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**GIAN PAOLO
BERETTA:**

I'd like to start, since today it's March 8, 2024. I don't know if it is politically correct. So maybe MIT OpenCourseWare, will cut this part. But I'd like to dedicate this to one special woman, for me, who is my mother, who would have been 100 today. So that's kind of a dedication, personal dedication.

Anyway, OK, so we are talking about exergies. And today, we will hopefully finish through this part. Remember, last time, we talked about what the exergies associated with a bulk flow state, which is this linear combination of enthalpy and entropy and which represents the maximum work you can extract from that bulk flow state in this situation. But it also represents the minimum work to produce from environmental condition that same bulk flow state.

So that means that if you combine those two ideas, and you consider a flow that you want to change from a bulk flow state 1 to another state 2, depending on how these states are, how they compare, you may be able to do that while extracting some work, or you may be needing to spend some work to do that change of state.

And the work involved always in a machine-- I mean, you can picture it in terms of a machinery that always has to interact with the environment, so as to perform the entropy exchanges that are needed so that the changes here are not accomplished by entropy production by reversibility, which would be a loss of efficiency. But the entropy changes are provided through the machinery by the environment.

So the maximum work you can extract, or the minimum work you need to spend, is equal to the difference in the exergies between the input and the output or the output and the input.

And as you see here, I've rearranged this expression collecting the $m \cdot \Delta h$ outside. And so this represents the energy change of your bulk flow. And so you could even call it heat.

It's not really a heat interaction because the bulk flow typically changes in temperature along the pipe or whatever pieces of equipment it flows through. So as it changes in temperature, if it experiences heat interactions, there are going to be many different heat interactions at different temperatures. But you could also represent this as an overall, or say, an equivalent heat interaction that would transfer the same amount of energy.

And so you get this, quote, "heat" multiplied by something that looks like a Carnot coefficient. If you interpret Δh over Δs as an equivalent heat temperature, then this becomes a Carnot coefficient. I'm mentioning that because often people like to trace back to find Carnot coefficients in all situations. And all right, so this is the correct Carnot coefficient in this situation.

So for example, this would be a heat pump setup. And then so you can define a first-law efficiency, which is the ratio of the heating of your flow to the work you spent. That's the COP. You can also call it first-law efficiency, but it can be greater than 1.

And you can define a second-law efficiency as the ratio of the exergy associated with the useful effect, which in this case, is heating up this flow. So this is Ex_2 minus Ex_1 on the numerator divided by the exergy of the resource that you have used, in this case, the work.

And again, you can represent these change in enthalpy as a heat. You can call it heat. And then that heat gets multiplied by its Carnot coefficient that transforms it to the associated exergy, as we have seen.

Now, it turns out that this temperature-- so it's the equivalent temperature of a heat source that would provide the same amount of energy, the change in enthalpy, and the same amount of entropy to the flow. And the ratio of that energy over entropy provided to the flow has dimensions of temperature. And we will see in a moment, that it corresponds in several cases, to the logarithmic mean temperature between the inlet and outlet.

Before I go to that, let me take up a very simple example, but which is anyway interesting. It's the exergy of a waterfall or of a hydraulic jump. Wherever there is such an opportunity, people have invested, and it's very hard to find a hydraulic jump that has not been transformed in a renewable energy source.

This waterfall is the highest in Europe. It's not the highest waterfall, but it's the highest artificial one. It was made up by the Romans, not for energy reasons. I think they only needed to avoid some floods related to a lake that was above there.

But as soon as it was possible, probably at the end of, I think, the 19th century, they started exploiting it. Now it is used almost every hour of the day, except when the groups of tourists are there. Then they stop. They let the water fall, so just to show it because it is an attraction.

So how do you analyze something like that? In an example like that, the bulk flow state is characterized, not only by the thermal aspect. So it is not only a change in the enthalpy. But you also have to remember that the bulk flow state also has the kinetic energy and the potential energy.

In this case, what's important is the potential energy, which of course, changes from this elevation to the elevation at the bottom where then the water goes away. You can neglect instead, the kinetic energy. So the energy balance reads like this. Presumably, this is when you don't just let it go as a waterfall, but you pass it through a proper turbine, hydraulic turbine to transform this energy into work, mechanical work.

Also, if you recall our formulas for the ideal fluid, ideal incompressible fluid, the change in the enthalpy is equal to the specific heat times the change in temperature. There would be, also, an additional term, which is plus the specific volume times the change in pressure. But here, the water up and down is that same atmospheric pressure. So that doesn't contribute.

So this is what we are left with. So the change in elevation, or the change in potential energy, can be transformed either in work if you're using it, or in an increase in temperature.

So if you do not exploit this thing and these energy, and you just let it run as a waterfall, there is no work. So this and that become equal. And if you put them equal, you find that there is an increase in temperature of the water, which per unit of jump or change in elevation is given by g , the gravitational constant divided by c , the specific heat of water, which is 4.2 kilojoules per kilogram, K.

