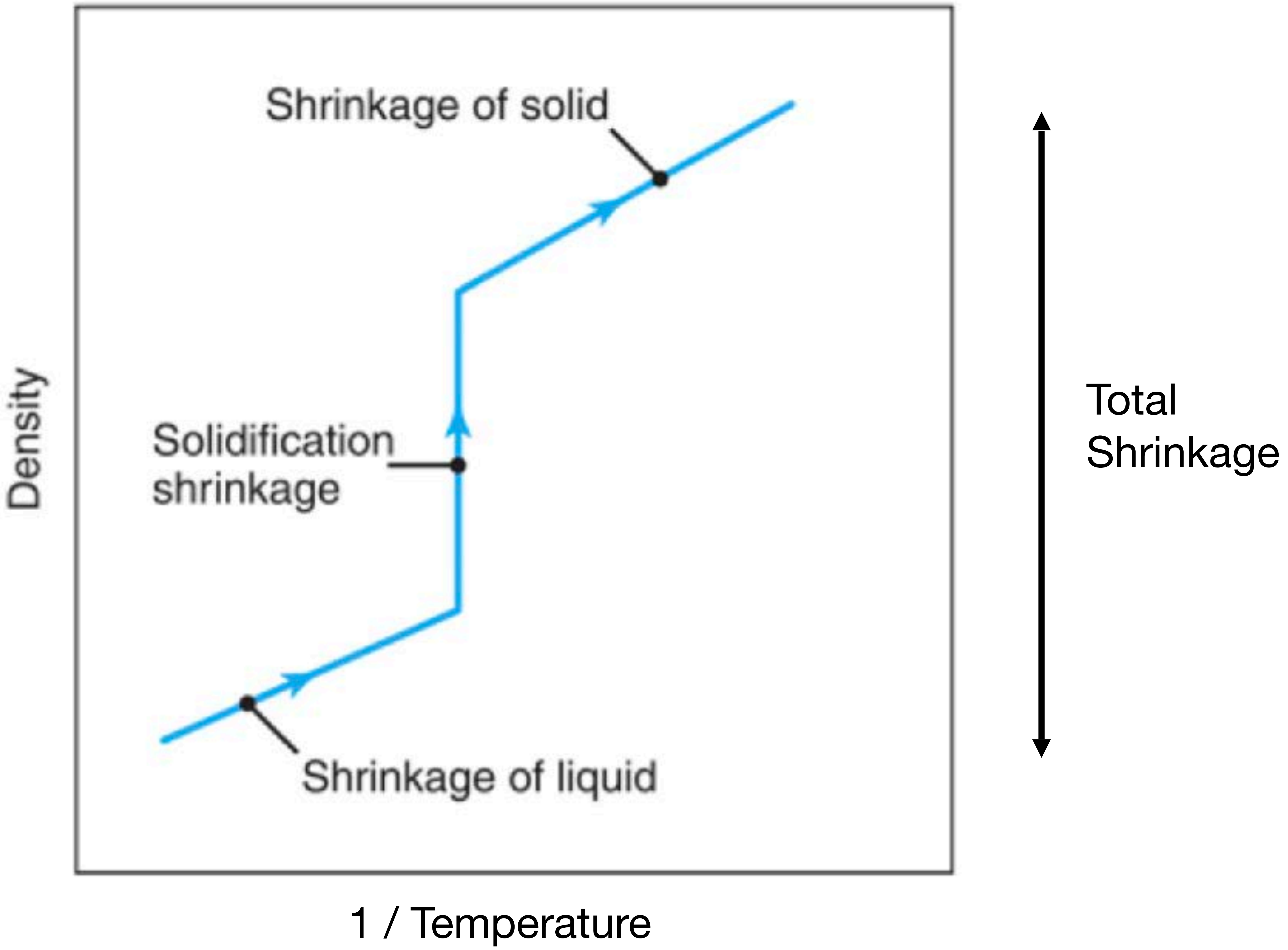
runner size?

riser?

expected flow?

Shinkage

Conservation of Mass: $\uparrow \rho \; means \; \downarrow V$



solid shrinkage: unavoidable and must be accounted for in final dimensions

liquid and solidification shrinkage: possible to counteract with proper riser design

Fluid Flow in Sand Casting

susceptible to turbulent flow: molten metal's viscosity is low

$$Re = \frac{\rho v_{run} L_c}{\mu}$$

oxidation, mold erosion, porosity, etc.

