Scores out of 4 possible points,

6.780 SEMICOND. MANUF. | Problem Set 6 Solutions | Spring 2003 D. Boning

Problem 1 (1/2 pt)

We have a circuit with $N = 100{,}000 = 10^5$ transistors.
The area of the gate for each is $A_g = 10\ \mu m \cdot 1\ \mu m$

$$= 10\,(10^{-4}\,cm)\cdot(10^{-4}\,cm)$$
$$= 10^{-7}\ cm^2$$

To achieve a yield of 95%, we need, assuming a Poisson model:

$$0.95 = e^{-N A_g D_0}$$

$$\ln 0.95 = -N A_g D_0$$

or $$D_0 = \frac{-\ln 0.95}{N \cdot A_g} = \frac{0.0513}{10^5 \cdot 10^{-7} cm^2} = 5.13\ \text{defects}/cm^2$$

Problem 2 (1/2 pt)

In this case we have a chip with area (all of which is assumed to be critical) of $A = 0.5\ cm^2$. We need to produce chips through three sequential process areas, followed by other steps which have their own yield, call it $Y_{assembly} = 0.95$

Define total yield Y as $Y = Y_1 \cdot Y_2 \cdot Y_3 \cdot Y_{assembly}$, where

$$Y_i = \left(1 + \frac{A_c D_{0i}}{\alpha}\right)^{-\alpha}, \text{ where } D_{0_1} = 0.9\ cm^{-2},\ D_{0_2} = 1.1\ cm^{-2},\ D_{0_3} = 1.3\ cm^{-3}$$

Or

$$Y_1 = \left(1 + \frac{0.5\,(0.9)}{2}\right)^{-2} = 0.666$$

$$Y_2 = \left(1 + \frac{0.5\,(1.1)}{2}\right)^{-2} = 0.615$$

$$Y_3 = \left(1 + \frac{0.5\,(1.3)}{2}\right)^{-2} = 0.57$$

So $Y = (0.666)(0.615)(0.57)(0.95) = 0.222$. Thus, to get 10,000 functional units, we need to "build"

$$\frac{10^4}{0.222} \text{ die, where we have 200 die/wafer,}$$

$\therefore$ # wafers to start is $\frac{10^4}{200(0.222)} = 225.2$ wafers.

- We could be a little more careful with this to differentiate between WAFER yield and FUNCTIONAL yield, as probably intended in this problem.

$Y_W = 0.95$ are the fraction of WAFERS that survive to the end of line.

$Y_F = Y_1 \cdot Y_2 \cdot Y_3$ is the die functional yield.

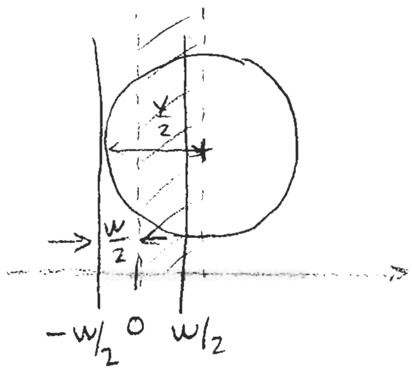
Then for 10,000 parts, we need to fabricate $10^4/Y_F$ or 42,833 parts. To get this we need $\frac{1}{200}$ of these in functional wafers or 214.16 or 215 wafers. But only 95% of start wafers make it, so we need to start $\frac{215}{0.95} = 226.3$ or $\underline{\underline{227}}$ wafers.

This is virtually identical to the earlier calculation, but more clearly differentiates the yield loss mechanisms.

Problem 3

This problem is confusing because the terminology used in May & Spanos, my problem statement wording, and the Stapper paper are mutually inconsistent. To make things clearer, let's consider three cases.

Case 1: Simple "line open" condition – we only fail when the entire line is interrupted.



