MIT 2.008 Design and Manufacturing II

Spring 2025
March 19th, 2025

- All work for CREDIT must be completed in this quiz document
- Closed Book, but you are allowed one double-sided 8.5” x 11” notes sheet
- Calculators are allowed, and we have provided them in the room.
- We have provided the following parts to inspect:
    - 2 lego bricks for questions 2-4
    - 1 metallic cribbage piece for question 5.
- We have provided a caliper to measure relevant dimensions of the parts, as needed. Due to the limited resolution, please report all measurements in **millimeters (mm)** and **NOT inches**.
- Please return the calculator, parts, and caliper at the end of the quiz.

General Notes

- *For qualitative answers, we’re not looking for long essays. Please answer using short (1-2 sentence per answer) bullet points.*
- *For quantitative answers, show your work as clearly as possible. When possible, keep answers in algebraic form until plugging in numbers at the very end; this way, it is much easier for graders to understand where you make mistakes and provide meaningful feedback **(and partial credit)**.*
- *Each subquestion (e.g. a, b, c) may have a few parts to it (**i, ii, iii**). Make sure you r**ead and answer all parts of the question.***

Name: ___________________________

Problem 1		Out of 12 points
Problem 2		Out of 29 points
Problem 3		Out of 15 points
Problem 4		Out of 30 points
Problem 5		Out of 14 points
Total		**100 points**

Problem 1 - Short questions (12 points)

1 point for each correct answer.

For the following prompts, **circle the correct choice** in the brackets, or cross out the incorrect choice(s).

a. To increase the thermoforming production rate, you can use a **(thin/thick)** plastic sheet with **(low/high)** heat capacity, **(low/high)** glass-transition temperature, and **(low/high)** absorptivity of the heater lamp.

Equation:

b. Circle the joining process(es) below that add(s) another entity (part or material) at the joint, in addition to the parts that are being joined:

   i. **Adhesive joining**

   ii. **Brazing**

   iii. **Soldering**

   iv. **Ultrasonic welding**

c. For a given specification, the following characteristics **increase** the chance that both control limits **(UCL and LCL) fall within the specification limits**:

   i) A process mean that is (**lower than/higher than/centered in between/off-centered from**) the USL and LSL
   ii) A process variation that is **(low/no effect/high)**
   iii) A sample size that is **(low/no effect/high)**
   iv) A process capability that is **(low/no effect/high)**

Equation:

For Problems 2-4, you are provided with 2 large "DUPLO" LEGO bricks (1 red and 1 blue). In the following problems, you will analyze the considerations of manufacturing, quality control, and assembly. While working through the questions, you would need to **measure the relevant dimensions using the calipers** provided. You may also refer to Appendix 1 for some formulae.

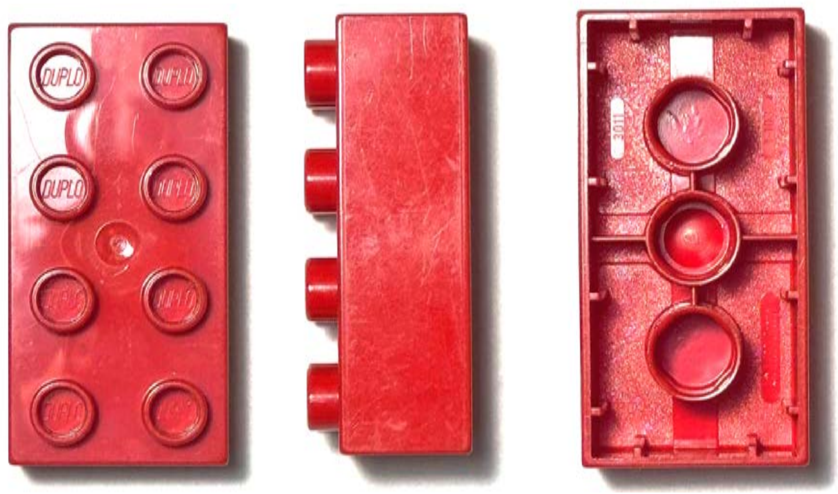


Figure 1 - Photographs of the DUPLO LEGO brick

The LEGO brick is made out of ABS which has the following material properties. For your convenience, the same table is shown in Appendix 2 in case you want to tear it off and use for reference.

Property	**Value**
Density	ρ = 1100 kg/m^3
Specific heat capacity (plastic)	c_p = 1700 J/kg-K
Thermal conductivity	k = 0.2 W/m-K
Glass transition temperature	T_g = 105°C
Melt temperature	T_m = 220°C
Viscosity of Molten ABS (@ 220 °C)	350 Pa•s
Elastic Modulus	2 GPa
Static Friction Coefficient	0.1

Problem 2 - Injection Molding (30 points)

a. Specify (and **mark on Fig. 1** where applicable) 3 features that indicate that the part was manufactured by injection molding, instead of thermoforming **(3 points)**

Gate mark, parting line, ejector pin mark, uniform thickness, good feature resolution, distinct markings on inner and outer surfaces, presence of ribs.

b. In terms of the injection molding process, **(5 points)**

i. Calculate the **flow path ratio.** Assume uniform thickness in the whole part

Correct measurement points and values of L and h +1
Using their values to calculate L/h +1

L = Length from gate to bottom corner = 34.6 mm + 19.1 mm = 53.7 mm
T or h = thickness of the walls = 1.5 mm
L/T or L/h = 35.8

ii. What kind of **defect** is likely to occur if the part is molded using standard injection molding process parameters?

This value is lower than the usual value of 120-280 for ABS. Flash is more likely to occur

iii. Suggest **two changes** to the injection molding process parameters that can be implemented to reduce the chance of this defect.

To prevent flash, compared to typical injection molding process, the following could be done:

- Lower injection pressure
- Lower the injection or mold temperature
- Using a material with a higher viscosity
- Using a material with a higher melting point.
- Increase clamping force

c. The ABS material is injected to fill the injection mold cavity to create the part in 1 second. Calculate the required **injection pressure** at the gate. **(4 points)**

$$\Delta P = \frac{12\mu}{t_{fill}}\left(\frac{L}{h}\right)^2$$

$$\Delta P = \frac{12*350}{1}(35.8)^2 = 5.38\ MPa$$

d. Calculate the **clamping force** required for a single LEGO brick **(4 points)**

Projected area = L*W = 63.7 mm * 31.7 mm = 2019.3 mm2
Correct measurement of projected area +2
Clamping force = $F_{clamp} = \Delta P\ A_{proj} = 10.9\ kN$
Correct equation and final answer

e. The injection molding machine has a rated clamping force of 45 kN.