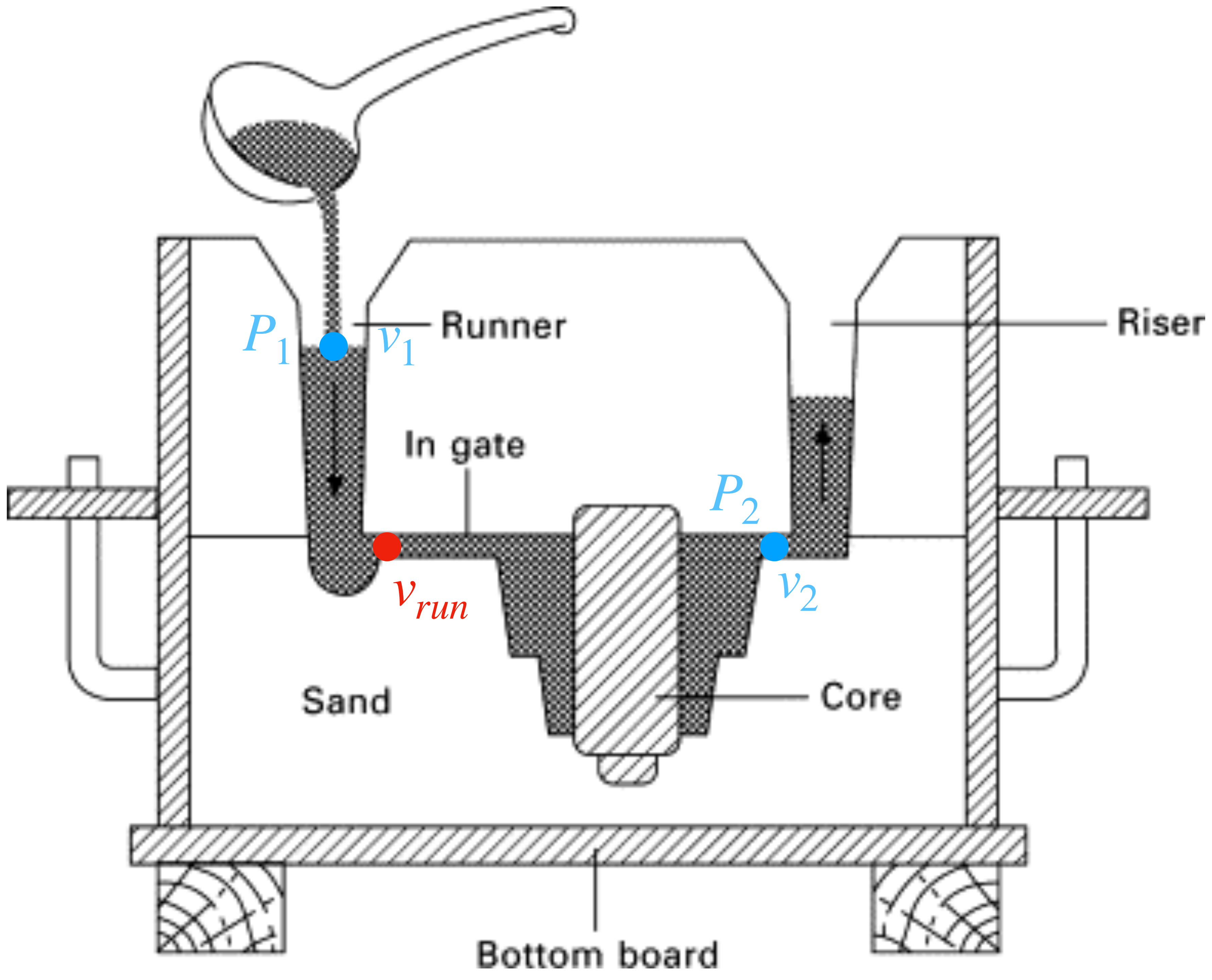
Bernoulli's Principle $P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 - frictional\ losses = P_2 + \frac{\rho v_2^2}{2} + \rho g h_2$

during flow: $P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 - frictional\ losses = P_2 + \frac{\rho v_2^2}{2} + \rho g h_2 \rightarrow$ $v_{run} = \sqrt{2gh_1}$

(gravity induced flow; reference)

after flow: $P_1 + \frac{\rho v_1^2}{2} + \rho g h_1 - frictional\ losses = P_2 + \frac{\rho v_2^2}{2} + \rho g h_2 \rightarrow$ $\Delta P_{static\ pressure} = \rho g h_1$