And if you represent it in these fancy units like degrees Celsius per meter, it says that if you have a waterfall higher than this one, 430 meters, the temperature increase below will be 1 degree Celsius, not much. Then you can compute, of course, the entropy produced by irreversibility because that's the entropy balance.

And again, you remember what for the compressible fluid model, the entropy goes as c times the logarithm of the temperatures. So you can rewrite it this way. And since the temperature increase is small, just one degree is small with respect to say, the absolute temperature which is 300 K. So $1/300$ is small with respect to 1, so you can approximate the logarithm of $1 + x$ as x , all right.

And so here is the approximation. And you can compute how much entropy is produced by irreversibility if this work is zero. This formula is also valid, also, if you get some work but not the entire maximum, the maximum you get, of course, when there is no entropy produced by irreversibility. So in as far as this is a good formula for the entropy production, it is set to zero when the work is equal to the potential energy change.

And that is, it's like per unit-- so per unit mass of water that falls and per unit of height, it's equal to g . And g can be written as, of course, 9.8 meters per seconds squared, but it can also be written in these other fancy units. It's 42 megajoules per ton over 4.3 kilometers.

Now 4.3 kilometers is about the height of the tallest mountain in Europe, Mont Blanc. So this means that if you take a ton of water and pull it down from the top of Mont Blanc to sea level, you get 42 megajoules. And 42 megajoules is an interesting number because it is the heating power of 1 kilogram of oil.

So that gives you the order of magnitude of these kind of things. So you need one ton and the entire height of Mont Blanc, to match what you can do with 1 kilogram of oil. That shows you the density of available power that is in hydrocarbons.

You can do the same in cooling. So I won't go much over the details of this slide. You find the same ratio here. And again, when you analyze problems like this, for example, you could have a situation in which your machinery, not only processes a flow of material, but maybe it also uses heat from some source at a given temperature and then uses work or produces work.

So this would require that you make an energy balance like this. So you have two heat interactions. One with the heat source at temperature T_Q , and the other with the heat source-- with the environment, which is always there to match, to balance the entropy.

And then you have your entropy balance. And then as you see, we've seen it several times now, if you combine these two balances by taking the entropy balance multiplied by the environmental temperature and subtracting it from the energy balance, you get the so-called exergy balance, where you have the various heats multiplied by their own Carnot coefficient.

In fact, the Carnot coefficient of the environmental temperature is zero here, so there is no exergy involved with this heat interaction. This is the Carnot coefficient that we just talked about associated with bulk flow changes and this is the exergy loss, or destroyed, due to the entropy production by irreversibility.

Now, when you compute the change in exergy for a bulk flow, and you compute it for particular substances, then you have to substitute the constitutive relations for those substances, those relations that relate the enthalpy and the entropy to the temperature and the pressure.

For example, if we are using a liquid, so this slide is for a liquid, as I said, the enthalpy change is equal to c specific heat times the temperature change times the specific volume. You can also write that as 1 over the density times the pressure change. And the entropy is c times the log of the ratio of the two temperatures.

Now OK, so you can substitute, collect the heat part, so that you put in evidence this Carnot coefficient looking-like term. And then there is a term here, which is connected with the pressure drop as the flow goes there. Now, if you can neglect this pressure drop, at least neglect this with respect to the thermal part of this change in exergy, then you get that your change in exergy is equal to the heat times the Carnot coefficient. And this temperature equivalent to TQ is given by this logarithmic mean temperature of the inlet-outlet temperatures.

I do the same for an ideal gas. So for the ideal gas, things change a bit because now the enthalpy is a function only of temperature, and the change in enthalpy is the specific heat at constant pressure times the change in temperature. And it's now the entropy that depends on pressure and temperature.

So it's $c_p \log$ of the ratio of the T 's minus $R \log$ of the ratio of the pressures. So you can put it in exactly the same formal relation, except that here you have c_p instead of c , and here you have the log of the ratio of the pressures. This is again, related to the pressure drop, namely, the work you need to spend in order to overcome the friction forces along the pipe in order to make the flow overcome those. But if you can neglect those, then we get, again, this formula and again, TQ is the log mean temperature.

And there is a third case in which you have a similar situation, which is whenever you are changing phase, your flow is maybe boiling or condensing, then the change in enthalpy is equal to the change in the quality of the-- so the vapor fraction that we defined last time. So the change in quality times the enthalpy of vaporization for h_2 minus h_1 , or the change in quality times the entropy of vaporization for the change in entropy.

So you can also do the same thing, put outside the energy change of your flow, call it Q , and what's left as a form of a Carnot ratio where the TQ is the ratio between the enthalpy and the entropy of vaporization, which is exactly the temperature at which the boiling and/or condensation occurs.

So for example, let's compute these number for the case of heating water, which is used for, for example, industrial applications. You often need steam or hot water for many processes or residential heating to heat up your homes.

All right, so what I want to compute is this Carnot coefficient, which you see, is therefore, the ratio between the exergy over the heat that is provided. So it's the exergy content of your heated water after you heat it. That's the exergy.