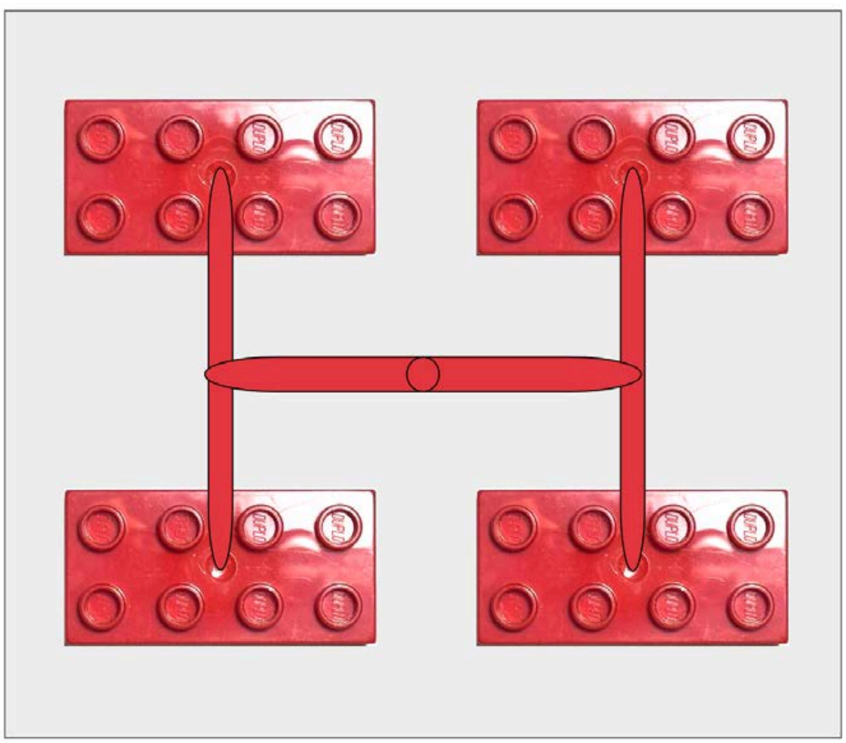
i. What's the maximum **number of bricks** that can be reliably injection-molded together **in a single mold**? **(1 points)**

The maximum number of bricks that can be molded together is floor(45/10.9) = 4 parts per mold.
Correct division using their calculated clamping force per part

ii. Sketch the ideal **arrangement of the parts in the multi-cavity mold** to ensure uniform injection pressure at all the gates. **(3 points)**

All parts top face facing the same direction +1
All parts have the same runner length +1
Showing the runner and gates +1

To maintain similar injection pressure at all 4 gates, the gates should be located at equal distance to the sprue, meaning that the length of the runner towards each gate should be the same. For a 4-cavity mold, we could achieve that using the "tree" configuration below, where the flow path splits into two twice. Directly branching the flow path into 4 runners of equal lengths is also feasible, wherein all LEGOs are fanning out radially from a central point. In contrast, a linear "manifold" configuration would result in different distances to some of the 4 gates.



f. Estimate the **cooling time** for the brick, assuming that the following condition is met: $(T_{melt} - T_{mold}) \approx 10\ (T_{ejection} - T_{mold})$ **(4 points)**

Cooling time: $t_{cool} \approx \frac{h^2}{4\alpha}$

Thermal diffusivity: $\alpha = \frac{k}{\rho c_p} = \frac{0.2}{1100*1700} = 1.07 \times 10^{-7} m^2 s^{-1} = 0.107\, mm^2 s^{-1}$

h = 1.5 mm

Cooling time: $t_{cool} \approx \frac{h^2}{4\alpha} = \frac{1.5^2}{4*0.107} = 5.26\, s$

g. Regular-sized LEGO bricks are smaller than these DUPLO LEGO bricks.

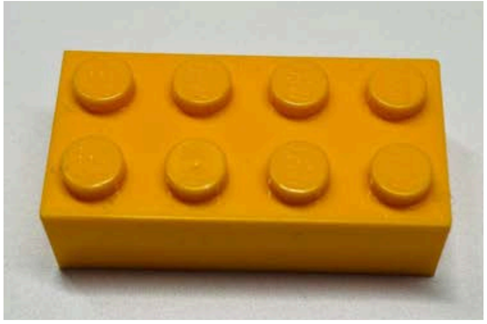


Figure 2 - Photograph of a regular-size LEGO brick.

Say that the regular-sized bricks are exactly **half the size of these DUPLO bricks in all dimensions: length, width, height, and wall thickness.** Assuming the same material and mold-filling time, estimate the ratio of the following parameters (e.g. 0.5x, 2x, etc.) to produce regular-sized bricks when compared to the DUPLO bricks: **(6 points)**

i. injection pressure at the gate

L' = 0.5L, W' = 0.5W, D' = 0.5D, h' = 0.5h.

Injection Pressure: $\Delta P' = \frac{12\mu}{t_{fill}}\left(\frac{L'}{h'}\right)^2 = \frac{12\mu}{t_{fill}}\left(\frac{0.5L}{0.5h}\right)^2 = \frac{12\mu}{t_{fill}}\left(\frac{L}{h}\right)^2 = \Delta P$

ii. clamping force per part

Clamping force:

$F'_{clamp} = \Delta P' A'_{proj} = \Delta P' L'W' = \Delta P' \times 0.5L \times 0.5W = 0.25\Delta PLW = 0.25F_{clamp}$

iii. cooling time

Cooling time: $t'_{cool} \approx \frac{h'^2}{4\alpha} = \frac{(0.5h)^2}{4\alpha} = 0.25\frac{h^2}{\alpha} = 0.25\, t_{cool}$

Problem 3 - Thermoforming (15 points)

DUPLO and regular LEGO bricks do not fit with each other. However, both of them fit on the LEGO "grass" base plate, as shown in the image below!