(no flow; no flow; reference)



where does the **clamping force** come from? The weight of the cope:

$$F_{clamp} = \Delta P A_{proj} = m_{cope} g = \rho V_{cope} g$$

Energy Contributions

for metal part + sprue/runners/riser:

$$E_{heat} = mc\Delta T = \rho Vc\Delta T$$

$$E_{melt} = mH = \rho VH$$

$$E_{total} = \rho V\Delta T + \rho VH$$

$$\boxed{E_{total} = \rho V(c\Delta T + H)}$$

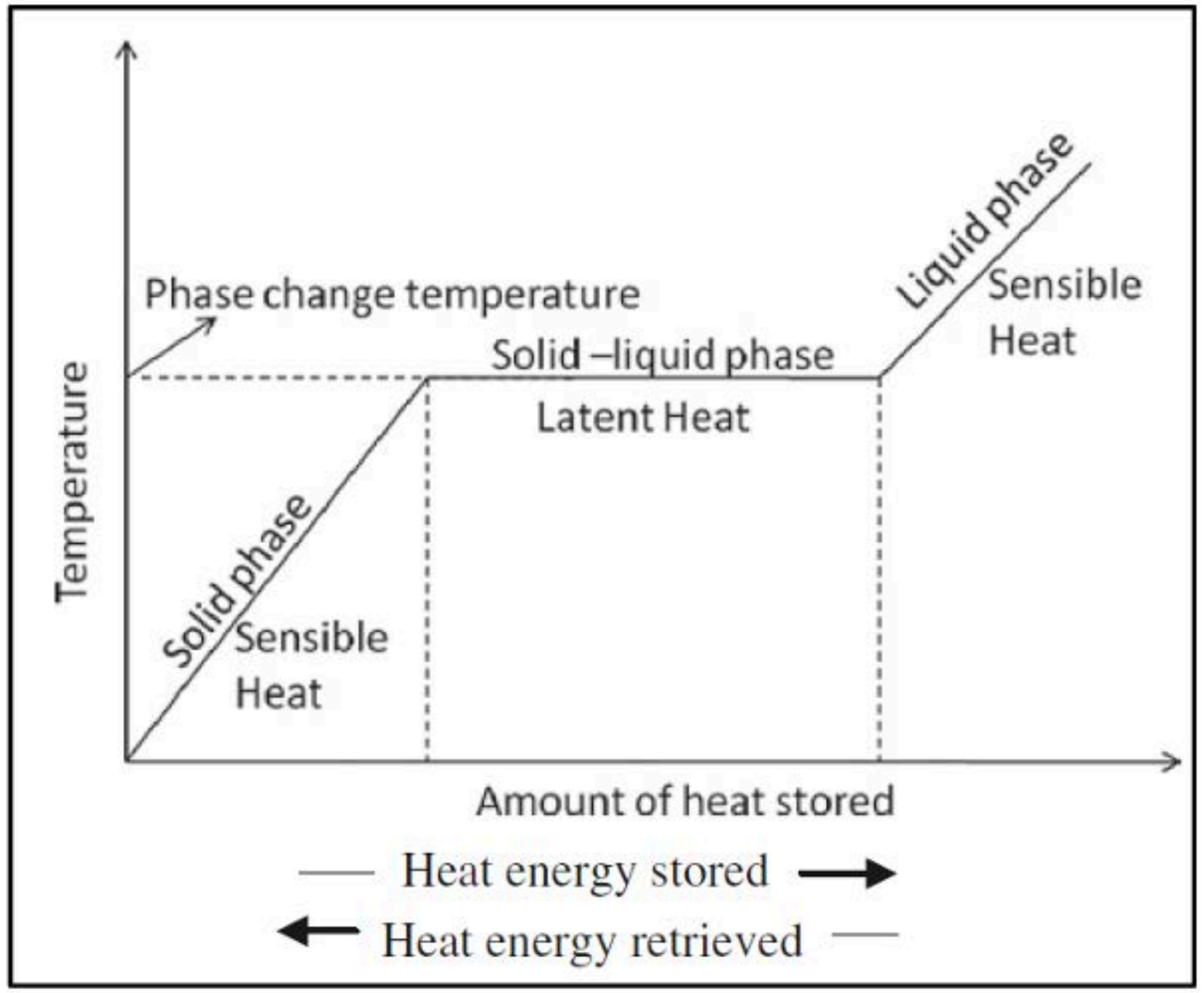


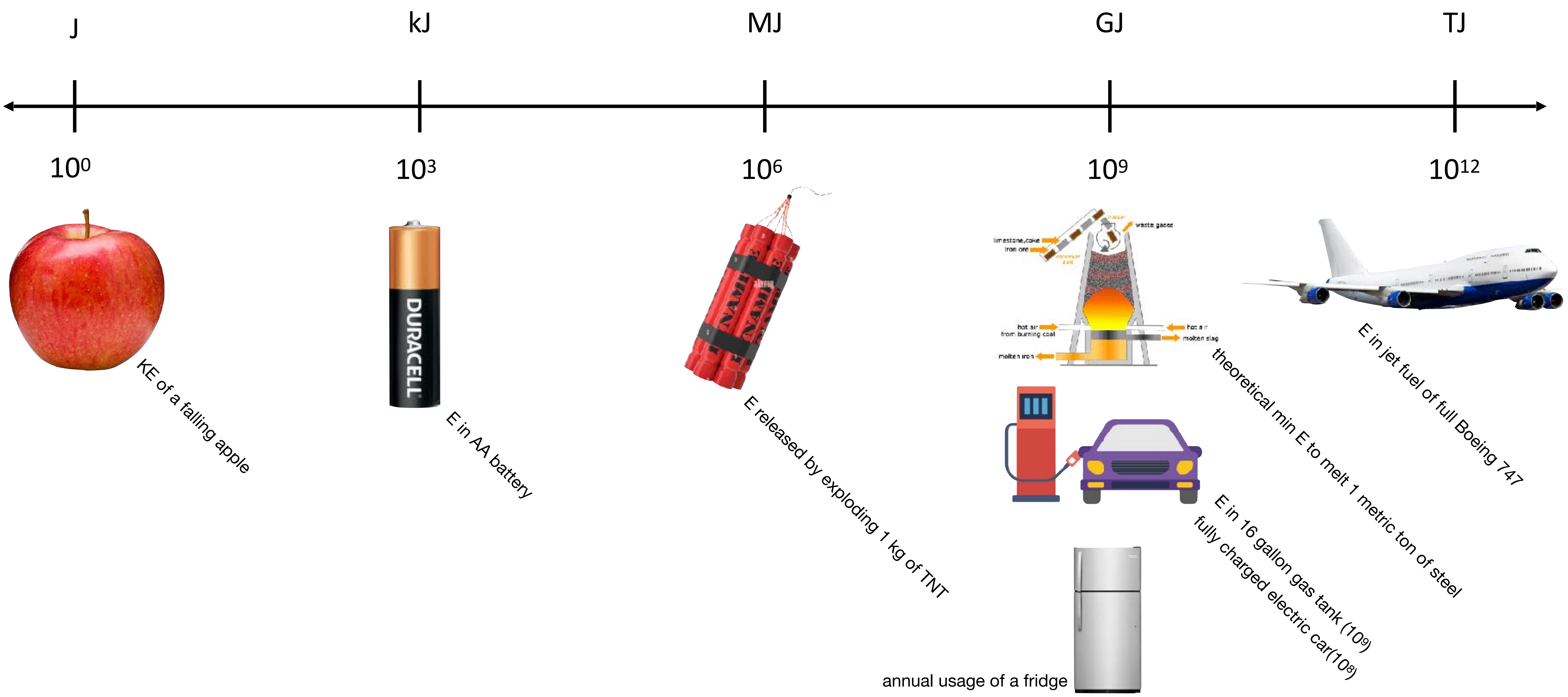
Fig-1: Principle of latent heat storage.

⬆temperature improves fluidity, but ⬆temp also ⬆cost

(requires more energy and takes longer to solidify + cool)

Shrinkage and Energy Calculations

Casting



Net Shape Considerations

Machining process – conventional mechanical processing

- Cutting of raw material
- Chipping of material to create the rough form
- Final machining by lathing, milling and drilling

Blank weight: **10.2 kg**
Finished part: **1.7 kg**
Material utilization: about **17%**

Manufacturing a casted part with subsequent mechanical processing

- Casting
- Final machining by lathing, milling and drilling

Blank weight: **2 kg**
Finished part: **1.5 kg**
Material utilization: **75%**

Costs advantages in the production of casting parts are achieved starting at a production quantity of 43 components.

Defects

Shrinkage and Energy Calculations

Casting

Image Credits

Slide 2:

Slide 20: