

# MIT 2.008 Design and Manufacturing II

Spring 2024  
March 20th, 2024

- Closed Book
- All work for CREDIT must be completed in this quiz document
- You are allowed one double-sided, hand written 8.5" x 11" notes sheet
- Calculators are allowed, and we have provided them in the room. Please return them at the end of the exam.

## General Notes

- *For qualitative answers, we're not looking for long essays. Please answer using short (1-2 sentence per answer) bullet points.*
- *For quantitative answers, show your work as clearly as possible. When possible, keep answers in algebraic form until plugging in numbers at the very end; this way, it is much easier for graders to understand where you make mistakes and provide meaningful feedback (**and partial credit**).*

Name: \_\_\_\_\_

|              |  |                   |
|--------------|--|-------------------|
| Problem 1    |  | Out of 12 points  |
| Problem 2    |  | Out of 28 points  |
| Problem 3    |  | Out of 8 points   |
| Problem 4    |  | Out of 24 points  |
| Problem 5    |  | Out of 28 points  |
| <b>Total</b> |  | <b>100 points</b> |

**Problem 1 - Manufacturing Relationships (12 points)**

**2 points each for parts a-f.**

For the following prompts, indicate the correct choice (highlighted) and provide a brief rationale.

- a) Amorphous materials are (**more/less**) prone to warpage compared to crystalline polymers.

Brief Rationale

Crystalline polymers experience a sharp drop in specific volume at melt temp, which means a higher chance of uneven cooling/thermal gradients than the smoother changes in amorphous polymers.

- b) The heating time for a plastic sheet to be thermoformed depends on (**thermal diffusivity, absorption coefficient**).

Brief Rationale

Polymers experience radiative heating for thermoforming, not conductive heating.

- c) You are turning a part on a lathe with a given spindle speed, feed, depth of cut and positive rake angle. If you decrease the rake angle, then chip thickness (**decreases / stays the same / increases**).

Brief Rationale

Smaller rake angle leads to more shearing of the material, and the chip thickness increases. A tradeoff, however, is that this also makes discontinuous chip formation more likely.

- d) Increasing the frequency at which you take samples and calculate sample means would decrease the gap between your UCL and LCL. **True / False**

Brief Rationale

The gap between UCL and LCL gets smaller as the subgroup size for samples increases, but the frequency of taking these samples does not affect the limits. What it does do is make it less likely that poor quality parts will escape detection, since you will catch losses in control more quickly.

- e) Your likelihood of systematic error (**increase/decreases/stays the same**) as  $C_{pk}$  gets closer to  $C_p$ .

Brief Rationale

$C_{pk}$  closer to  $C_p$  implies that the manufacturing process mean is closer to the center between the specification limits, which means that variations are more likely random instead of due to an assignable, controllable cause.

- f) UCL/LCL are parameters that are provided by designers to manufacturers to ensure parts are always within specifications. **True / False**

Brief Rationale

UCL/LCL are indicators of the consistency of the manufacturing process. USL/LSL are the design parameters which reflect the functional requirements of the part being made.



*Figure 1 - Disassembled components of Surebonder glue gun*

For Problems 2-4, you will inspect the Surebonder glue gun, analyzing the considerations of manufacturing the nozzle, packaging, and main housing. A view of disassembled components (other than the packaging) is shown above.

Note: physical examples are provided for your reference, but you should not need to disassemble the glue gun in order to answer any of the questions. The blue tape can be pulled back to access the parts in question. The yellow tape should not be removed, to keep the supplementary electronics in place. All parts referenced in the problems should be easily removable. If you have any questions, please ask. Please try to reassemble anything you inspect at the end of the quiz, but if you are tight on time the course staff are happy to assist you.

### Problem 2 - Cutting (28 points)

Figure 2 shows one of the nozzles of the SureBonder glue gun. The nozzle gets machined out of a  $D_0 = 0.5''$  maximal diameter hexagonal aluminum rod.

Please keep in mind the area of a hexagon is  $A_{hex} = \frac{3\sqrt{3}}{2}s^2$ ,

where  $s$  is the side length, which is half the maximal diameter.

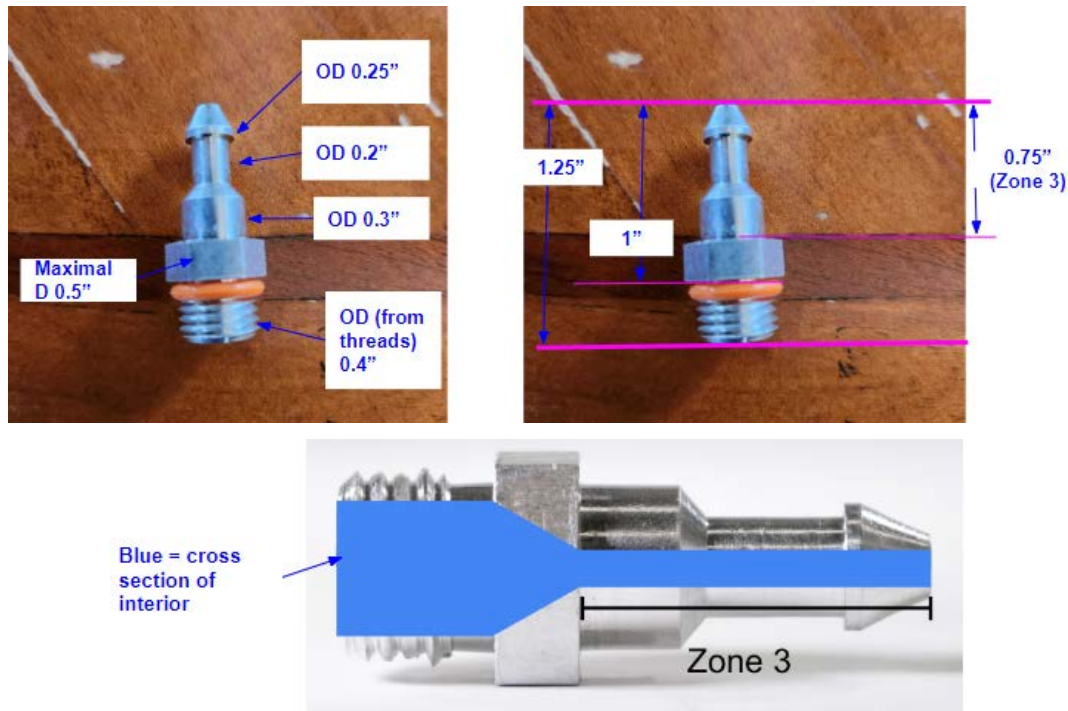


Figure 2 - Nozzle Diameter Dimensions

TABLE 21.2

**Approximate Range of Energy Requirements in Cutting Operations at the Drive Motor of the Machine Tool (for Dull Tools, Multiply by 1.25)**