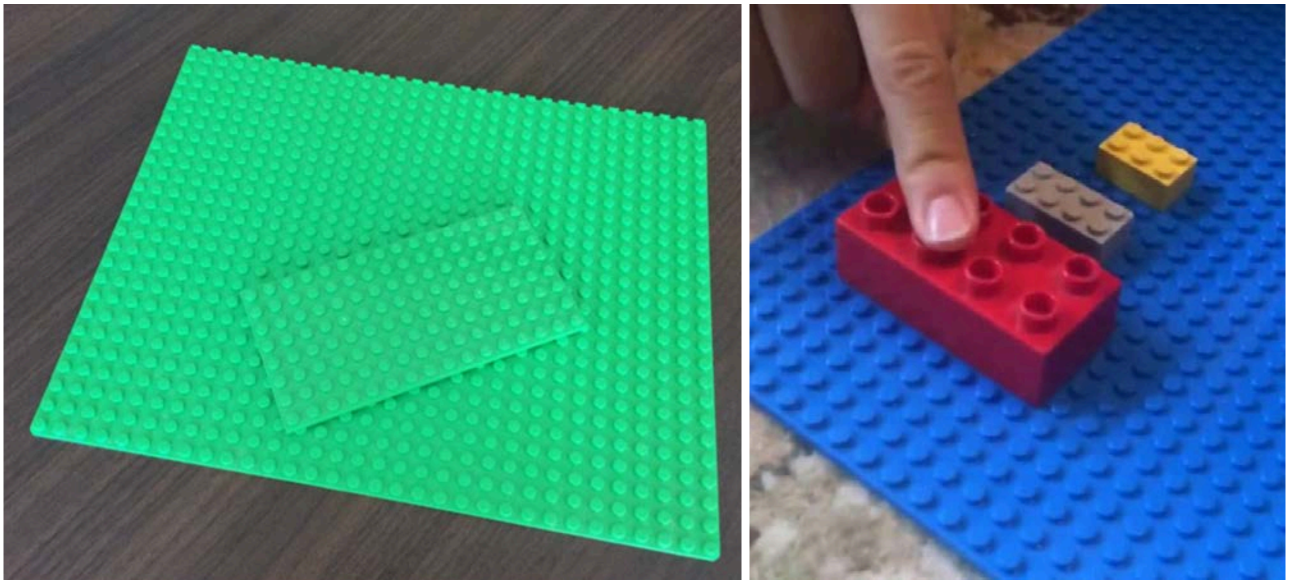


Figure 3 - LEGO base plates (left), DUPLO and regular bricks are compatible with the base plate (right)

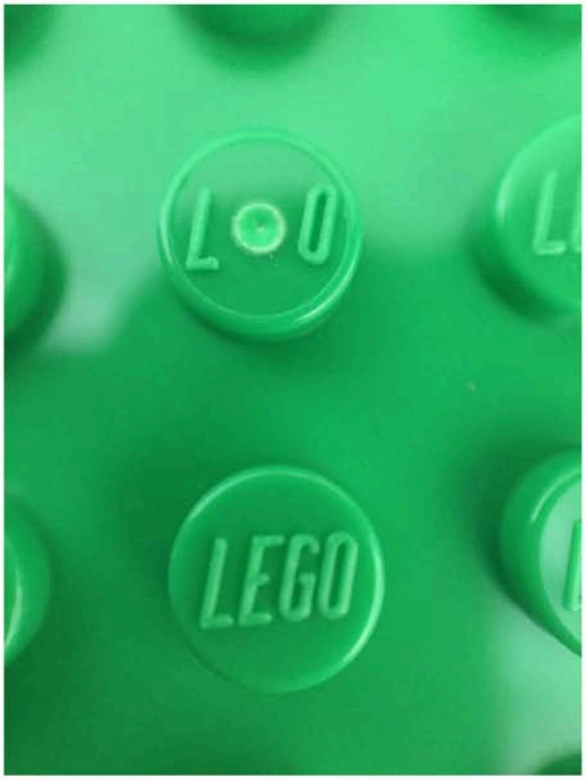
a. Have you ever noticed that smaller base plates are quite rigid, whereas the larger base plates are flexible? Now that you've taken 2.008, you can appreciate that they are made with two different processes: injection molding and thermoforming! Just using your knowledge of these two processes, which process do you think is used to make the large flexible base plate? Give **two reasons** why this process is used instead of the other process? **(3 points)**

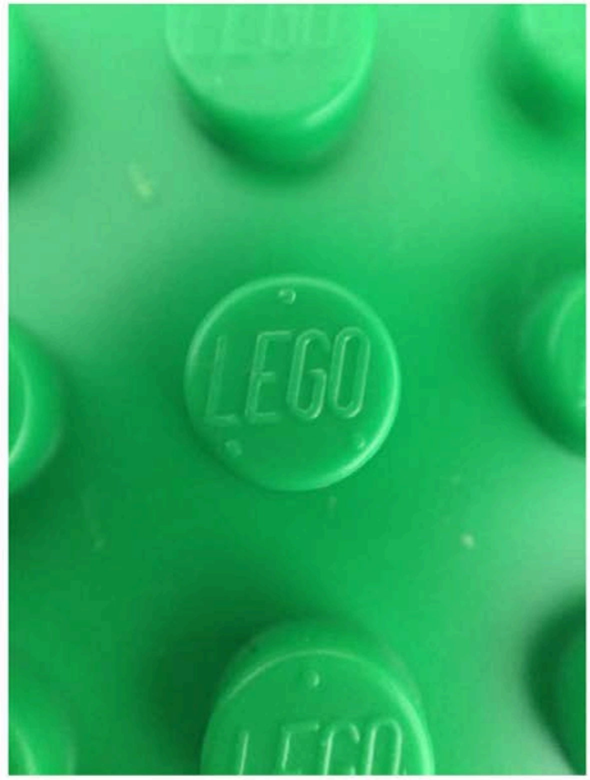
The larger flexible base plate is made with thermoforming. We know that the materials used for thermoforming are softer than materials used in injection molding because of the lower glass transition temperature (amorphous). The flexibility also comes from the fact that there are no ribs present. Thermoforming is suitable to make larger plates because injection molding would struggle to create parts with extremely high flow ratio. Injection molding would require quite high injection pressure as well as clamping force due to the large area. The large area also limits the productivity of the injection-molding process.

Identify the large flexible base plate made with thermoforming +1

Any two reasons: less stiff material, no ribs present, large L/h, large area +2

b. Let's inspect the two kinds of base plate in more detail. From the magnified top face shown below, **identify and label the process used** to make each of the two parts. Give **one identifier for each** that led to your choice **(3 points)**





Injection Molding / **Thermoforming**

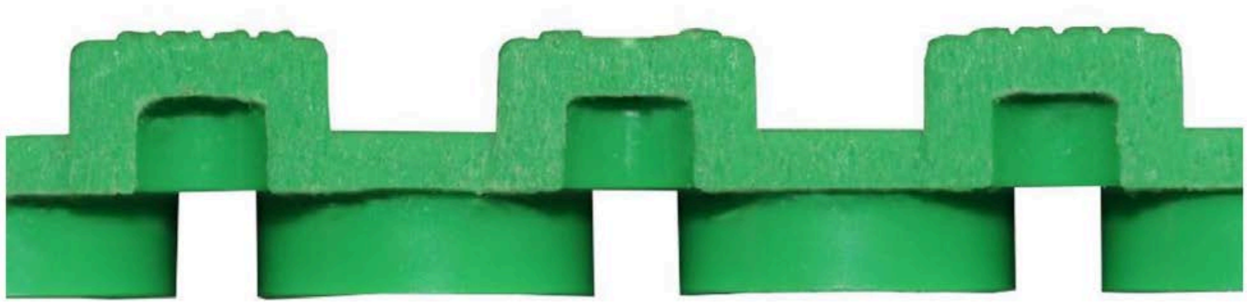
Injection Molding / **Thermoforming**

Left: injection molding. Gate mark, logo is thicker and more defined

Right: thermoforming: vacuum hole marks. Shallow logo.

c. Now we shall further inspect the cross-section of the two base plates, as shown below.
   i. Similar to question (b), identify which process is used for each picture. Give one reason each for choosing that process. **(3 points)**
   ii. On the cross-section of the part that you believe to be thermoformed, draw the thermoforming tooling around it. Indicate the key features in the tooling necessary for thermoforming the part, based on what you observe about this geometry (while working on this question or the questions above). **(4 points)**

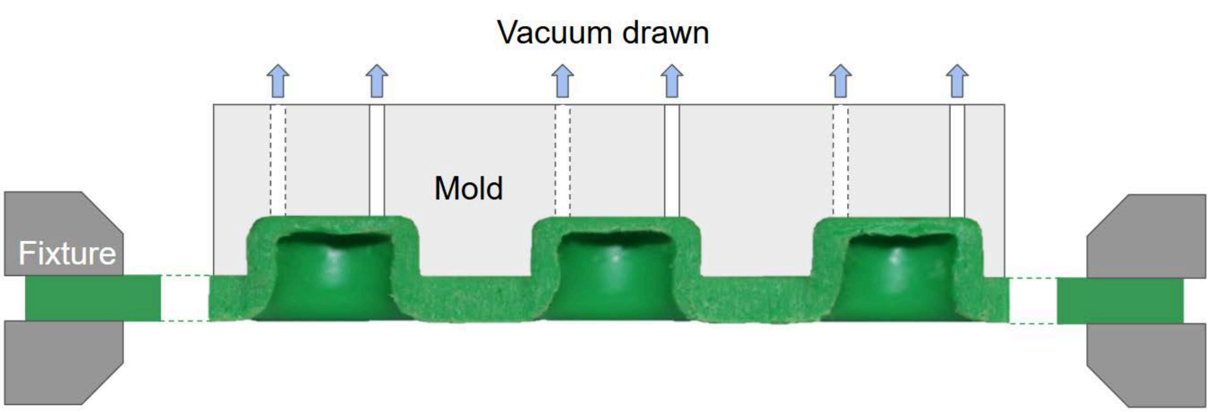
Injection molded. Regular thickness, sharper features (especially bottom face), extra features at the bottom (not possible by thermoforming of a single sheet with uniform thickness)



Injection Molding / **Thermoforming**

Injection Molding / **Thermoforming**

Thermoformed. Varying thickness due to drawing. Poorly-defined features on the bottom side allude to a one-sided mold on the top.



1 point each for including the following features: mold, vacuum holes, fixture

1 point for placing the mold on the top face.

d. Can the DUPLO bricks you have also be thermoformed? Justify why or why not. **(2 points)**

No.

The base plate only required tight tolerances on one side so it can be thermoformed with a one-sided mold. Meanwhile, the bricks need tight tolerances on both the top and bottom features to fit well with other pieces. While it is possible to do thermoforming with molds on both sides, it would be very impractical due to the high draw ratio that would result in making the side walls.

The internal ribs and cylinders of the DUPLO brick not only add structural rigidity, but is necessary to create the press-fit between lego bricks. The current rib design cannot be created with thermoforming process, whereas a complete redesign may compromise the fit and feel of the press-fit between bricks.

Problem 4 - Process Control and Assembly (19 points)

Arguably the two most important features on a LEGO brick are the “stud” (positive boss, shown as green circle on the red brick in Fig. 4) and the “anti-stud” (cavity, shown as a yellow circle on the blue brick in Fig. 4) as they directly influence how bricks fit with each other. Hence, it is crucial to keep these features within the specification. *For simplicity, we define the geometry of the anti-stud as a circle that the stud fits inside of.*

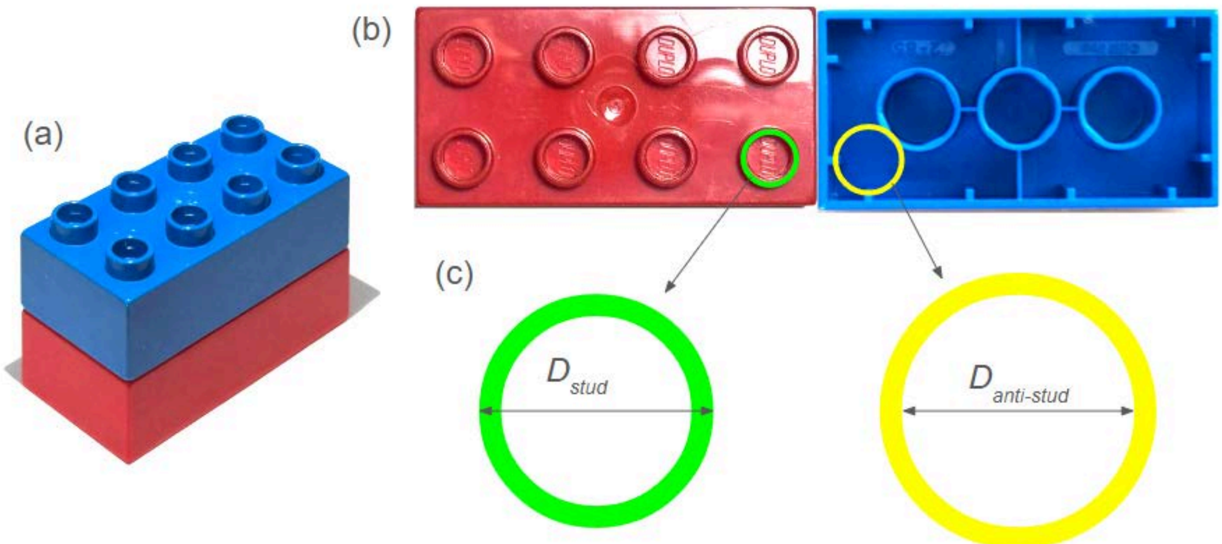


Figure 4 – (a) Two bricks assembled together. (b) Top and bottom views of the bricks showing the stud (green circle on red brick) and anti-stud (yellow circle on blue brick). (c) Press-fit model of the positive and negative boss

The **stud outer diameter** has a specification of $\mathbf{D_{stud}}$ **= 9.2 mm +0.03/+0.05 mm**, while the **anti-stud inner diameter** has a specification of $\mathbf{D_{anti\text{-}stud}}$ **= 9.2 mm -0.05/-0.03 mm**.

a. Explain how these nominal diameter and tolerance values ensure that two bricks that meet the specifications will stay together when they are assembled. **(2 points)**

Press-fit +1

Positive interference +1

The stud and anti-stud is held together by press-fit. Both features have the same nominal dimension of 9.2 mm, but different tolerances:

- The stud outer diameter has both the upper and lower specification limits above the nominal diameter
- The anti-stud inner diameter has both the upper and lower specification limits below the nominal diameter.

This means that if both parts meet specification, there will be positive interference between the features. This positive interference has a USL of +0.05-(-0.05) = 0.10 mm and an LSL of 0.03-(-0.3) = 0.06

mm. This positive interference of +0.06/+0.10 mm creates normal forces at the interface which generates friction that holds the two pieces together.

For questions b-g, we will focus on the dimension of the **stud outer diameter** which has a specification of **D_{stud} = 9.2 mm +0.03/+0.05 mm**

b. What are the USL and LSL of the **stud outer diameter**? **(2 points)**

USL = 9.2 + 0.05 = 9.25 mm
LSL = 9.2 + 0.03 = 9.23 mm

c. LEGO's injection molding subcontractor claims that their **process** can create bricks with the **stud outer diameter** dimensions having a $C_p = C_{pk} = 1.33$. **(6 points)**

i. What does the fact that $C_p = C_{pk}$ say about the process distribution curve relative to the specification limits?

$C_p = C_{pk}$ means that the sample mean is centered between the USL and LSL. process mean in same as specification center.

ii. Calculate the process mean and standard deviation of the stud outer diameter.

Since the mean is centered in between the USL and LSL, the process mean is equal to:

$$\bar{x} = \frac{USL + LSL}{2} = 9.24\ mm$$

We can use either the C_p or C_{pk} equation to calculate the process standard deviation.

$$C_p = \frac{USL - LSL}{6\sigma_{process}}$$

$$\sigma_{process} = \frac{USL - LSL}{6\,C_p} = \frac{9.25 - 9.23}{6 \times 1.33} = 0.0025\ mm$$

d. For statistical process control, 9 parts are sampled for every 1000 bricks molded. Calculate the appropriate **upper and lower control limits (UCL and LCL)** for the stud outer diameter using the process described above. **(4 points)**

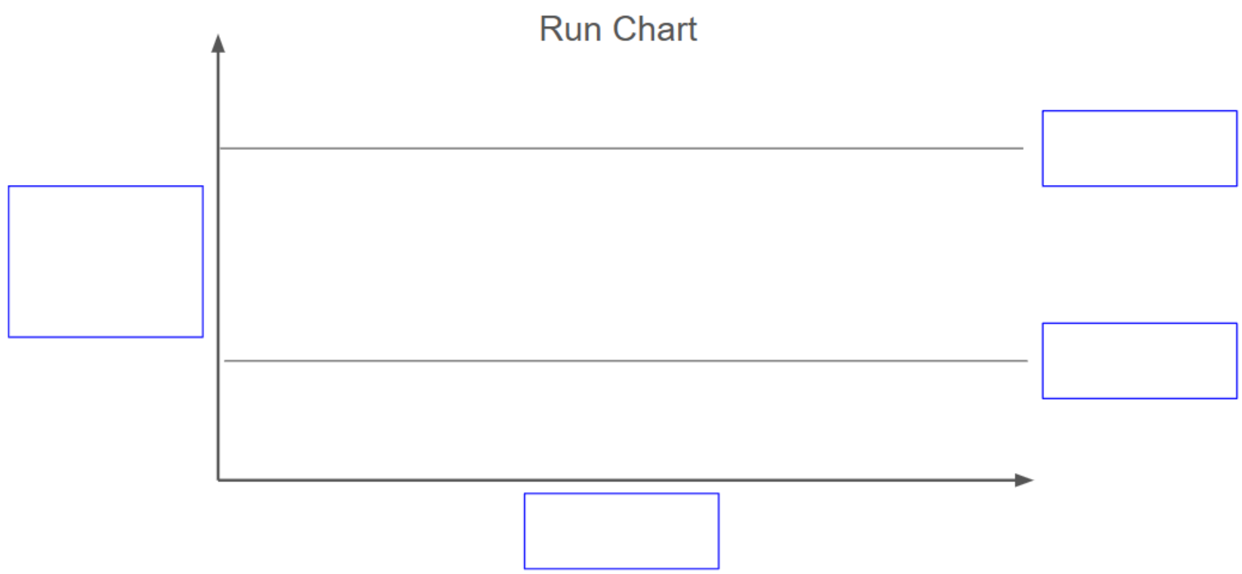
Sample size: n=9
identify correct sample size +1
Use the right equations +1

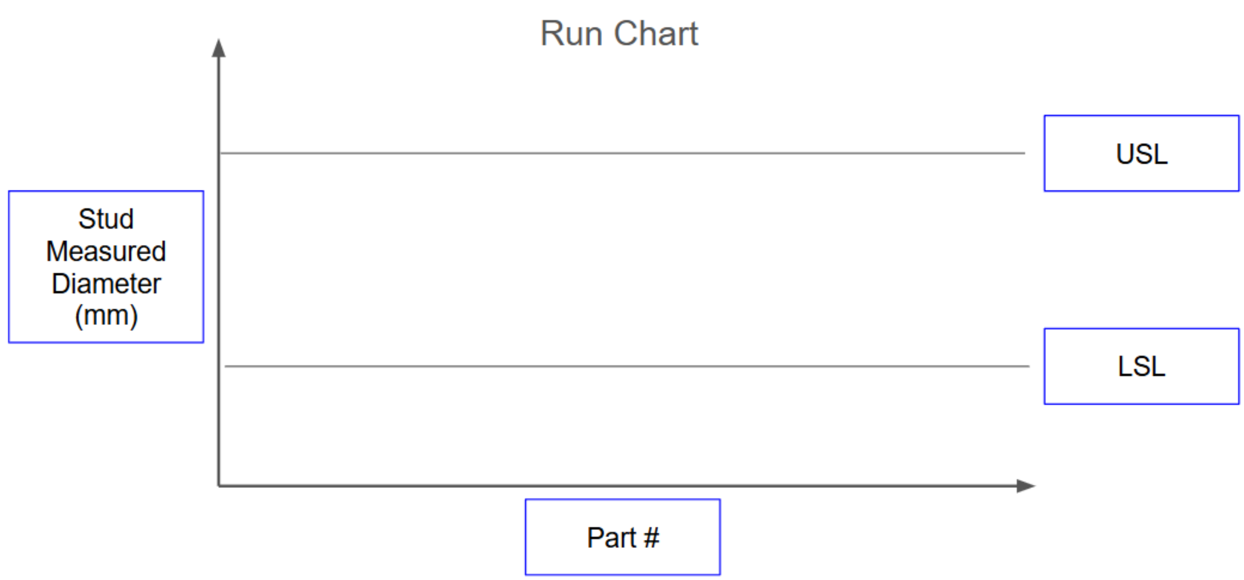
$$UCL = \bar{x} + 3\frac{\sigma_{process}}{\sqrt{n}} = 9.24 + 3\frac{0.0025}{\sqrt{9}} = 9.2425\ mm$$

$$LCL = \bar{x} - 3\frac{\sigma_{process}}{\sqrt{n}} = 9.24 - 3\frac{0.0025}{\sqrt{9}} = 9.2375\ mm$$

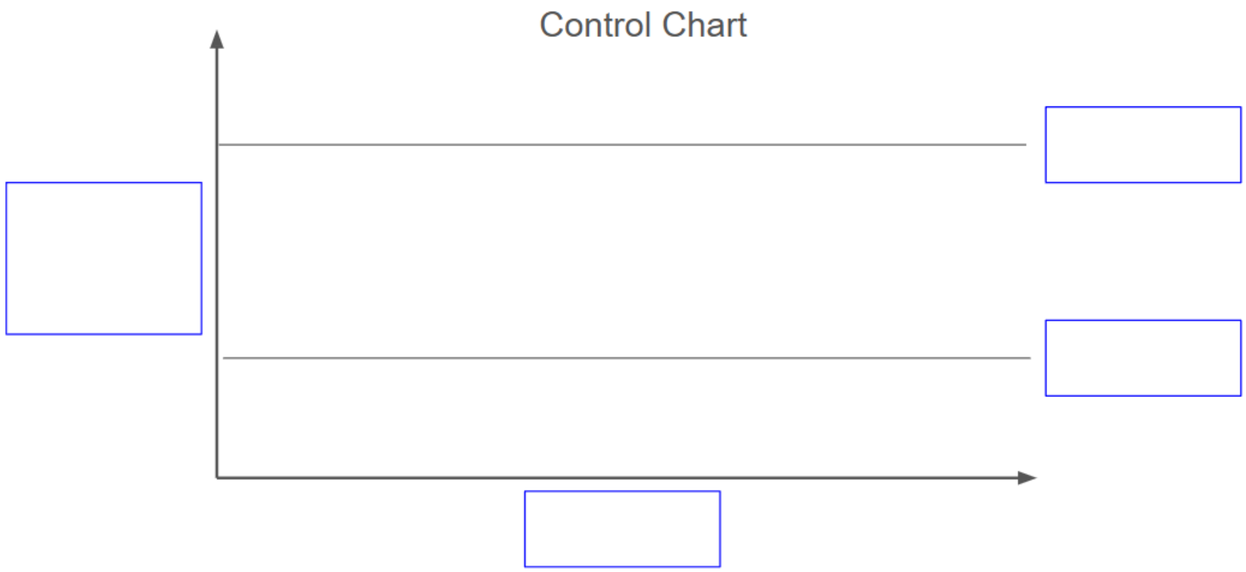
Correct answer for each 2*+1

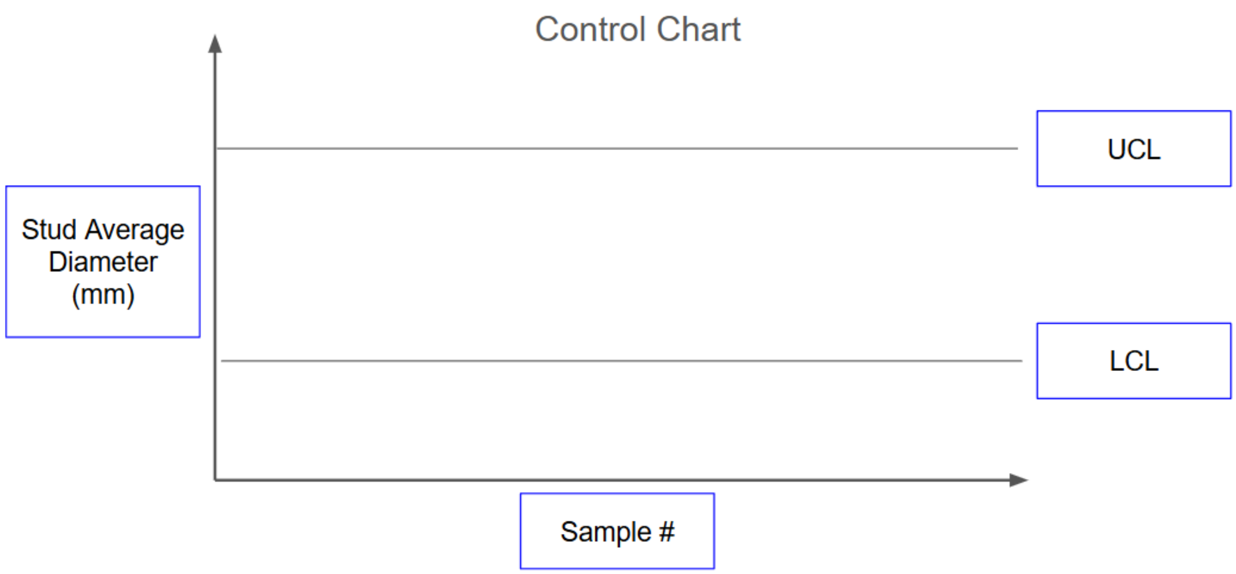
e. You are preparing to plot a run chart of this process. Please **label the axes and limits** inside the blue boxes in the run chart below. **(4 points)**





f. Separately, you are also preparing to plot a control chart. Please **label the axes and limits** inside the blue boxes in the control chart below. **(4 points)**





g. What would realistically happen to the fit of the LEGO bricks and the force needed to assemble and disassemble them if the stud outer diameter is: **(3 points)**

i. Smaller than the specification

A smaller stud outer diameter may cause the interference to become too low. The press-fit will become too loose and the two bricks cannot stay assembled together. The required force to assemble and disassemble will be too low.

+1 too loose/lower force

ii. Slightly larger than the specification

Studs with slightly larger outer diameters may still form a press-fit with the anti-stud, but the interference is higher. If the interference is too high, the press-fit will become too tight, and a higher force would be required to assemble and disassemble the bricks.