- when $x < W$, $A_c = 0$ ... we never lose entire line
- when $x > W$, the furthest away from our zero origin (at center of line) that the defect can be is pictured

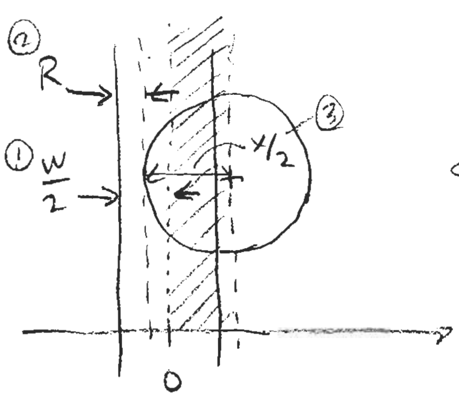
- So A_c to the "right" of origin, denoted $A_{c\,right}$, is the shaded area above, or

$$A_{c\,right} = \left(\frac{x}{2} - \frac{W}{2}\right) L$$

So $A_c = A_{c\,right} \cdot 2 = (x - W) L$

Case 2: R is Min Remaining Thickness

Now we'll (carefully) define R as the minimum width of the line remaining. This is a common situation as a reliability constraint – a minimum wire width is needed to ensure it doesn't fail in the field due to electromigration, for instance.



$$A_{c\,right} = \left(\overset{①}{-\frac{W}{2}} + \overset{②}{R} + \overset{③}{\frac{x}{2}}\right) L$$

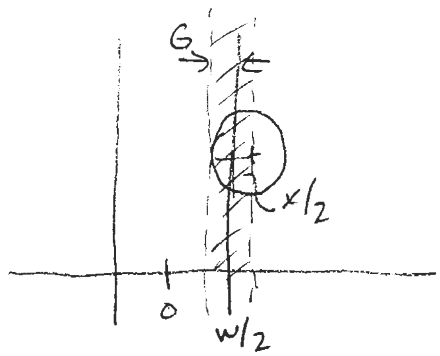
So

$$A_c = (x - W + 2R) L \quad \text{when } x > W$$

NOTE: THIS IS DIFFERENT THAN IN MAY/SPANOS and STAPPER

case 3: G IS MAX GAP IN LINE

Finally, suppose we instead have a constraint on an allowable gap (amount of line that can be "lost" without failure). Now the picture is different



Now $A_c(\text{right}) = \left(\frac{w}{2} - G + \frac{x}{2}\right) L$

So

$$A_c = (x + w - 2G) L \quad \text{for} \quad x > G$$

NOTE: This formula is consistent with the stapper formula, although does not match the stapper terminology.

For the rest of this problem we could proceed with either case 2 or case 3 interpretation. I will use case 3 in the following, but use G instead of R to emphasize this point.

Part a Evaluate $A_{AV} = \int_{X_L}^{X_U} A_c(x)\, g(x)\, dx$

Here I will assume that $X_L > G$, with $g(x) = \frac{X_U^2 X_L^2}{x^3 (X_U^2 - X_L^2)} = \frac{K}{x^3}$

So
$$A_{AV} = \int_{X_L}^{X_U} L(x + w - 2G) \cdot \frac{K}{x^3}\, dx$$

$$= \int_{X_L}^{X_U} L(w-2G)K \cdot \frac{1}{x^3}\, dx + \int L \cdot K \frac{1}{x^2}\, dx$$

$$= L(w-2G)K\left(\frac{-1}{2}\right)\frac{1}{x^2}\Bigg]_{X_L}^{X_U} + L \cdot K (-1)\frac{1}{x}\Bigg]_{X_L}^{X_U}$$

$$= \frac{L(w-2G)K}{2}\left(\frac{1}{X_L^2} - \frac{1}{X_U^2}\right) + LK\left(\frac{1}{X_L} - \frac{1}{X_U}\right)$$

$$= \frac{L(w-2G)K}{2}\left(\frac{X_U^2 - X_L^2}{X_L^2 \cdot X_U^2}\right) + LK\left(\frac{X_U - X_L}{X_L \cdot X_U}\right)$$