| Material                | Specific energy<br>$W \cdot s/mm^3$ |
|-------------------------|-------------------------------------|
| Aluminum alloys         | 0.4–1                               |
| Cast irons              | 1.1–5.4                             |
| Copper alloys           | 1.4–3.2                             |
| High-temperature alloys | 3.2–8                               |
| Magnesium alloys        | 0.3–0.6                             |
| Nickel alloys           | 4.8–6.7                             |
| Refractory alloys       | 3–9                                 |
| Stainless steels        | 2–5                                 |
| Steels                  | 2–9                                 |
| Titanium alloys         | 2–5                                 |

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We will consider only the first orthogonal cutting step of the process plan. This starts with raw hex stock and turns down to 0.3" cylindrical diameter for the entire length of Zone 3 through a single pass. The spindle speed N is 1200 RPM and the feed f is 0.025" per revolution.

a) Given the conditions described above, how long will it take to complete the operation? (4 points)

$$\text{Linear speed along part: } v = Nf = 1200 \frac{\text{rev}}{\text{min}} * \frac{1 \text{ min}}{60 \text{ s}} * \frac{0.025 \text{ in}}{\text{rev}} = 0.5 \text{ in/s}$$

Define Zone 3 length as L. Then,

$$\text{Time } t = \frac{L}{v} = \frac{L}{Nf} = \frac{0.75 \text{ in}}{0.5 \text{ in/s}} = 1.5 \text{ seconds}$$

b) Given your answer to part (a), calculate the MRR (in in<sup>3</sup>/s). (6 points)

Define volume removed as VR. Then,

$$MRR = \frac{VR}{t} = \frac{VR * N * f}{L}$$

$$\text{For this operation, we are converting from a hexagon to a cylinder: } VR = \frac{3\sqrt{3}}{2} \left( \frac{D_0}{2} \right)^2 (L) - \pi \left( \frac{D_f}{4} \right)^2 (L)$$

$$MRR = \frac{L}{t} \left( \frac{3\sqrt{3}}{8} D_0^2 - \frac{\pi}{4} D_f^2 \right) = Nf \left( \frac{3\sqrt{3}}{8} D_0^2 - \frac{\pi}{4} D_f^2 \right)$$

$$= 1200 \frac{\text{rev}}{\text{min}} * \frac{1 \text{ min}}{60 \text{ s}} * \frac{0.025 \text{ in}}{\text{rev}} \left( \frac{3\sqrt{3}}{8} (0.5 \text{ in})^2 - \frac{\pi}{4} (0.30 \text{ in})^2 \right) = 0.0458 \text{ in}^3/\text{second}$$

c) Calculate the minimum and maximum estimated power (in W) required to complete the operation.

Assume 1 inch = 25.4 mm. (5 points)

From table 21.2: Aluminum specific energy = 0.4 - 1 W\*s/mm<sup>3</sup>

$$\text{Power} = u * MRR = u Nf \left( \frac{3\sqrt{3}}{8} D_0^2 - \frac{\pi}{4} D_f^2 \right)$$

$$\text{Power} = 0.4 \frac{\text{W-s}}{\text{mm}^3} * \frac{(25.4 \text{ mm})^3}{\text{in}^3} * 0.0458 \frac{\text{in}^3}{\text{s}} = 300 \text{ W (Lower estimate)}$$

$$\text{Power} = 1 \frac{\text{W-s}}{\text{mm}^3} * \frac{(25.4 \text{ mm})^3}{\text{in}^3} * 0.0458 \frac{\text{in}^3}{\text{s}} = 751 \text{ W (Upper estimate)}$$

d) Estimate the minimum and maximum average cutting force (in N) during the operation. Note that for

a hexagon, the average distance of an edge from the center is  $\left( \frac{1}{4} + \frac{\sqrt{3}}{8} \right) D_0 = 0.4665 D_0$ . (7 points)

$$F_{\text{cut}} = \frac{\text{Power}}{v_{\text{cut}}} \text{ or } F_{\text{cut}} = \frac{\tau_{\text{cut}}}{r_{\text{avg}}} \Rightarrow \text{either becomes } F_{\text{cut}} = \frac{\text{Power}}{N * 2\pi * r_{\text{avg}}} = \frac{u f \left( \frac{3\sqrt{3}}{8} D_0^2 - \frac{\pi}{4} D_f^2 \right)}{2\pi r_{\text{avg}}}$$

$$r_{\text{avg}} = \frac{r_{0,\text{avg}} + r_f}{2} = \frac{0.4665 D_0}{2} + \frac{D_f}{4} = \frac{0.4665(0.5 \text{ in})}{2} + \frac{0.3 \text{ in}}{4} = 0.192 \text{ in. So,}$$

$$F_{\text{cut}} = \frac{\text{Power}}{N * 2\pi * r_{\text{avg}}} = \frac{300 \text{ W}}{1200 \frac{\text{rev}}{\text{min}} * \frac{1 \text{ min}}{60 \text{ s}} * 2\pi * 0.192 \text{ in}} * \frac{\text{in}}{25.4 \text{ mm}} * \frac{1000 \text{ mm}}{\text{m}} = 491 \text{ N (Lower estimate)}$$

$$F_{\text{cut}} = \frac{\text{Power}}{N * 2\pi * r_{\text{avg}}} = \frac{751 \text{ W}}{1200 \frac{\text{rev}}{\text{min}} * \frac{1 \text{ min}}{60 \text{ s}} * 2\pi * 0.192 \text{ in}} * \frac{\text{in}}{25.4 \text{ mm}} * \frac{1000 \text{ mm}}{\text{m}} = 1226 \text{ N (Upper estimate)}$$

e) You have been asked to reduce the power requirement needed to make this nozzle. Assuming the dimensions of the nozzle are customer specifications and cannot change, what are 3 parameters you would change that may reduce the power needed? (6 points)