+1 too tight/higher force

iii. Much larger than the specification

If the studs are extremely large, it may not fit into an anti-stud at all. If the parts are forced to fit together, the bricks may experience plastic deformation and will no longer fit the press-fit equation that is based on elastic deformation. The plastic deformation will cause irreversible change in the dimension of the bosses. As a result, the press-fit may become too loose afterwards.

+1 not fit/break part/plastically deform

h. For a given feature dimension within the specifications, how would the force needed to assemble and disassemble the LEGO bricks be affected by the following changes to the injection molding process: **(4 points)**

i. Molding the LEGO with a different material

Changing the material may change the elastic modulus. For the same interference value, the press-fit pressure, normal force, and friction force will increase with increasing elastic modulus of the LEGO material

ii. Polishing the mold to achieve a smoother molding surface

The surface of the injection mold will directly influence the surface finish of the molded part. A smooth surface will have a lower coefficient of friction than a rough surface, and result in lower friction force. Ultimately, this results in less force required to assemble and disassemble bricks.

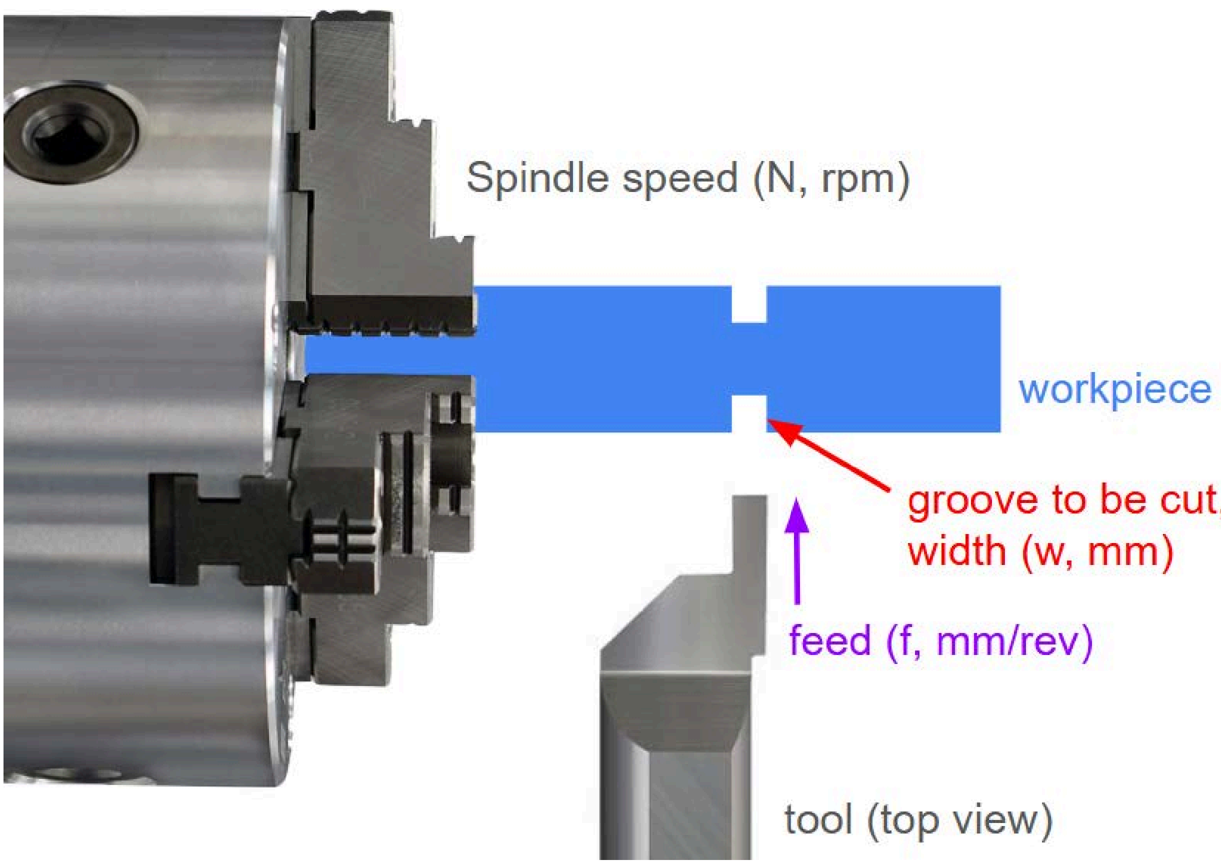
If excessive material is polished off, the mold dimension would change (the surfaces will recede), resulting in the studs being larger than the specification. This would increase the press-fit interference and the force to disassembled the bricks.

Problem 5 - Cutting (14 points)

Figure 5 – Photograph of an aluminum cribbage piece (the color is not indicative of the bulk material).

You want to **cut a single groove** in the aluminum cribbage piece. Instead of a regular lathe that has a wide cutting edge and feeds in the axial direction, you are using a custom lathe that has:

- a cutting tool edge width equal to the width of the groove.
- feeds in the radial direction**.**