And the exergy, remember, is the minimum work required to do that job. So this ratio, exergy change over heat, represents the minimum work per unit energy of your heating. And so that is useful. So it's the best you can do.

So if you compute it for these temperatures from, say, heating water from 60 degrees to 120, which corresponds to an industrial application and assume an environment at 300 K, that's 27 degrees C, here's the number. You compute that number, you get 17.2%. If instead you're heating water from 20 to 60 degrees Celsius, in the same environment, that would be 4%.

So this is interesting because, for example, let me ask you, with this number, can you tell me what is the second-law efficiency of doing, for example, this heating with an electric boiler? An electric boiler uses electricity.

Electricity is pure exergy.

So how do you define the second-law efficiency. It's η_2 is equal to the exergy of your result, useful effect, times the exergy consumed, all right. So in this case, you're using work. The exergy consumed is equal to the work that you're using in your electric boiler.

And the exergy up there is $Q \times (1 - T_a / T_Q)$, where T_Q is the logarithmic mean temperature. So it's precisely what we have computed here. Now, in an electric boiler, your work is completely transformed into heat by joule effect. So even if your boiler were 100% efficient, very well insulated you get that Q is equal to W , see.

So we are left that the second-law efficiency is equal, in this case, to 4%. So 4%, and this is the kind of efficiency that signals how much room for improvement is there. So there is 96% room for improvement.

Nevertheless, we do use electric boilers. But where? You use them where you don't need this-- to heat water all the time. For example, in the South of Italy, where I have a small house with olive trees, and I go there just for the summer, I have electric boilers because I use them, maybe, one month per year.

So the investment cost wouldn't pay if I buy a more efficient equipment, like, would be a heat pump or even a gas, which is intermediate. Because the more efficient is the equipment, the more complicated is the machinery, so the higher the cost. And so if I use it for only a few months a year, it doesn't pay to invest that much.

But still, you have to keep that in mind that this is the idea. And maybe I could have invested in solar thermal heating, which would even save me the cost of electricity at the expense of investment cost for the equipment. But also that has a problem in some countries and especially in the countryside where this equipment is left outside with nobody watching. It is easily stolen, so that's another issue, security.

All right. So let me say a little more about fuels because we will in the second part of the course, we will do systems with chemical reactions. And so we will return to this in much more detail.

But for the moment, let me just say that if I were to heat up something or water with a gas boiler, I would need to put here the exergy of the fuel that I have consumed. So how do you compute exergy for a fuel? The idea is the following.

You take your fuel at environmental conditions, and you mix it, or keep it separate with just sufficient air. So the stoichiometric amount of air that is needed to oxidize that fuel. And then you have a piece of equipment, which you try to make as close to reversible as possible.

And while this equipment produces the combustion or the oxidation of your fuels, and therefore produces products of combustion, it produces also work. It needs, as usual, an interaction with the environment, so as to exchange the entropy that is needed to balance the entropy without the need of irreversibility. And so this result is the exergy of your fuel, the maximum work you can extract while your products of combustion, notice, are down in mutual equilibrium with the environment. So are again, environmental conditions.

Typically, we will see that the exergy of the fuel will be given by this expression. We will define an enthalpy of the combustion reaction and an entropy of combustion. So this combination is the Gibbs free energy of the combustion at standard conditions of pressure and temperature. That's the meaning of the naught up there, the superscript. And with a minus in front, that will be the most important contribution to the exergy of the fuel. There will be some term related to mixing that we will see later.

Regarding the heat of-- the heating value, the heating value if I go back to this previous slide. Now, suppose I don't have any work extraction here, and all I have is an interaction with the environment. And so I burn my fuel and then exit the product, so cool down the products of combustion at environmental temperature. In order to cool-- doing that cooling down, I need to extract heat. That heat is the heating value of the fuel. Again, we'll return to those.

The message here, for practical applications in which you need to compute the exergy of a fuel, the rule of thumb is that it turns out that for all hydrocarbons, we will see the detailed list, the ratio of this part of the contribution to that part is small, plus or minus 2.5%.

So if you accept that kind of approximation, and typically in exergy computations, you can afford that kind of precision, you can simply see you can neglect this part. And so see that the exergy and the heating value are approximately equal. So you can say that the exergy of the fuel is approximately equal to the heating value-- to the yes, to the heating value of the fuel, which is, in turn, approximately equal to the lower heating value.

As you know, there is a distinction between lower heating value and higher heating value, which has to do with the fact that in the products of combustion, there is water. And if you do the cooling of that water, of that stream of products of combustion, down to environmental conditions, some of the water will condense. And so the condensing of the water releases the enthalpy of vaporization. And that is additional energy that you can, in principle, extract.

So you call lower heating value when you pretend that water doesn't condense even if you're going down to 25 Celsius. And the higher heating value is the opposite case. You pretend that it all condenses, even though, part of the water remains as a form of vapor mixed with the products of combustion. So these are two extremes.

For example, methane, has a lower heating value of 50 megajoules per kilogram and a higher heating value of 55. So it's not an unimportant effect. It's like 10% the difference, so the contribution of condensation.