And plugging back in for $K = \frac{x_U^2 x_L^2}{x_U^2 - x_L^2}$, which cancels constants:

$$A_{AV} = \frac{L(W-2G)}{2} + L \cdot \frac{x_U^{\cancel{2}} \cdot x_L^{\cancel{2}}}{\underbrace{x_U^2 - x_L^2}_{\cancel{(x_U - x_L)}(x_U + x_L)}} \cdot \frac{\cancel{(x_U - x_L)}}{\cancel{x_L \cdot x_U}}$$

$$\boxed{A_{AV} = L\left\{\frac{W}{2} - G + \frac{x_U \cdot x_L}{x_U + x_L}\right\}}$$

Part b Limit as $x_U \to \infty$ of A_{AV}:

$$A_{AV} \to \frac{L(W-2G)}{2} + L \cdot x_L = \boxed{L\left(x_L + \frac{W}{2} - G\right)}$$

As $G \to 0$, the A_{AV} increases, which makes sense since we allow ourselves to lose less and less of of the line,

The expression in May and Spanos diverges as $R \to 0$, which is not consistent with expectations.

Under CASE 2: An alternative derivation, under the R = MIN REMAINING THICKNESS, follows

Part a $$A_{AV} = \int_{x_L}^{x_U} L(x - W + 2R)\frac{K}{x^3}\,dx$$

where we will assume $x_L < W - R$, so

$$A_{AV} = -LK \cdot \frac{1}{x}\Big]_{x_L}^{x_U} + L(-W+2R)K\left(-\frac{1}{2}\right)\frac{1}{x^2}\Big]_{x_L}^{x_U}$$

$$= L \cdot K\left(\frac{1}{x_L} - \frac{1}{x_U}\right) + L(-W+2R)\cancel{\frac{K}{2}}\left(\cancel{\frac{1}{x_L^2} - \frac{1}{x_U^2}}\right)$$

$$= L \cdot \frac{\cancel{x_U^2 x_L^2}}{(\cancel{x_U - x_L})(x_U + x_L)}\left(\frac{\cancel{x_U - x_L}}{\cancel{x_L \cdot x_U}}\right) + L\left(\frac{-W+2R}{2}\right)$$

$$\boxed{A_{AV} = L\left\{\frac{x_U x_L}{x_U + x_L} + \left(R - \frac{W}{2}\right)\right\}}$$

- Close to the "G" interpretation...

Part b Now in the limit as $x_U \to \infty$

$$\boxed{A_{AV} \to L\left\{x_L + R - \frac{W}{2}\right\}}$$

Now, as $R \to 0$ this means we can survive as long as there is ANY remaining line... and A_{AV} decreases as we let the remaining thickness requirement shrink.

Also, for $R \to W$ (that is, we want the ~~entire~~ entire line to be present, so any defect encroaching on line causes failure), we have $A_{AV} \to L\left\{x_L + \frac{W}{2}\right\}$, which is the same limit as earlier for $G \to 0$.

Indeed, we can recognize that $R = W - G$, so

$$A_{AV} = L\left(x_L + R - \frac{W}{2}\right) = L\left(x_L + \frac{W}{2} - G\right)$$

and the two interpretations are now consistent.

$\boxed{\text{Problem 4}}$

We have 200 mm wafers, but dice are only good in center 190 mm. Chips are 5mm x 5 mm in size, with yield of 80%.

Part a

Assuming a Poisson model, the yield is

$$Y = Y_o e^{-A_c D_o}$$ where Y_o is the "gross yield loss" from the bad outer ring

or

$$\frac{\ln Y/Y_o}{A_c} = -D_o \implies D_o = \frac{1}{A_c} \ln \frac{Y_o}{Y}$$

- So here $A_c = (5 \times 10^{-1}\text{ cm})(5 \times 10^{-1}\text{ cm})$
  $= 25 \times 10^{-2}\text{ cm}^2 = 0.25\text{ cm}^2$

Y_o is fraction of area in center 190 mm $= \dfrac{\pi (190/2)^2}{\pi (200/2)^2} = \left(\dfrac{19}{20}\right)^2$

$$Y_o = 0.903$$

- Thus, if we observe a total yield $Y = 0.8$, we must have a defect density:

$$D_o = \frac{1}{0.25\text{ cm}^2} \ln\left(\frac{0.903}{0.8}\right) = 0.482\text{ defects/cm}^2$$