Recalling the equation derived in part c,  $Power = u * MRR = uNf(\frac{3\sqrt{3}}{8}D_0^2 - \frac{\pi}{4}D_f^2)$

Since the nozzle dimensions are fixed, there are 3 parameters above which can be reduced:

- Specific cutting energy  $u$  (only magnesium has a lower  $u$  compared to aluminum in the table)
- Spindle speed  $N$
- Feed  $f$

Other acceptable answers include those from the similar question in HW2:

- Increasing the rake angle/increase shear angle
- Reducing the tool radius (by sharpening the tool edge)
- Do multiple passes/reduce depth of cut for each pass
- Add coolant/lubricant to reduce losses due to friction.

### **Problem 3 - Thermoforming (8 points)**

Now, we will consider the packaging of the glue gun.



*Figure 3 - Thermoformed Packaging*

- a) Is the packaging thermoformed on a positive or negative mold? Use the reference image below showing a rough sketch of the cross section A-A of the packaging to justify your answer. Physical examples are provided if you would like to inspect the packaging yourself. **(4 points)**

The radius of curvature at the bottom of the sheet is much smaller than the radius of curvature on the top features. The packaging as a whole also fits over the glue gun in a convex fashion. This suggests a positive mold. Vacuum hole marks are also present on the bottom surface.

- b) Retailers are complaining that the glue gun is too tight inside of the packaging. As the packaging manufacturer, you suggest that this is due to excess shrinkage during cooling. Provide 2 solutions which may resolve this issue. **(4 points)**
1. Reduce forming temperature or choose a material with a lower glass transition temp; a lower temperature drop during cooling means less shrinkage.

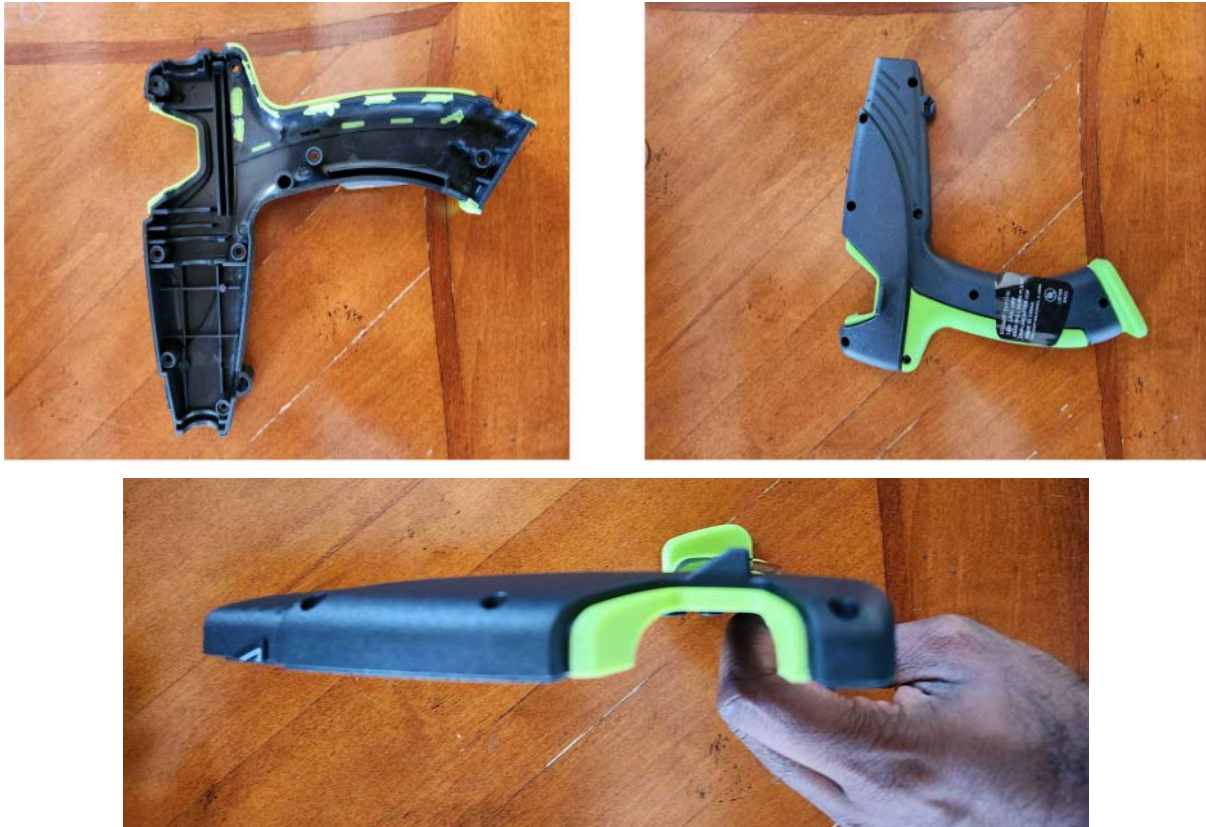


2. Increase the dimensions of the mold; this allows forming with the same degree of allowable strain (because the forming temperature is the same) on the sheet and compensates for the resulting shrinkage.
3. Use a forced cooling method such as quenching, so the part temperature can be reduced before removing it from the mold.
4. Allow the part to cool on the mold for longer.
5. Use less crystalline and more amorphous material.

**Problem 4 - Injection Molding (Glue Gun) (24 points)**

The housing shown in Figure 4 is made out of Polypropylene.

*Note: Ignore the overmolded green part; for this problem, we can treat the housing as a single part.*



*Figure 4 - Glue Gun Housing*

Note: For the purposes of this problem, you can ignore curves and small features in the housing, and instead treat its dimensions as a series of rectangular prisms with a top view as shown in Figure 5 below.