In this table, I listed some heating values of some typical fuels. Yes, there are still technical units around. In Europe it's kilocalories per kilogram. Here, you get still BTUs per pound. International system of units is more like joules. And this is megajoules per kilogram.

And you see that more or less when you present in terms of these units per kilogram, the fuels have different definitely heating values, but they are within a limited range. They're all around 40. 42 is crude oil. I mean, a typical value because crude oil varies a lot from place to place. But the canonical standard is 42 megajoules. That number that corresponds that we've seen, in this waterfall example, to lifting one ton of water up to the top of Mont Blanc from sea level.

The same energy is what is needed to heat up one ton of water by 10 degrees Celsius or to evaporate, not one ton, but 18 kilograms of water at atmospheric pressure. So that's also oxidizing one kilogram of oil. So that's the heating value of one kilogram.

If you compare it to the power density in some other-- natural-- I mean resource for-- primary energy, fission of uranium isotope 233, a half milligram of that will give you that same amount. So you see how different concentrations of energies we have.

Now, for reasons that we will see later, but also just for fun, you can plot on this graph the history of efficiencies, second-law efficiencies, of technologies that mankind has developed over the years to convert fuel exergy into work, starting from the early-- the beginning of the Industrial Revolution in England, where you would use vapor to pump water out of the coal mines.

And then you see, recognize here some names. See where efficiency of the order of these numbers-- OK, I'm sorry. The efficiency are on this side of the graph, so 1%. Today, you go up. We are of the order of 60%, 65%, almost 65%. The best available technology is reaching this much.

The diagram is made a straight line by plotting, not the linear. This scale is not linear in the second-law efficiency. It's linear in the efficiency over the 1 minus the efficiency. So the efficiency divided by the room for improvement. This ratio is called the logistic scale. You could plot the same diagram on a linear scale, and you would get a typical S-shaped curve typical of any learning process.

And in fact, you see, and I can even write the differential equation for this to obtain this formula, from which it is interesting, too, that you extract the time constant, the characteristic time of this learning. And this time constant is 60 years. I think you can prove that it's related to this slope here, the maximum height.

So in other words, it took us 300 years to go from 1% to 65%, and we are not done yet. And because this is the second-law efficiency, the upper limit is 1. So that means that we will continue. So at the end of this century, if our learning process goes on, we will be at 85%, And still it will go. At least this is what history tells us.

But since the correlation is very remarkable here, it's very likely that we do-- since this is typical of any human-- There is an entire section of the literature about the studies of the history of technologies that represent many other technologies on these logistic graphs, and they all show a similar behavior.

You could also plot here the number of words that a child learns from when he doesn't speak to when he starts speaking, and it follows the same curve. And here, also, for fun, I just put the arrows on the timeline where the various names that are associated with thermodynamics, see Rankine, Boltzmann, and so on, are placed with respect to this graph.

Somewhere, I think, somewhere here, or is it there? Yeah, no, probably here, that name. Mollier who gave the name of to the enthalpy versus entropy graph that I discussed the other day, is here and is also important because his student was Nusselt. Nusselt is another name you might have heard from heat transfer.

So he was a student of Mollier. And actually, I think we pronounce it Mollier thinking that it is French. But he was German, so I don't know how the German pronounce Mollier, but I'll let you figure it out.

In any case, I also put this slide because I wanted to advertise-- if you click here, you go to the proceedings of a conference that we held 2008 in memory of Professor Kenan. And so we edited this volume, and there are contributions about the state of thermodynamics at that time, which is not very different from the current one with big names, including Steven Chu, Max Tegmark, and many others that are considered big names in the field of thermodynamics and energy.

All right, so, why am I talking about that? All right, well, because. we've seen that history tells us that we are going to go to 85% efficient by the end of the century. How can we do that? Because currently, the point of this slide is that we cannot do it with current technology. At least not with the current technology that uses the fuel to burn in a flame, in a combustion process.

The reason is because this is the philosophy of our traditional power plants, flame-based, is that you burn the fuel. And with the hot products of combustion, you heat some working fluid, and you make it work in a thermal cycle.

The thermal cycle, for example, in a combined gas turbine and steam thermal cycle, has an efficiency of about 85%. So it means that your irreversibility in your power plant wastes only 15% of the exergy that thermal part of your power plant receives from the hot products of combustion. And then you have the flame. The flame per se, by itself, without losing one BTU of energy wastes, because it's an irreversible process, wastes 25% of the exergy.

So if you multiply the two efficiencies here, 75 times 85, you get about 64. And that's the explanation of why. So 64, it means that we are wasting 36% of the exergy. And where is that wasted? OK, 25 here, and another 15, or more or less, here.

So you see, in order to get to 85%, we must we must get rid of the waste that we have in combustion. We must use the fuel, not by burning it but in some other way.

And as we will treat combustion in a few weeks, we will end up drawing this diagram in which you have the enthalpy versus entropy. It looks like an energy versus entropy diagram. You have the fuel-air mixture here, and the products of combustion there.