- Assuming that all die "fit" in the good area without any loss from fractional die, we can calculate the total yield as a function of $S = \sqrt{A_c}$ where A_c is the die area.

$$\text{TOTAL DIE} = \frac{\pi\left(\frac{19\text{ cm}}{2}\right)^2}{S^2} = \frac{283.5}{S^2}, \quad S \text{ in cm}$$

$$\text{GOOD DIE} = \text{TOTAL DIE} \cdot Y$$

$$Y = 0.903\, e^{-S^2 \cdot 0.482}$$

A plot of these is shown on pg 9.

Part b Now use a negative binomial yield model, with $\alpha = 1.5$.

Now $Y = Y_0 \left(1 + \frac{A_c \cdot D_0}{\alpha}\right)^{-\alpha}$

So $\frac{Y_0}{Y} = \left(1 + \frac{A_c \cdot D_0}{\alpha}\right)^{\alpha}$

$$D_0 = \frac{\alpha}{A_c}\left\{\left(\frac{Y_0}{Y}\right)^{1/\alpha} - 1\right\} = \frac{1.5}{0.25\ cm^2}\left\{\left(\frac{0.908}{0.8}\right)^{1/1.5} - 1\right\} =$$

$\doteq 0.529$ defects/cm^2 // (slightly larger than in part a)

• Now our GOOD DIE as function of S goes as

$$\text{GOOD DIE} = \text{TOTAL DIE} \cdot Y$$
$$= \text{TOTAL DIE} \cdot 0.903\left(1 + \frac{S^2 \cdot 0.529}{1.5}\right)^{-1.5}, \quad S \text{ in cm}$$

~ These are also plotted on page 9.

Part c Under a "black-white" model, then

$$Y = Y_0 \cdot f = (\text{fraction in 190 mm}) \cdot (\text{fraction good in rest of wafer})$$

$0.80 = 0.903 \cdot f \Rightarrow f = 0.886$ or about 88.6 of the center 190 mm of wafer is "good" to yield total good fraction of 0.8

GOOD DIE = TOTAL DIE $\cdot f$

~ This model is also plotted on pg. 9.

Part d What D_0 needed to achieve over yield of 50% for die with $S = 15$ mm?

(a) Poisson: $Y = Y_0 e^{-A_c \cdot D_0}$, $A_c = (1.5\ cm)^2$

$$\ln \frac{Y_0}{Y} = A_c \cdot D_0$$

$$D_0 = \frac{1}{A_c} \ln\left(\frac{Y_0}{Y}\right) = \frac{1}{(1.5\,cm)^2} \ln\left(\frac{0.903}{0.5}\right)$$

Poisson: $D_0 = 0.263\ cm^{-2}$ //

(b) Negative Binomial:

$$D_0 = \frac{\alpha}{A_c}\left\{\left(\frac{Y_0}{Y}\right)^{1/\alpha} - 1\right\} = \frac{1.5}{(1.5\,cm)^2}\left\{\left(\frac{0.908}{0.5}\right)^{1/1.5} - 1\right\}$$

Neg. Bin: $D_0 = 0.326\ cm^{-2}$ //

(c) Black-white

$$f = \frac{Y}{Y_0} = \frac{0.5}{0.903} = 0.554 \ //$$

≥ Actually a "worsening" since this model has no defect density notion.

Problem 5

We have two critical layers

metal 3 : $D_0 = 1\ cm^{-2}$, $A_{MEM} = 3\ mm^2$, $A_{LOG} = 5\ mm^2$
via 3 : $\lambda = 10^{-7}$, $N_{MEM} = 2 \times 10^6$, $N_{LOG} = 10^5$

Chip area $A = 2\ mm \times 4\ mm$, at \$10/chip.