Note: Assume all dimensions provided are **exclusive** of the part thickness (other than the thickness dimension itself); in other words, they reflect the dimensions in **contact with the mold**.

Note: the part has a height of 0.7". Instead of doing the calculations yourself based on this height, you can assume that this height adds a surface area of **17in<sup>2</sup>** in contact with the side of the mold (NOT in the direction of clamping force).

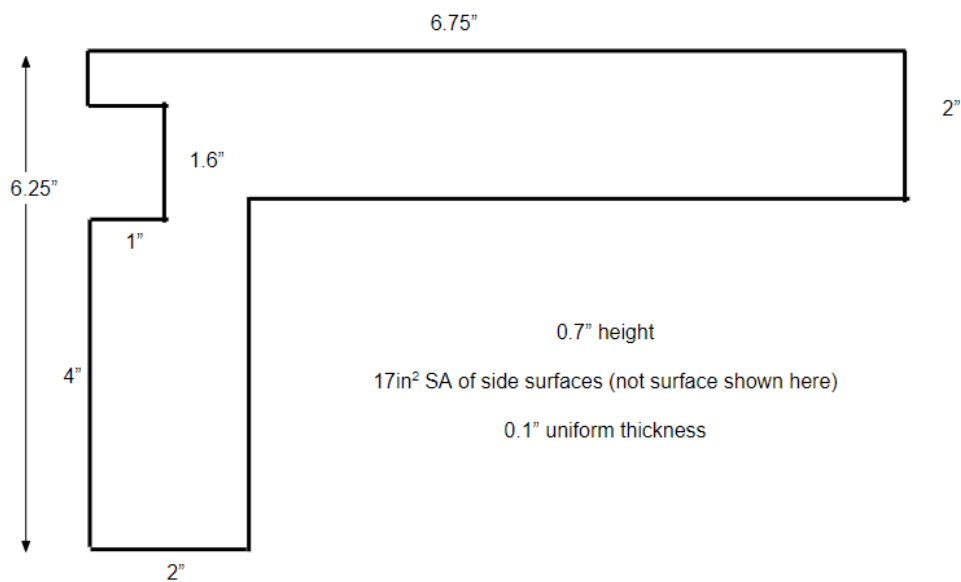


Figure 5 - Glue Gun Housing Dimensions for Purposes of the Quiz Problem

The following are the material properties for Polypropylene.

| Property                                   | Value   |
|--|---|
| Density                                    | $\rho = 1200\text{kg/m}^3 = 0.694\text{ oz/in}^3$ |
| Specific heat capacity                     | $c_p = 2130\text{ J/kg-K}$                        |
| Thermal conductivity                       | $k = 0.2\text{ W/m-K}$                            |
| Glass transition temperature               | $T_g = 108^\circ\text{C}$                         |
| Melt temperature                           | $T_m = 200^\circ\text{C}$                         |
| Viscosity of Molten Polyethylene (@ 220 C) | 600 Pa•s  |



|                             |         |
|-----------------------------|---------|
| Acceptable flow path ratios | 150-280 |
|-----------------------------|---------|

- a) Estimate the clamping force for a single part. Assume the pressure needed to injection mold this part is 30 MPa. (5 points)

$$F_{clamp} = \Delta P A$$

$$A = \text{Left section} + \text{top section} - \text{gap section.}$$

$$= (6.25''(2'') + (6.75'' - 2'')(2'') - (1.6'')(1'')) = 20.4 \text{ in}^2$$

Keep in mind that the 17in<sup>2</sup> of side surface area contacting the mold is relevant for determining the part volume, but is not relevant to clamping force which is only applied on the surface indicated in Figure 5.

$$F_{clamp} = \Delta P A = 30 \text{ MPa} * 20.4 \text{ in}^2 * \frac{(25.4 \text{ mm})^2}{\text{in}^2} * \frac{\text{m}^2}{(1000 \text{ mm})^2} = 395 \text{ kN}$$

- b) Which machine/machines are capable of producing this part? (8 points)

Note: 1kN ~ 0.113 US tons

Note: You can ignore the volume of edges; in other words, assume the part volume is just surface area in contact with the mold, times part thickness.







| Machine type/<br>Clamping force |   | Clearance<br>between tie bars   | max. shot<br>weight (PS) |
|---------------------------------|---|---------------------------------|--------------------------|
| BOY XXS<br>6.93 US tons         |    | 6.3 inch /<br>8.1 inch diagonal | 0.33 oz                  |
| BOY XS<br>11.0 US tons          |  | 6.3 inch /<br>8.1 inch diagonal | 0.26 oz                  |
| BOY 22 A<br>24.2 US tons        |  | 10 inch                         | 2.05 oz                  |
| BOY 25 E<br>27.5 US tons        |  | 10 inch                         | 2.45 oz                  |
| BOY 35 E<br>38.5 US tons        |  | 11 x 10 inch<br>(h x v)         | 2.45 oz                  |
| BOY 50 E<br>55.0 US tons        |  | 14.2 x 13.2 inch<br>(h x v)     | 2.45 oz                  |
| BOY 60 E<br>66.0 US tons        |  | 14.2 x 13.2 inch<br>(h x v)     | 5.56 oz                  |

Figure 6 - Injection Molded Machine Specifications

Which machine can make the part depends on 2 things: whether the machine can supply the necessary clamping force, and whether it has sufficient shot weight to make the entire part volume per injection.

$$F_{clamp} = 395 \text{ kN} * \frac{0.113 \text{ US ton}}{\text{kN}} = 44.62 \text{ US tons. Only the Boy 50 E and Boy 60 E can make this.}$$

$$\text{Volume } V = A_{total} * \text{thickness } h.$$

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Now, the extra 17in<sup>2</sup> in contact with the mold due to the part's height is relevant.

$$V = (20.4\text{in}^2 + 17\text{in}^2) * 0.1\text{in} = 3.74\text{in}^3$$

Possible shot weight =  $3.74\text{in}^3 * 0.694\text{oz/in}^3 = 2.59\text{oz}$ . **Only the Boy 60 E can accommodate this.**

- c) Estimate the maximum number of housings that you could produce in an hour assuming that all other cycle time contributions besides the cooling time are negligible and that  $(T_{\text{melt}} - T_{\text{mold}}) \approx 10(T_{\text{ejection}} - T_{\text{mold}})$ . **(5 points)**