And it turns out that, when you burn your fuel in a flame, what you do is to move from this to that, and you produce entropy by irreversibility. Whereas, before burning it, your fuel has an exergy which is represented by this much. And you have now here, it as an exergy that is represented by this much. So you've wasted this much exergy because of this irreversibility.

So the flames are the bottleneck of efficiency in fuel usage to power conversion. And this is why-- people realize that, of course, and they are working on developing alternatives to this way of burning instead of the flame.

For example, with the fuel cells, you oxidize your fuel while you are also extracting electrical energy. Because you see, in order to avoid the irreversibilities, you need to be quick. For example, in an internal combustion engine, when you light the spark at the top dead center, and the fuel air mixture starts to burn, there is a flame which propagates. So you can view it in slow motion. It takes a while to entrain the entire combustion chamber. But in reality, that happens in milliseconds.

So you have to be quicker than that if you want to do it at that level, or you have to develop alternatives to flame. For example, you can determine a different path to oxidize the fuel with intermediate steps so that you can control the intermediate steps and use them to avoid the irreversibilities.

Or you can use the fuel cell, which does it with electrochemistry, and then you can integrate your fuel cell in a thermal cycle that uses the thermal energy that is released. So the entropy that needs to be taken away from the fuel cell, is going to be used by a bottoming thermal cycle. And just doing that, on paper, I list here some efficiencies that can already be reached, which go beyond the limit of flames.

And this takes me to this last topic of the first part, which is also going to be the topic of the homework that I will put out today. Regarding that, I'm going to put a deadline, which is just before the spring break. But I did that because I want you to be free for the spring break.

But if you need to take more time, feel free to do it. I don't want to spoil your spring break. But even if you want an extra 10 days, do take them. I don't really care. It's your choice. So there is an official deadline, but it's not. It can be broken.

All right, so allocation, why is this problem interesting? OK, what's the question that it answers? What fraction of the fuel consumed in a co-generation facility should be allocated to the heat produced? Now, in a co-generation facility, we use a fuel. And it produces, not only electricity, but it also produces heat, which can be used for industrial or for residential uses.

That heat that is produced this way, goes into the homes. And it is important to know how much primary energy was used to produce it. Why is it important? Because of real estate and efficiency regulations.

Now, almost all countries, and I listed here the names of these regulations in various countries, see, it goes from Europe almost anywhere, including, of course, the US. Energy performance certificate of a building or energy score, home energy score.

It essentially tells you, of your house, what's the level of efficiency. And that level of efficiency determines the value of your house. So if I go buy a house, I care about how much it consumes because that's the bills I will receive to heat it up. And so if it is an efficient house, it has more value than an inefficient.

So this problem, in fact, since in order to compute this energy score, you need to know, among the various parameters, it's not only how good are the windows and the walls and so on, and the roof. But it's also how efficient is the equipment that you're using in order to produce the heating?

And if you live in a city like Brescia, or if you live at MIT now, I know that also here we have the same problem, that you have a co-generation facility which is not in your house, but it's somewhere else in town. And then through pipes, they send heat to you. In order to do that calculation of the home energy score, you need to know how much energy was consumed, or what is behind that heat that you are using, which substitutes your local boiler or gas boiler that you have in a standard house.

So in my condominium, I still have the gas boiler, so the question is easily answered. I look up how much heat has been produced over one year, how much gas I paid, and that tells me how much energy is behind my BTU of heat produced. But if it is co-produced, co-generated in a heat and power facility, Combined Heat and Power, CHP facility, then the question is not that simple.

So here is a scheme. Here is the facility which produces work and heat and uses fuel. Now, normally, that work would be produced in a separate production facility that uses just some fuel and produces only work. And that heat would be produced, like in my gas burner, by a separate production equipment that uses some fuel and produces that.

So the question is, how do we compare these separate production scenario with a co-generation facility? And you see, the reason is that we know before we start, I mean, without any additional discussion, we can compute how much fuel we saved with respect to the separate production.

That is called primary energy savings. It is defined as the ratio of the difference between what I would consume in a separate production, what I actually consume in my co-generation facility, divided by what I consumed with the reference production. So this is the ratio.

And we know that typical co-generation facilities allow you to save between 20% and 40%. So we're talking about large amounts. That's why it is pursued and done. So we know how much that's the overall savings.

But the problem is that in order to do that calculation for the home energy parameter, you need another number. You need, essentially, the COP for the heat. It's how much the efficiency. It says, you see, you need to know this denominator. How much of the fuel, which has been used in the co-generation facility has to be assigned to the production of heat. And there is some arbitrariness into how we define this.

It is no-- Yeah, some people claim there is a thermodynamic-- we'll see that exergy-- The exergy people will claim that that's the method that you should use. But in our discussion, we will show that that's not appropriate, not yet.

So we need to define allocation factors. So suppose we have these allocation factors, α , they add up to 1. So essentially, it's the percentage of fuel that we think is being used for, say, α_W for the work production line and for the heat production line. So you think of your co-generation facility as an equivalent separate production facility in which-- or as having two internal lines.