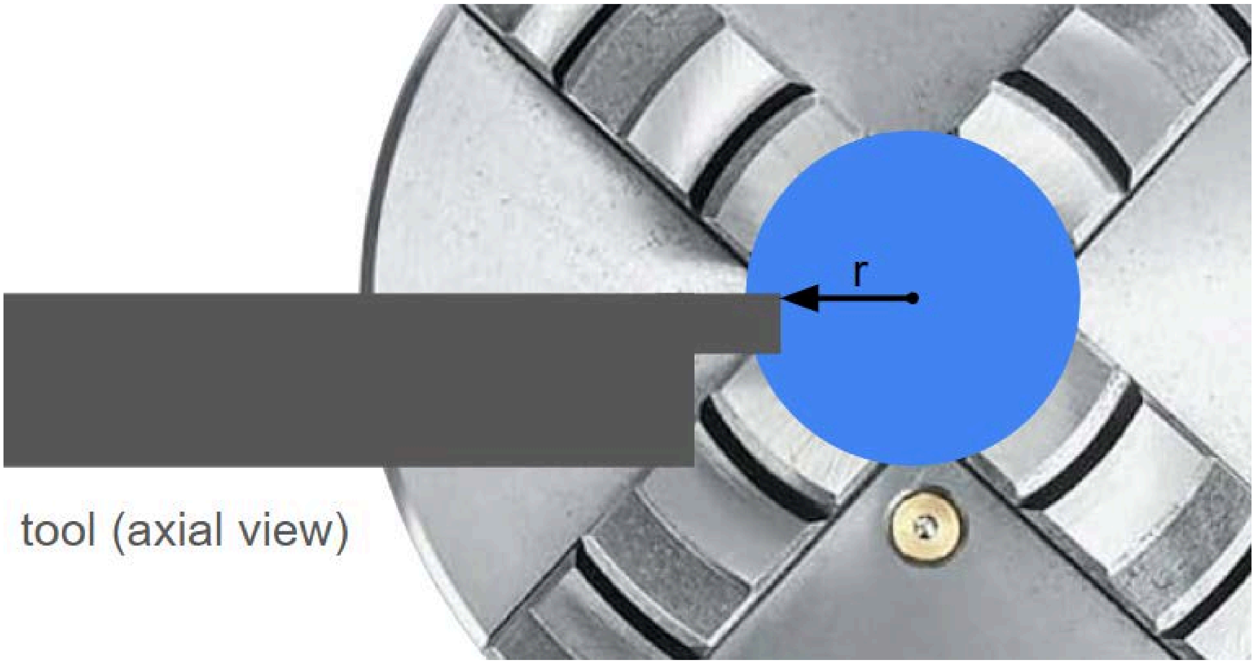


Figure 6 – Top-down (top image) and down-the-axis (bottom image) diagrams showing the cutting of the groove of cribbage piece on the lathe tool

a. During the lecture, we provided an equation for average MRR based on the average diameter of the part during the turning process. Now, we want to formulate the instantaneous MRR equation as a function of the current workpiece radius (r) at the region being cut. Using the variables defined in the figures above, write the equation for **MRR(r)**. The answer should be in variables r, other variables listed in the figure (N, f, w), whichever are relevant, and constants. **(6 points)**

MRR = volume/time

$MRR = f \times w \times v\ (mm^3/s)$

Note the different feed direction

+1 for thinking in the 3 dimensions given

f=f (mm/rev)
+1 for including f

d=w (mm)
+1 for including w

$$v = V_{cut} = \pi D \frac{N}{(60 sec/min)} = 2\pi r \frac{N}{60} = \frac{1}{30}\pi r N \ (mm/s)$$

+1 for recognizing relation between v, r, and N

$$MRR(r) = w \times f \times \frac{1}{30}\pi r N = \frac{\pi N w f}{30} r \ (mm^3/s)$$

+2 for full equation

Note that even though we are feeding in a different direction, the MRR equation ends up being the same as turning an outer diameter. The only difference is that v varies as the tool cuts at a radius r. Therefore this same equation can be found using the specific MRR turning equation and adjusting the variables slightly.

MRR = v_cut*f*d
MRR = 2*pi*r*N*f*d
MRR = 2*pi*r*N*f*w

b. At what point during the cutting operation will the machine need to output the highest power? **(2 points)**

P = u*MRR
Max power = Max MRR +1

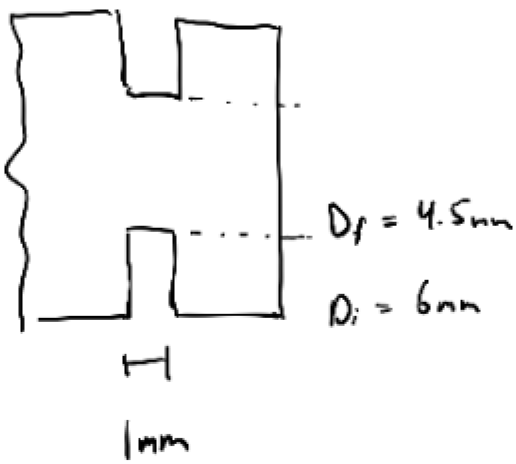
At the beginning of the cut, at max r
Max MRR = Max r +1

The power required decreases with r over time until it's fully cut. If you graphed this over time it would decrease each time you change r.

c. Given the cutting parameters below, calculate this **highest output power**. (Measure the cribbage dimensions as needed). Would this stall the lathe at LMP? **(4 points)**

Spindle speed (N) = 2500 RPM
Feed (f) = 0.5 mm/rev
Aluminum Specific cutting energy (u_s) = 0.7 W.s/mm^3

Measure the cribbage piece diameter as 6mm meaning that r_outer is 3mm



$MRR(r = 3mm) = \frac{\pi N w f}{30} r = \frac{\pi \times 2500 \times 1 \times 0.5}{30} 3 = 392.7\ mm^3/s$

measurement of max diameter 6mm and conversion to r max of 3mm +1

To properly evaluate the max power required we need to calculate the MRR under the maximum Vcut (maximum N and maximum r)

MRR calculation +1

Power = us*MRR = 0.7*392.7 mm^3/s = 275W

Power calculation +1

As shown in the lecture notes and in the demo, the LMP lathe has a rated output power of 3 hp = 2.2 kW. Therefore, it should be able to easily handle this cutting operation.

Correct final answer and comparison +1

There were alternative ways to solve this such as calculating other values at max power rating and seeing if above or below machining parameters. In addition, mostly full credit was given for any carry through errors from the previous question.

d. Assume you are not going to stall the machine. Calculate the cutting force. **(2 points)**

$$F_{cut} = \frac{Power}{v_{cut}} = \frac{u_s \times MRR}{v_{cut}} = \frac{u_s \times f w v_{cut}}{v_{cut}} = u_s f w = 0.7(Ws/mm^3) \times 0.5(mm) \times 1(mm) \times 1000(mm/m) = 350\ N$$

Correct cutting force equation, given the specific cutting energy is known +1

Correct answer +1

Appendix 1. Formulae

Injection Pressure

$$\Delta P = \frac{12\mu}{t_{fill}}\left(\frac{L}{h}\right)^2$$

μ = viscosity of plastic

t_{fill} = time needed to fill mold

Clamping Force

$$F_{clamp} = \Delta P\, A_{proj}$$

Cooling Time

$t_{cool} \approx \frac{h^2}{4\alpha}$ with $\alpha = \frac{k}{\rho\, c_p}$ if the following is satisfied: $(T_{melt} - T_{mold}) \approx 10\ (T_{ejection} - T_{mold})$

α = thermal diffusivity

k = thermal conductivity

c_p = specific heat capacity

ρ = density

h = characteristic part thickness

Appendix 2: Physical properties of ABS used in the injection-molding of the LEGO DUPLO brick.

Property	**Value**
Density	ρ = 1100 kg/m^3
Specific heat capacity (plastic)	c_p = 1700 J/kg-K
Thermal conductivity	k = 0.2 W/m-K
Glass transition temperature	T_g = 105^{o}C
Melt temperature	T_m = 220^{o}C
Viscosity of Molten ABS (@ 220 ^{o}C)	350 Pa•s
Elastic Modulus	2 GPa
Static friction coefficient	0.1