Part a Yield impact matrix

	metal 3	via 3
Memory Block	$e^{-A_{MEM} D_0}$	$e^{-\lambda N_{MEM}}$
Logic Block	$e^{-A_{LOG} D_0}$	$e^{-\lambda N_{LOG}}$

Plugging in, we get the following. The total yield loss in the memory and logic are at far right; the total loss from the metal3 and via3 layers is at bottom and the total product yield is at bottom right in the table below.

Note that $e^{-\lambda N_{MEM}} \cong (1-\lambda)^{N_{MEM}}$ when λ is very small and N_{MEM} is very large. The binomial expression, however, takes a value very near to 1 to a high power, which can have numerical precision problems. This is the whole reason for the Poisson distribution/approximation.

	metal3	via 3	
Memory Block	0.970	0.819	0.794
Logic Block	0.951	0.990	0.941
	0.922	0.811	0.748

Clearly, the memory block is giving us the majority of yield loss, and most of that is in the via3 process. Next we consider a way to address this problem.

Part b

- Adding a border improves via yield, but takes more critical area and thus decreases the metal yield. Each 0.1 μm added decreases λ by 10x. Denote border as b (in units of 0.1 μm); then the new via failure rate is

$$\lambda^* = \frac{\lambda}{10^b}$$

- At the same time, the critical area increases by 10% for each $b * 0.1\mu m$: $A^* = A(1.1)^b$

- The total memory yield is

$$Y_{MEM} = \{ e^{-A^*_{MEM} \cdot D_0} \}\{ e^{-\lambda^* N_{MEM}} \} = e^{-A^*_{MEM} D_0 - \lambda^* N_{MEM}}$$

$$-\ln Y_{MEM} = A^*_{MEM} \cdot D_0 + \lambda^* N_{MEM}$$

$$C = -\ln Y_{MEM} = A_{MEM} \cdot D_0 (1.1)^b + \lambda N_{MEM} 10^{-b}$$

$$= (0.03)(1.1)^b + (0.2) 10^{-b}$$

The yield is maximized when C is minimized with respect to b

Recall that $\frac{d}{dx} a^{u(x)} = a^{u(x)} \ln a \frac{du(x)}{dx}$

So

$$\frac{dc}{db} = 0.03 (1.1)^b \ln 1.1 + (0.2) 10^{-b} \ln 10 (-1) = 0$$

So $(0.03) \ln 1.1 (1.1)^b = (0.2) \ln 10 (10)^{-b}$

$$11^b = \frac{(0.2) \ln 10}{(0.03) \ln 1.1} = 161.06$$

$$b = \ln 161.06 / \ln 11 = 2.119$$

So the total border to maximize memory block yield is $b\,(0.1\mu m) = 0.21\ \mu m$.

Part c

Total die size also increases 10% for each 0.1 μm of border added. Compared to the existing design, which earns more money per wafer?

- Previous design $Y_{TOTAL} = 0.749$, die size = 2 mm x 4 mm. So the good die per wafer is

$$M = \frac{\pi (100\,mm)^2}{8\,mm^2} \cdot Y_{TOT} = 3{,}927 \cdot 0.749 = 2{,}941 \text{ good die}$$

at $10/die, gives $29,410 per wafer.

- With new design having border of 0.21 μm, we have new yield of

$$Y_{TOT}^* = Y_{mem}^* \cdot Y_{LOGIC} = Y_{MEM}^* \cdot 0.9418$$

$$= e^{-0.03(1.1)^{2.1}} e^{-0.2(10)^{-2.1}} \cdot 0.9418$$

$$= 0.964 \cdot 0.998 \cdot 0.9418 = 0.906$$

But area of chip has increased:

$$M^* = \frac{\pi (100\,mm)^2}{8\,mm^2 (1.1)^{2.1}} \cdot Y_{TOT}^* =$$

$$= 3{,}214.66 \cdot 0.906 = 2{,}912 \text{ good die}$$

at $10/die, gives $29,120

So, although the yield goes up, the extra die size means we earn $290 less per wafer.

Note: this analysis assumes that the borders are not applied to logic vias, & critical area for logic (and thus yield for logic) does not change.