If in fact, if your combined production was simply that you have two separate lines, then you would easily find what the α is. You just use whatever it goes on one line and whatever fuel goes on the other line.

But combined heat and power is best when you don't have separate, but you're using the various levels of temperature at which your working fluids are, to do what is appropriate to reduce the irreversibilities. And so you have a lot of equipment here which play the game of both-- They are both on the side of the work production and of the heat production.

But in any case, suppose we have these α s, and then, for example, we could define the separate efficiency, so the partial efficiencies. Partial efficiency of work production would be the work, divided by the amount of fuel only the part that I have associated with that production. And the COP for heat is the heat divided by the complement to that, so that's the fuel energy associated with the heat production.

And of course, you can also compute partial primary energy savings by comparing what-- the amount of fuel that is-- the amount of fuel that is used in separate production, you subtract what you actually consume in your co-generation facility, so that's the numerator and compare it to the separate production. So that gives you the savings of the co-generation with respect to-- partial to the production of electricity and the same for heat. And if you have that number, α_Q that number α_Q is what is needed for the home index, efficiency index.

So in the beginning of co-generation, people reasoned and defined allocation factors in this way. We call this method the incremental electricity-centered allocation. It basically says, all right, I, in my co-generation facility, I produce this much work.

If I produced it in the reference separate production facility with this reference efficiency, I would have consumed this much fuel, W over the reference efficiency. So that in my co-generation plant, I produce more than-- I'm sorry, I consume more than that fuel. I consume F .

So if I subtract what I would have consumed for the production of that work, it means that the extra fuel I'm consuming is attributed to the heat that I'm producing. And therefore, you would put at the denominator here. You would assign to that heat, this extra fuel with respect to the separate production of the electricity that you have done.

But this, of course, is unfair because it assigns all the primary energy savings to the heat and zero to the work. If it is true that my co-generation facility saves 40% of the fuel, it means that there is a benefit of co-generation. And that benefit, is it really all due to the production of heat, or is it should be distributed between the two products?

So another more balanced way of reasoning, which took over later, is to define-- maybe some local or state authority, energy authority, defines or states in some standards-- reference values for the efficiencies of separate production of heat and work. So they define some reference values.

And then they say, all right, use as allocation factors, the weighted fraction of what you would have consumed by producing the same W and same Q in separate facilities. So this defines-- because this W over $\eta_{\text{reference}}$ is the fuel you would have consumed if you produced this amount of electricity in separate production, and Q over $\text{COP}_{\text{reference}}$ is the fuel that you would have consumed if you have produced that amount of Q , the same as your co-generation facility in a separate production.

The good thing about these allocation factors is that it distributes equally the primary energy savings to the heat and the work, so that the partial primary energy savings for work is equal to the partial energy savings for heat, and it's equal to the overall partial energy savings.

The question is, what reference values should the authority select? OK, one method is to simply do it arbitrarily, or maybe do it by some logic. You publish it, and it stays there for some years. Then maybe you update those reference values.

The other extreme is you select the exergy reasoning. So you say, well, that's used as reference values, the best you could do. Namely the reversible processes. Yes.

AUDIENCE: If I'm understanding this correctly, so the first method was used earlier because it didn't require these select reference values to be calculated. It just used the efficiency of the work producing part of the co-generation plant itself.

GIAN PAOLO BERETTA: Yes, at the beginning, certainly, it was an easy way of reasoning. It didn't require an authority to define. And also you didn't have the motivation. The motivation came, and this is how we came with this question, and this is how-- I'll give you the reference to a paper with your advisor, Professor Ghoneim.

We worked together on this problem because the utility which produces district heating in Brescia came to us and asked, OK, we have a problem now. Because we are giving these numbers, but with these numbers, every apartment goes into the top class. So we are assigning so low consumption of fuel to the heat that we are providing them that they are so happy they are in class A+, even with the windows open.

So it doesn't make sense. That's why they came, and so I said, we need something more. And so they also, after these discussions, they also went to the European community and had them change the regulations.

AUDIENCE: Thanks

GIAN PAOLO BERETTA: Yeah. And so the exergy idea is, let's take as reference the best we could do. And the best we could do is the reversible case. So what is the best we could do in terms of efficiency for work production?

If I have to produce this much work, and I'm using a fuel, with an exergy that is this much, Ex , well, if there are no irreversibilities in my machinery, so it's reversible, the exergy gets all transformed. There is no loss of exergy, so the exergy of the fuel goes all into exergy of the product, so it's 1 to 1, so W_{max} , or let's say rev , is equal to the exergy of the fuel. But the exergy of the fuel, we said, is approximately equal to the heating value. Let's call it Q_F to simplify.

So that means that this machinery, reversible machinery, that could do that, has a first-law efficiency, which is what we call $\eta_{W ref}$, which is the work produced. First law. So it's divided by the heating value of the fuel, the energy that has been consumed. But this is also approximately equal to the exergy of the fuel. And as you see here, this is approximately 1.