Because of the assumption provided about the differences between the temperatures, we can use the simplified estimate for cooling time:

$$t_{\text{cool}} = \frac{h^2}{4\alpha} = \frac{h^2 \rho c_p}{4k} = \frac{(0.1\text{in})^2 (1200\text{kg/m}^3) (2130\text{J/kg-K})}{4(0.2\text{W/m-K})} * \frac{(\frac{25.4}{1000}\text{m})^2}{\text{in}^2} = 20.61 \text{ seconds}$$

$$\text{Parts produced in an hour} = \frac{\text{part}}{20.61\text{s}} * \frac{3600\text{s}}{\text{hr}} = 174 \text{ parts}$$

- d) Assume that multiple gates are positioned such that the longest dimension provided (6.75") is the characteristic length. Which is a bigger risk, flash or short-shot? Why? Propose 2 (qualitative) ways to fix that failure mode using molding parameters and material properties. **(6 points)**

|                              |  |
|------------------------------|--|
| <b>Bigger risk &amp; why</b> | <p>Max flow ratio <math>L/T = 6.75/0.1 = 67.5</math></p> <p>This is far below the given acceptable range of 150-280, which means the part is thick enough compared to its length to risk overrunning the mold. Therefore, flash is the bigger risk.</p>  |
| <b>Fix 1</b>                 | Reduce the thickness - a little over 60% reduction will bring the flow ratio into acceptable range   |
| <b>Fix 2</b>                 | <p>Acceptable answers may include</p> <ul style="list-style-type: none"> <li>- Reducing injection pressure</li> <li>- Reducing packing/holding pressure</li> <li>- Having fewer gates to increase characteristic length or limit weld lines where flash may occur</li> <li>- Using a material with lower acceptable flow ratios</li> <li>- Use a material with higher viscosity/lower injection temp.</li> <li>- Increase clamp force</li> </ul> |



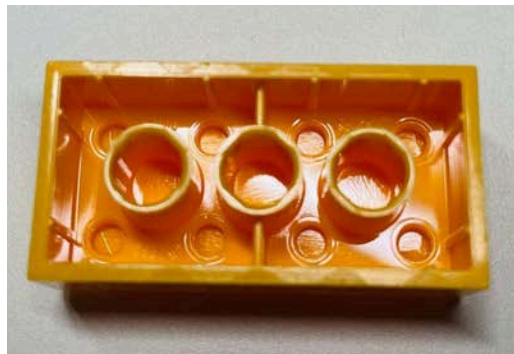
**Problem 5 - IM/Assembly/Quality (Lego Bricks) (28 points)**

You are a manufacturing engineer working for a manufacturing company that is a supplier for LEGO company. Figure 2 and 3 shows the prototype of the manufactured LEGO block and the dimensions respectively. As you answer the following questions, you must keep the following quality requirements in mind:

- i) The critical surfaces that may affect assembly of the LEGO bricks must not have imperfections/marks.
- ii) The part needs to maintain its symmetry.

The LEGO brick is made out of ABS which has the following material properties.

| Property                              | Value                        |
|---------------------------------------|------------------------------|
| Density                               | $\rho = 1100 \text{ kg/m}^3$ |
| Specific heat capacity (plastic)      | $c_p = 1700 \text{ J/kg-K}$  |
| Thermal conductivity                  | $k = 0.2 \text{ W/m-K}$      |
| Glass transition temperature          | $T_g = 105^\circ\text{C}$    |
| Melt temperature                      | $T_m = 220^\circ\text{C}$    |
| Viscosity of Molten ABS<br>(@ 220 °C) | 350 Pa•s                     |