Whereas, the same story, if I use my fuel, and I want to produce heat at the temperature T_Q , the exergy of that heat is this. And if I have no irreversibility all that goes into this.

So the η , let's call it COP, would be the Q divided by the energy of the fuel because COP is a first-law idea, so the energy of the fuel. And the energy of the fuel is Q_F . But Q_F , again, is approximately Ex_F .

So this is Q divided by the exergy. The exergy gets all transformed in that. So here in the denominator, I put $Q \frac{1 - T_a}{T_Q}$. OK, so the COP is one over the Carnot coefficient calculated at the temperature of our production, heat production. So you certainly can do that. So I'll give you a numerical example for this in a moment.

So here's a paper with Professor Ghoneim and another colleague of mine in Brescia, Paolo Iora, which we addressed the issue and acknowledged that there is no correct way to do it. It's hand waving, and it's arbitrary in some sense.

But the idea that we introduced was this. OK, suppose this local authority has to decide what efficiencies, reference efficiencies to set. What criterion should they use?

Well, they would look at their local area. And by local area, it could be a small district, it could be a town, it could be a state, it could be a nation, It could be Europe. It could do this at various levels because there are standards, local standards, and so on.

In the homework, I'll give you reference to a European standard, which was 2007, which had the issue. Frankly, I didn't follow up, but I'm sure that they've improved a lot with more recent standards. So that means that the problem is of interest also at the European level, which is a congregation or a community of several nations.

OK, So you do take the data that come from various facilities in your local area. Every facility is always obliged to communicate to some energy authority, their consumptions and their productions yearly, at least yearly. And so take those data and look at your overall system as a co-generation system because after all, it produces work and heat.

And you know how much fuel has been used for each of them. So if I have assigned a value for the allocation factors, like α_W and α_Q . So if I'm the authority. I decided these numbers.

With these numbers, those who have combined heat and power facilities have used them in order to compute their own partial efficiencies. So they will tell me how much fuel they have used or is behind this work and how much is behind the heat. So I compute the overall efficiency for the heat production and for the work production for the local facility. I mean, for the local area.

But then I can also apply my same rules or allocation coefficients or allocation fractions to the local area viewed overall, as an overall combined heat and power facility. And if I have given the correct values of α , I should get the same number, whether I look at it locally with small facilities or the overall local area system.

So that was the method that we have provided, and here is an example with some numbers. So suppose we have a local area and suppose that we have separate production facilities that have 52% efficiency for energy, for electricity, and 90%, so a 0.9 COP for the production of heat, like industrial boilers.

And here, the combined heat and power plant is a combined cycle. This is also a back pressure steam turbine data. But I'm going to discuss only this one. And these are two parameters that characterize a combined cycle. So it's the ratio of the work to the heat that is produced. And the first-law efficiency, so 78.

So suppose that the reference values are taken as the values of the separate productions in my local area, 52 and 90. So look at just the first line. Here are the numbers that come out with the three methods that we discussed so far.

So the electricity centered keeps, essentially, the same efficiency for the electricity as separate production because it gives all the advantage to the heat. So it would make it appear that you are producing heat with a COP of 354. That created that problem that I told you of unfair and unacceptable, too much advantage to the heat. In a sense, if you kept that, everybody would want the district heating. So it would be a fight to have your houses to be district heated.

If you do the exergy, which is the other extreme method, the electricity efficiency goes down. And the COP is still very high. If you use the separate production method with these references, this gets more reasonable.

What's reasonable is that there is an increment in the separate production-- I mean, in the partial efficiency for work and also the same increment for heat. So going from 52 to 63 is an increment of maybe, what, 20%, well also going from 90 to 109 is 20%.

So you have evenly distributed, you have treated equally, work and heat. So you gave to both work and heat the same weight or the same advantage of the co-generation.

But a problem is, that when you now compute for your local area, the local average of the efficiencies, you don't find the same values. For in this example here, so you have to define how important co-generation is in your local area.

And so you have to define an electric index, which is 50% In this case. That means that heat is twice the work and also that your co-generation facilities cover 40% of the heat production in the local area. With these numbers, you can compute the local averages. And these local averages are 62 for work and 96.8 for heat. For work it's not too far, but for heat, it's far because 109 is very far from 96.

So what we propose is simply to say, all right, let's use the local area's numbers and use them as reference. So if I use them as reference, of course, numbers change, and the local numbers get closer. But not yet, I have to keep doing. So. If I iterate at a certain stage, I get to a place in which, yeah, in principle, I should get that these two numbers become equal and also, so there's no more improvement.

And so the local area, so the-- yeah, the local area is-- I'm not sure these numbers are really correct because this is 90.8. This is 92. So there may be something wrong in these numbers. Because the idea was, I believe, but I may be wrong now, so you check on me because that this should become almost equal in the long run. All right, so that's the idea.

But the story is not finished because now this same issue of allocation comes up, also, when you have hybrid facilities, hybrid power plants. For example. you know that we could use solar energy for heating water.

And one way, of course, so you have solar energy. And suppose you have one of those parabolic mirrors that focus the solar energy onto a pipe. And in this pipe, you run water or you run a mixture of molten salts so that you can go a higher temperature. So you run diathermal oil so that it doesn't boil. So you run a fluid that takes up that heat. Depending on what fluid you use, you can reach different temperatures.

For example, with air and water, if you pressurize it, you can go 120 degrees C, maybe higher, but you have to go higher pressures. With diathermal oils, you can go up to 300, 350 because then at higher temperatures, the oil disgregates. And so you reach some technological limits there. Molten salts, you can reach maybe 500, 550.

So people have built power plants with this idea of concentrating solar energy. These mirrors follow the Sun so that you use it efficiently. There are other architectures of similar ideas, like, you have lots of mirrors on your field, and there is a tower, and your pipe is there. So the sunlight, with the mirrors, heats up this pipe that is up there. And that way you concentrate, and you get high temperatures.

And then the rest, once you have these high temperatures, you can use your standard thermal cycle and convert thermal energy into mechanical and electrical work. So no problem about that. This belongs to the realm of renewable facilities that obtain important incentives from state, governments, and-- And so those are very important because they allow you to pay for the equipment and some of these helps are, for example, here I gave examples of tax credits, cash grants, loan programs, all sorts of different things. And this is just only something in the United States. Every country has its own way of doing this.

It's similar to what you do for photovoltaic. So photovoltaic is a direct conversion of solar energy into electricity. This is less direct. It goes through heat and then you have a power plant. And in the end, you also produce electricity.

So what is the problem? The problem is that sometimes when you do these power plants, it is also useful to hybridize it. In other words, use this heat, yes, in your facility, but use also some fuel. Use that heat, but use also some fuel.

Because that allows you to use the facility more hours per year, for example, or maybe it allows you to increment the temperatures and improve on the overall efficiency. And so it's either that you start from a solar facility and hybridize it using fuel, or sometimes, which also happens very often, you start from a traditional facility, fuel based, and you hybridize it by putting a solar field on the side to, for example, provide the low-temperature heating that the fuel does in a less efficient way or do some other kind of integration.

So in all these hybrid problems, the important aspect is that-- the important question is, all right, I produce work, electricity. The question is, how much of that electricity that I produce can be said to be renewable, and how much is not? Because the renewable fraction gets the incentives. The other doesn't.

So even here, we have a problem, which is entirely dual to the one that we have discussed. And that's why I'm going to go fast on this. But I will ask you, in your homework, which, by the way, this is a heavier homework. So it's also more work.

So it will have two parts, two videos. One with a problem on co-generation, and the other is put words in these slides about this part of the hybrid facilities. So in any case, the duality stands on the fact that in the previous problem, we had the alphas on the left-hand side. Here, we have the betas on the right-hand side. So β_R is the renewable fraction of electricity. So β_R is the key. It's the number that will give you access to these economic incentives.

Also, here, you can talk about primary energy savings and also talk about partial efficiencies, where you put up there-- you see, F is the fuel. It's the hydrocarbon that you're using, and R is the solar energy.

So again, once you have these betas, you can think that your hybrid facility has two lines. One is the one that produces electricity starting from the fuel and the other that produces other electricity starting from solar, and you want to do this splitting.

So you can define the primary energy savings with respect to separate production. So separate production will be, I only have one solar facility and one fossil fuel facility, separate. Apparently, it is better if you combine them. You get higher efficiencies. I don't have experience, so I don't have the numbers of if it is of the same order, maybe 40% is too much. But for sure, there is some improvement in the overall efficiency.

Also here, again, at the beginning and perhaps even still now, because I don't know if this problem has evolved enough, there is one way, that we call an incremental fossil-centered, where you give priority to the renewable. And so you say, all right, I use that fuel. With that fuel, I usually produce this much electricity. Fine, so the extra electricity is attributed to the solar.

But that's unfair because it doesn't distribute evenly the primary energy savings. So again, you can use a separate production method like we did for the other case. And again, you could use exergy. Here are only one point I need to explain, is that, what's the exergy of solar energy? That's a chapter in itself.

But you could argue that there are some papers, I can give you references to those papers. We also contributed some. A good number is this. Of course, there are formulas that get you to this one, meaning that energy coming from the Sun comes from a very high temperature source, the order of these many K, Kelvin. So the photons that come from the Sun carry not only energy, but also entropy.

But then there is a problem that as it gets to the atmosphere, and you get to the surface of the Earth, you get some direct lights and some rays and some indirect through refraction, and yeah, so from the sky, the sky has a certain temperature. So there are lots of reasoning that go behind a number like this. And so, again, this is a chapter in itself.

But in any case, the problem here is that with the exergy method, you're using the reversible heat engine as a reference. And that is not feasible.

Actually, I see that the time is over, so maybe we'll return just with few words at the beginning of next lecture about this because there's still something I want to say that I didn't. And it is important also. So hold on, don't do the homework yet, OK. All right. So I'll see you next time. See you Tuesday.