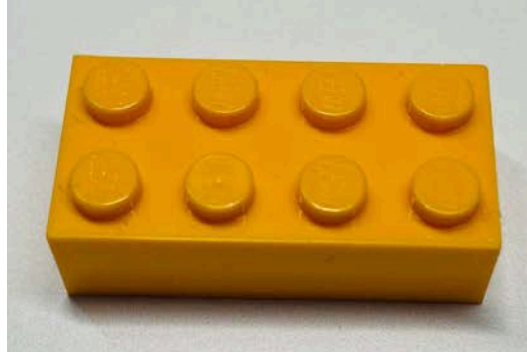


Figure 7 - Manufactured Lego Block

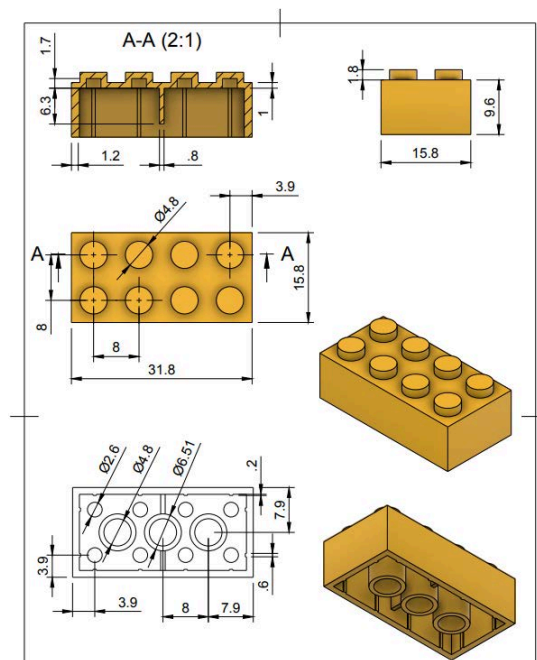


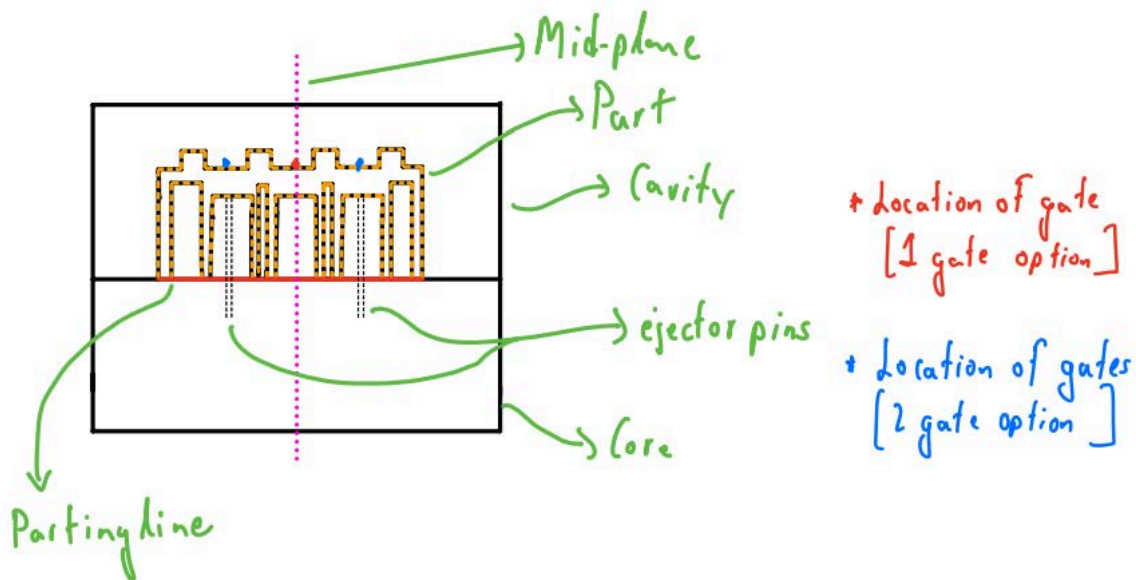
Figure 8 - Dimensions of Lego Brick Design (\*All dimensions are in mm)

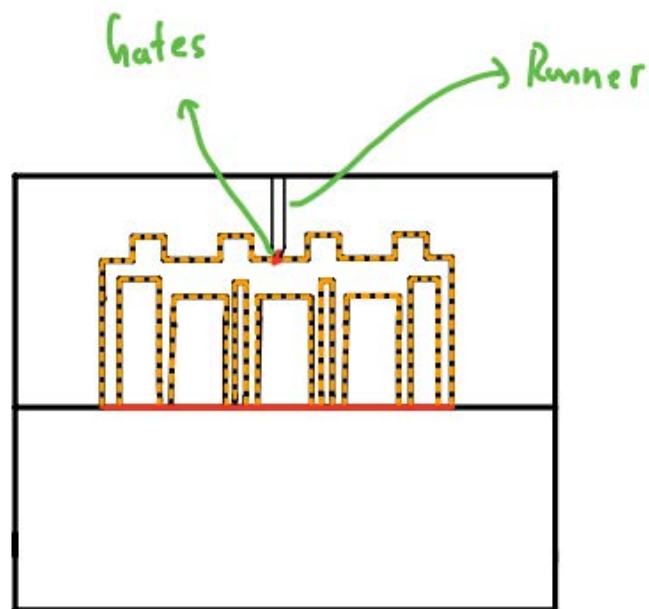
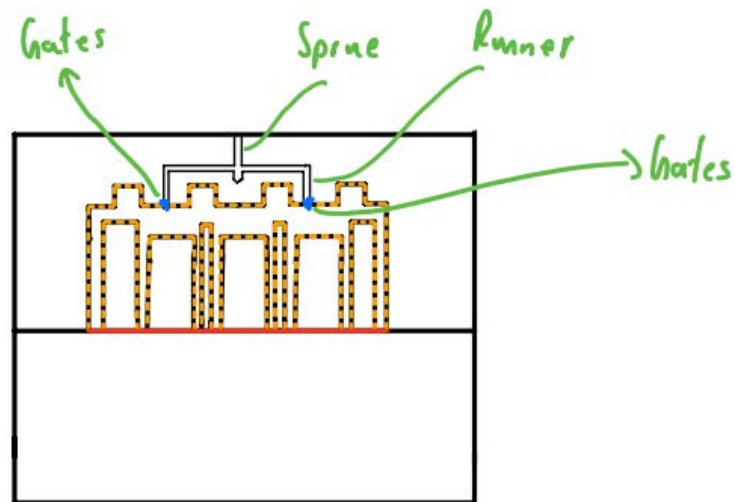
- a) Given the quality requirements and the assumption that there is no post-processing (to get rid of gate/ejector pin marks on surfaces), we need to decide where to put the gate(s) and ejector pins. For the one- and two-gate scenarios below, indicate where you would ideally put the gate/gates and ejector pins, and explain one advantage and disadvantage behind choosing the specified number of gates. Note: You can also indicate in any of the existing pictures provided or draw your own to show when you would put the gate/ejector pins. (6 points)

|        |   |
|--------|---|
| 1 gate | Location: Near the midplane of the part, to maintain symmetry of flows exiting the gate |
|--------|---|

|         |  |
|---------|--|
|         | Advantage: Minimizes the possibility of weld lines, assuming high pressure. It is also simpler/cheaper mold to produce. Maintains symmetry of flows exiting the gate, therefore making a symmetric part. Fewer gate marks. |
|         | Disadvantage: May have higher chances of short shot if the pressure is not proper relative to two gates.   |
| 2 gates | Locations: Symmetrically located from the lego block midplane, each gate equidistant from the midplane point.  |
|         | Advantage: IM pressure needs to be lower as the molten plastic from each of the gates needs to travel a shorter distance. Less chance of short shot.   |
|         | Disadvantage: Greater risk of loss of symmetry if gates are not correctly placed, higher chance of weld lines affecting part quality, greater risk of flash. More complicated/expensive mold to make. More gate marks.     |

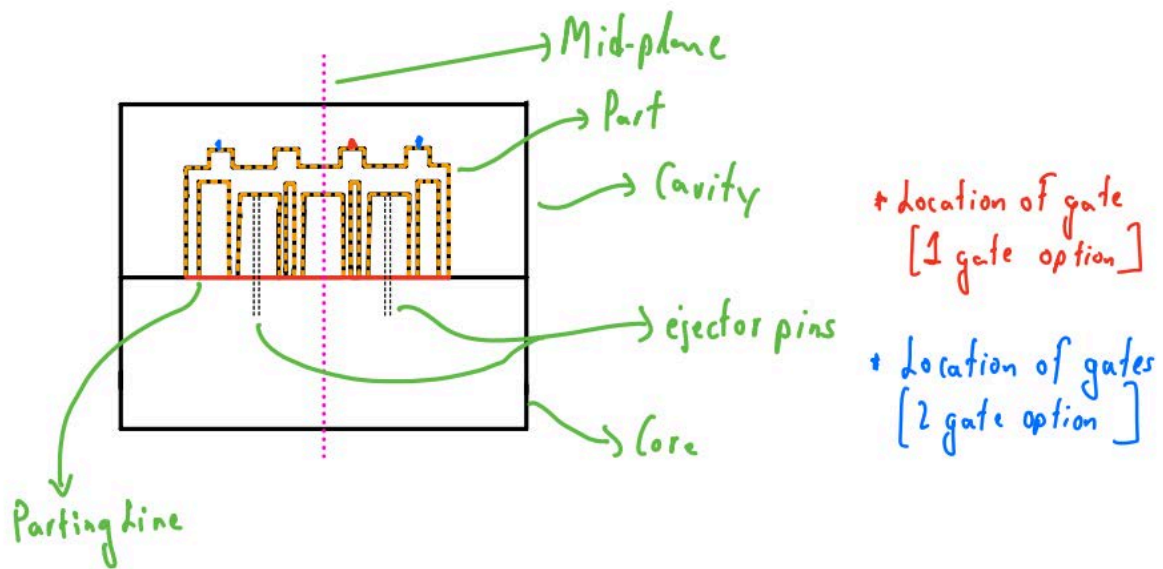
- b) Sketch a cross section of the mold used to make this part, **identifying and labeling** all critical features of the part and components of the mold. *Note: Minimum the drawing should include gate(s), ejector pin(s), core, cavity, runner(s) and parting line.* (6 points)



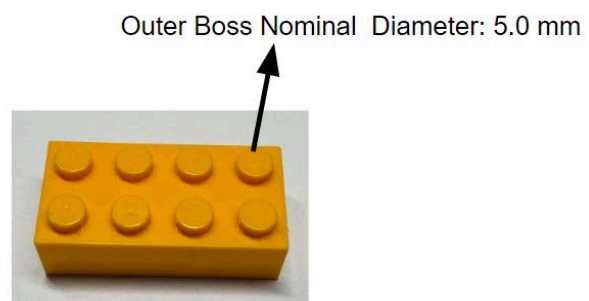


**Alternative Gate Locations:**





For the rest of this problem, we will consider the assembly of two lego bricks with each other. Two bricks will fit together through interference between the outer bosses of the lower piece and the internal bosses of the upper piece, as shown below.



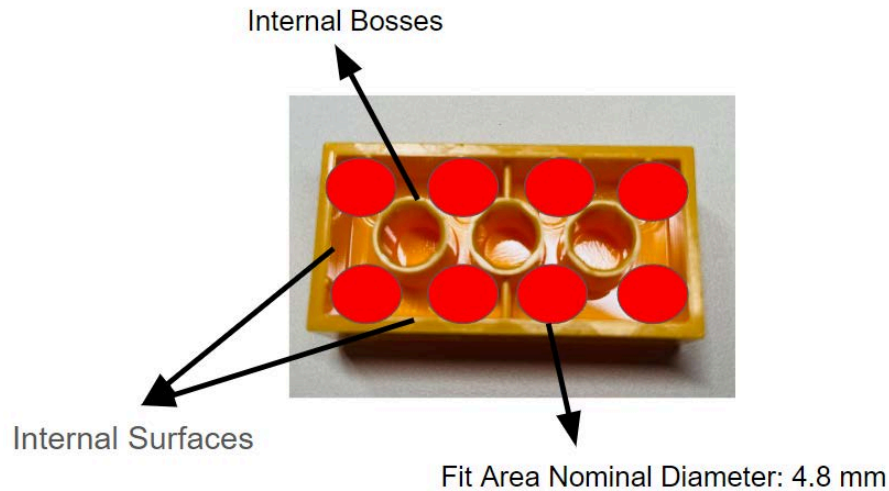


Figure 9 - Assembly of 2 Lego Bricks

Note: Fit area (red circles) is the area within which the outer boss from another brick fits into when two LEGO bricks are being assembled together.

| Dimension           | Mean [mm] | Standard Deviation [mm] |
|---------------------|-----------|-------------------------|
| Outer Boss Diameter | 4.95      | 0.15                    |
| Fit area Diameter   | 4.70      | 0.25                    |

- c) You find the outer boss of the LEGO bricks to have a mean diameter of 4.95 mm with a standard deviation of 0.15mm. If your specification for this feature is 5.00 +/- 0.10 mm, what is the capability  $C_p$  and capability index  $C_{pk}$  of this process? (7 points)

$$USL = 5.10 \text{ mm}$$

$$LSL = 4.90 \text{ mm}$$

$$\mu \text{ (mean)} = 4.95 \text{ mm}$$

$$\sigma \text{ (standard deviation)} = 0.05 \text{ mm}$$

$$Z_{high} = \frac{USL - \mu}{\sigma} = \frac{5.10 - 4.95}{0.05} = 3$$

$$Z_{low} = \frac{\mu - LSL}{\sigma} = \frac{4.95 - 4.90}{0.05} = 1$$

$$Z_{min} = \min(|Z_{high}|, |Z_{low}|) = 1$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{5.10 - 4.90}{6 \cdot 0.05} = 0.66$$

$$C_{pk} = \frac{Z_{min}}{3} = \frac{1}{3} = 0.33$$

In the initial version of the quiz, we had set outer boss  $\sigma$  (standard deviation) = 0.15 mm instead of 0.05 mm. This follows the same process but would change the answer as follows:

$$Z_{high} = \frac{USL - \mu}{\sigma} = \frac{5.10 - 4.95}{0.15} = 1$$

$$Z_{low} = \frac{\mu - LSL}{\sigma} = \frac{4.95 - 4.90}{0.15} = 0.33$$

$$Z_{min} = \min(|Z_{high}|, |Z_{low}|) = 0.33$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{5.10 - 4.90}{6 \cdot 0.15} = 0.22$$

$$C_{pk} = \frac{Z_{min}}{3} = \frac{0.33}{3} = 0.11$$

- d) During the manufacturing of the brick, quality engineers determine the process statistics by measuring samples of the outer boss diameter. Parts are measured in sample sizes of 20 each in order to check whether the process is stable. What is the Upper Control Limit (UCL) and Lower Control Limit (LCL) used during this sampling? (3 points)

$$UCL = \bar{x} + 3\sigma_{subgroup} = \bar{x} + 3 \frac{\sigma_{process}}{\sqrt{n}} = 4.95 \text{ mm} + \frac{3 \cdot 0.05 \text{ mm}}{\sqrt{20}} = 4.984 \text{ mm}$$

$$LCL = \bar{x} - 3\sigma_{subgroup} = \bar{x} - 3 \frac{\sigma_{process}}{\sqrt{n}} = 4.95 \text{ mm} - \frac{3 \cdot 0.15 \text{ mm}}{\sqrt{20}} = 4.916 \text{ mm}$$

In the initial version of the quiz, we had set outer boss  $\sigma$  (standard deviation) = 0.15 mm instead of 0.05 mm. This follows the same process but would change the answer as follows:

$$UCL = \bar{x} + 3\sigma_{subgroup} = \bar{x} + 3 \frac{\sigma_{process}}{\sqrt{n}} = 4.95 \text{ mm} + \frac{3 \cdot 0.15 \text{ mm}}{\sqrt{20}} = 5.05 \text{ mm}$$

$$LCL = \bar{x} - 3\sigma_{subgroup} = \bar{x} - 3 \frac{\sigma_{process}}{\sqrt{n}} = 4.95 \text{ mm} - \frac{3 \cdot 0.15 \text{ mm}}{\sqrt{20}} = 4.85 \text{ mm}$$

- e) Assume that the fit area diameter is specified to be 4.80 +/- 0.10 mm (USL 4.90, LSL 4.70), and the outer boss diameter is specified to be 5.00 +/- 0.10 mm (USL 5.10, LSL 4.90). This means a specified interference fit of 0.20 +/- 0.20 mm (USL 0.40, LSL 0.00) for when these two parts are press-fit together.

## Combining Normally Distributed Data Sets

added data sets:

$$\mu_{new} = \mu_1 + \mu_2$$

$$\sigma_{new} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

subtracted data sets:

$$\mu_{new} = \mu_1 - \mu_2$$

$$\sigma_{new} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

If you randomly select two lego pieces, what is the probability that they will fit together within the specified range of interference? *Note: For simplicity, assume 7 of the 8 fit areas/outer bosses will fit for sure and the question is asking about one of the fit areas/outer bosses. (4 points)*

$$\mu_{interference} = \mu_{outer\ boss\ diameter} - \mu_{fit\ area\ diameter} = 4.95mm - 4.70mm = 0.25mm$$

$$\sigma_{interference} = \sqrt{\sigma_{body\ OD}^2 + \sigma_{ring\ ID}^2} = \sqrt{0.05^2 + 0.25^2} = 0.255\ mm$$

$$Z_{high} = \frac{USL - \mu}{\sigma} = \frac{0.40 - 0.25}{0.255} = 0.588$$

72.24% of parts fall below the upper specification limit.

$$Z_{low} = \frac{LSL - \mu}{\sigma} = \frac{0.00 - (0.25)}{0.255} = -0.980$$

16.35% of parts fall below the lower specification limit.

The percentage of assemblies that fit the specification is 72.24% - 16.35% = 55.89%

In the initial version of the quiz, we had set outer boss  $\sigma$  (standard deviation) = 0.15 mm instead of 0.05 mm. This follows the same process but would change the answer as follows:

$$\sigma_{interference} = \sqrt{\sigma_{body\ OD}^2 + \sigma_{ring\ ID}^2} = \sqrt{0.15^2 + 0.25^2} = 0.292\ mm$$

$$Z_{high} = \frac{USL - \mu}{\sigma} = \frac{0.40 - 0.25}{0.292} = 0.514$$

69.50% of parts fall below the upper specification limit.

$$Z_{low} = \frac{LSL - \mu}{\sigma} = \frac{0.00 - (0.25)}{0.255} = -0.856$$

19.77% of parts fall below the lower specification limit.

The percentage of assemblies that fit the specification is 69.50% - 19.77% = 49.73%

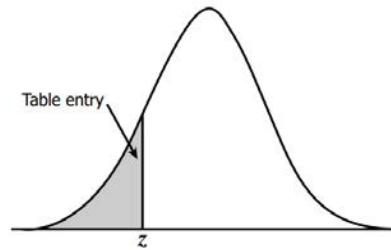
- f) What are two manufacturing changes you would implement to increase this probability? (2 points)

Some manufacturing changes that could be implemented to increase this probability are:

- Reduce variation of injection molding process by controlling room temperature/ensuring temperature fluctuations are not affecting random errors, ensuring ABS pellet material properties are uniform. That is increase  $C_p$
- Centering the process by removing systematic errors like mold wear, or IM parameters like plastic packing pressure, cooling fluid temp, cooling time/shrinkage is managed well . That is to increase  $C_{pK}$ .

## Appendix I: Z-score Table

$$z = \frac{x - \mu}{\sigma}$$



| <i>z</i> | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4     | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3     | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2     | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1     | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0     | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9     | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8     | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7     | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6     | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5     | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4     | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3     | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2     | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1     | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0     | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9     | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8     | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7     | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6     | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5     | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4     | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3     | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2     | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1     | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0     | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9     | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8     | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7     | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6     | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5     | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4     | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3     | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2     | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1     | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0     | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

| <i>z</i> | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0      | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1      | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2      | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3      | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4      | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5      | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6      | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7      | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8      | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9      | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0      | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1      | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2      | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3      | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4      | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5      | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6      | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7      | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8      | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9      | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0      | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1      | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2      | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3      | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4      | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5      | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6      | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7      | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8      | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9      | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0      | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1      | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2      | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3      | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4      | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |



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